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列車の走行によって引き起こされるトンネル内の音場

大阪大 基礎工 杉 本 信 正 (Nobumasa SUGIMOTO)

ABSTRACT

This paper deals with a sound field in a tunnel generated by traveling of a high-speed train. A magnitude of pressure disturbances is estimated by using the linear acoustic theory. The train's motion is taken into account through a source term of the wave equation in the form of a pair of acoustic monopoles or an acoustic dipole. One-dimensional analysis is first given for two cases. One is the sound field generated by the impulsive motion of the train in an infinitely long tunnel and the other is the one by the entry motion into a semi-infinite tunnel. The magnitude of the pressure pulse radiated is derived on assuming the source strength averaged over the tunnel's cross-section. To justify this assumption, the three-dimensional analysis is developed for the impulsive motion of the dipole along the axis of the tunnel of circular cross-section. The sound field is solved in a closed form and this solution endorses the results based on the one-dimensional analysis.

1. Introduction

When a high-speed train travels inside of a long tunnel, it happens that pressure disturbances generated give rise to an acoustic shock wave even if the train speed is well subsonic. As this shock wave is radiated from a tunnel exit, it brings about an environmental noise problem just like the one due to a sonic boom by supersonic flight. This problem becomes severer with increase in train speed so it will be vitally important for magnetically levitated trains whose project is now under way. The radiation of the shock wave is not the only problem. Emergence of the shock wave itself is undesirable, of course, for durability and performance of trains as well as tunnels. To inhibit the shock wave, the author has proposed to connect an array of Helmholtz resonators, more generally, an array of cavities along the tunnel as side branches and has demonstrated its effectiveness by the numerical simulations (Sugimoto, 1992, 1993).

This paper considers, on returning to the starting point of the problem, a sound field in a tunnel generated by traveling of a train to estimate a maximum pressure level. Because the train speed is well subsonic, evolution of pressure disturbances may be treated in a framework of the acoustic theory. It is assumed to consist of the following three stages. The first stage is an inner flow field rather than a sound field around the train in which pressure disturbances are generated originally. The next is a near sound field in which the pressure disturbances from the flow field are radiated in the form of sound but the linear acoustic theory is still applicable to the lowest approximation. This is the region with which the present paper is concerned. The final stage is a far sound field in which the shock wave emerges. Evolution in this field cannot be described appropriately without invoking the nonlinear theory. The results in the near field are to be employed in prescribing the pressure disturbances for further evolution in the far field.

2. Modeling of the problem

Before embarking on analysis, we discuss how the problem is modeled. First of all, we note that a train is such a slender and stremlined body that it has an extremely long axial dimension relative to a lateral one. In fact, the train stretches axially from about 20 to a few hundred meters against the lateral dimension, say, 3 m. Here we define a 'long train' and a 'short train' according as the train's axial length is much longer than the tunnel's typical diameter or comparable with it. Similarly, the tunnel has also an extreme ratio of axial length to diameter. The following analysis assumes a long tunnel so no effects of reflection at the tunnel exit are taken into account.

When the train travels in the tunnel, it gives rise to pressure disturbances in the vicinity of the train. Because of the extremely slender geometry, twofold length-scales are associated with the flow field, i.e., the train's length l and the tunnel's diameter D. Here the train's lateral dimension s is small relatively to D but is regarded as being comparable with it. With the train speed U as a typical one, two time-scales are therefore involved, l/U and D/U. Because the train speed is slower than the sound speed a (for example, U is assumed to be 150 m/s (540 km/h) for the magnetically levitated trains so that the train's Mach number M $(= U/a)$ is 0.44), the flow field is weakly compressible so that its effect may still be neglected to the lowest approximation with respect to the Mach number. In fact, the magnitude of pressure fluctuations is small relative to the atmospheric one. Such a region is identified as the inner flow field.

With distances away from the train, the pressure disturbances are radiated in the form of sound. As its typical wavelength is of course the train's length, a typical time-scale is determined by the sound speed to be l/a. With l ranging from 20 to a few hundred meters long, this time is long enough as sound that the pressure disturbances fall under the category of so-called infra-sound. In fact, their frequency spectra observed for the present high-speed train (Shinkansen) are below 10 Hz. This near sound field is characterized as follows. The tunnel plays a role of a waveguide so that the sound can be propagated without any geometrical spreading. The waveguide allows the sould field to be represented by a superposition of the lowest non-dispersive mode and higher dispersive modes. Since the group velocity of the higher mode is slower than the sound speed, the fastest disturbances are propagated in the lowest mode so that the wavefront becomes plane toward the far field.

For this near field, the effect of the inner flow field is taken into account through a source term of the linear wave equation as follows (Crighton *et al.*, 1992):

$$\frac{\partial^2 p}{\partial t^2} - a^2 \Delta p = a^2 \frac{\partial q}{\partial t}, \tag{1}$$

where p and q denote, respectively, the excess pressure over the atmospheric pressure p_0 and the source term reflecting the near flow field; Δ stands for the three-dimensional Laplacian, t being the time. The train's motion creates the mass outflow in front and the inflow of the same magnitude at rear. At the same time, the train's motion exerts force to the surrounding fluid, i.e., air. Since the train is not acoustically compact axially but compact laterally, its motion may be modeled by distribution of point sources along a finite axial extent. But a long train having a small lateral dimension may be modeled substantially by a pair of the acoustic monopoles of strength $\rho_0 SU$ but with opposite sign where ρ_0 is the density of the fluid in equilibrium and S is the train's cross-sectional area. In unbounded space, the pair of the moving monopoles defines the slender Rankine ovoid provided the distance between the monopoles is long enough, so that an effect of the tunnel wall becomes weaker as s becomes smaller.

As opposed to the long train, the short train may be modeled by an acoustic dipole. The dipole strength is given by the sum of the momentum $\rho_0 SUl$ as if the fluid displaced by the train were moving with U and the impulse imparted to the fluid (Lighthill, 1986). The impulse is also the momentum associated with the virtual mass of the train. For the Rankine ovoid, the virtual mass in the axial direction is given by $2\pi\rho_0 s^3/3$, where s is now taken to be the radius of the cylindrical part in the middle of the ovoid. But this mass is negligibly small compared with that of the displaced fluid $\pi\rho_0 s^2 l$. Even for a short train with $l = 20$ m and $s = 1.5$ m, the fraction of the virtual mass in the dipole strength is as small as 5%. Thus the impulse may be neglected. Even if the dipole is assumed, it does not necessarily mean a sphere, for which the dipole strength is given by $3\rho_0 V/2$ $(= 2\pi\rho_0 s^3)$ where V is the volume of the sphere of radius s.

In the following, the one-dimensinal version of Eq.(1) is first solved by taking, as the source term, a pair of monopoles with opposite sign and then a dipole. It is assumed that the source strength can be averaged over the tunnel's cross-section so that the monopole strength and the dipole strength are taken to be $\rho_0\chi U$ and $\rho_0\chi Ul$, respectively, where χ denotes the ratio of the train's cross-sectional area to the tunnel's one. We then solve the sound field for two cases, one in which the train moves impulsively with constant speed in an infinitely long tunnel and the other in which the train rushes into a semi-infinitely long tunnel. The maximum pressure level in each case is derived. To justify the averaging, a three-dimensional sound field is solved by taking the case of the impulsive motion of the dipole.

3. One-dimensional analysis

Let us first examine a sound field due to the impulsive motion of a one-dimensional monopole in an infinitely long tunnel. The sound field due to a pair of the monopoles can easily be constructed by the principle of superposition. The pressure field is governed by the following equation:

$$\frac{\partial^2 p}{\partial t^2} - a^2 \frac{\partial^2 p}{\partial x^2} = a^2 \frac{\partial}{\partial t}\left[mh(t)\delta(x - Ut)\right], \tag{2}$$

where m designates the monopole strength $\rho_0 \chi U$ and $h(t)$ and $\delta(x - Ut)$ denote, respectively, the unit step function and the delta function, x being the axial coordinate along the tunnel. Executing the differentiation in the source term, it consists of two parts, $m\delta(t)\delta(x - Ut)$ and $-mUh(t)\delta'(x - Ut)$ where the prime implies the differentiation with respect to the argument. The former represents the impulsive effect while the latter represents the steadily moving effect.

Upon introducing new coordinates ξ $(= x - Ut)$ and τ $(= t)$, Eq.(2) can be solved by using the Fourier transform with respect to ξ and the Laplace transform with respect to τ defined, respectively, as follows:

$$\hat{p} = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} p\,(\xi, \tau)\,\mathrm{e}^{ik\xi}\,\mathrm{d}\xi \quad \text{and} \quad \tilde{p} = \int_0^{\infty} p\,(\xi, \tau)\,\mathrm{e}^{-\sigma\tau}\,\mathrm{d}\tau. \tag{3}$$

Then Eq.(2) is transformed into

$$\tilde{\hat{p}} = \frac{ma^2}{(2\pi)^{1/2}} \frac{(\sigma + iUk)}{\sigma[(\sigma + iUk)^2 + a^2k^2]}. \tag{4}$$

Effecting the respective inverse transforms and reverting to the original variables x and t, p, designated as p_{+m}, is obtained as

$$p_{+m} = \frac{ma}{2}\left[-\frac{1}{1-M}h(x - at) + \frac{1}{1+M}h(x + at) + \frac{2M}{1-M^2}\,h(x - Ut)\right]h(t). \tag{5}$$

The pressure field due to the positive and negative monopoles positioned by the axial distance l apart can be derived by adding to (5), p_{-m} with x replaced by $x+l$. The resulting pressure field is shown in Fig.1. The positive square pulse of magnitude $ma/[2(1 - M)]$ is propagated into the positive direction of x with the sound speed, while the negative pulse of magnitude $-ma/[2(1 + M)]$ is propagated into the negative direction. Also the negative pulse of magnitude $maM/(1 - M^2)$ is moving with the train. With $m = \rho_0 \chi U$, the magnitude of the positive pressure pulse Δp_+ is given relative to the atmospheric pressure p_0 as follows:

$$\frac{\Delta p_+}{p_0} = \frac{ma}{2(1 - M)p_0} = \frac{\gamma}{2}\frac{\chi M}{(1 - M)}, \tag{6}$$

where γ is the ratio of the specific heats and $a = (\gamma p_0/\rho_0)^{1/2}$. The magnitude of the

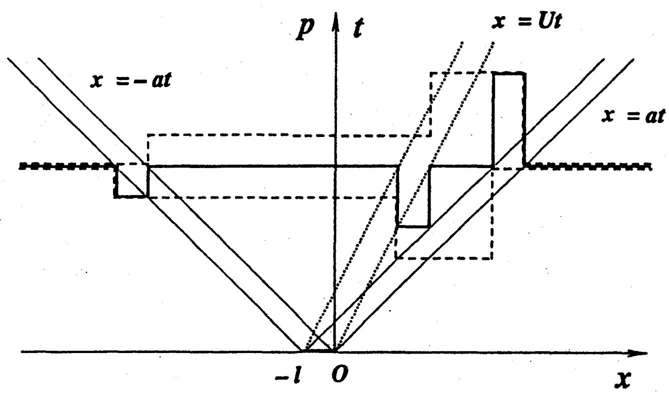

Figure 1: Pressure field due to the impulsive motion of the pair of the monopoles in the infinitely long tunnel where the bold solid line shows the total pressure profile while the broken lines show the pressure profiles due to each monopole located at $x = 0$ and $x = -l$ at $t = 0$, respectively, and the dotted lines are the paths of the train's head and tail.

negative pulse propagating into the negative direction is provided by (6) with the Doppler factor $1-M$ replaced by $-(1+M)$. Also the magnitude of the negative pulse Δp_0 moving with the train is given by

$$\frac{\Delta p_0}{p_0} = -\frac{\gamma\chi M^2}{1-M^2}. \tag{7}$$

If the limit $l \to 0$ is taken with ml ($= \mu$) fixed, the pressure field due to the impulsive motion of the dipole can be derived. With the source terms $q = -\mu h(t)\delta'(x - Ut)$, the solution is easily obtainable by differentiating (5) with respect to x and changing the sign as follows:

$$p = \frac{\mu a}{2}\left[\frac{1}{1-M}\delta(x-at) - \frac{1}{1+M}\delta(x+at) - \frac{2M}{1-M^2}\delta(x-Ut)\right] h(t). \tag{8}$$

The pressure field is given just as in Fig.1 with each square pulse replaced by the delta function of the corresponding strength. Once the solution due to the dipole is known, it can be extended to the case with the pair of monopoles. Then the delta function may be regarded not only as the limit of the square pulse but also as that of various sequences of appropriate functions such as the following Gaussian function:

$$\delta(x) = \lim_{l\to 0}\frac{1}{l}\ \exp\left[-\left(\frac{x}{l/\sqrt{\pi}}\right)^2\right]. \tag{9}$$

Next we consider the sound field due to the entry of the train into the semi-infinite tunnel extending in $x \geq 0$. In this case as well, we first consider the case of the monopole moving with constant speed U. Unlike the preceding case, the monopole strength does not change with time so the source term q is taken as $m\delta(x - Ut)$. Rather the boundary condition $p = 0$ is imposed at $x = 0$. Then the solution, denoted as p_{+m} here as well, is

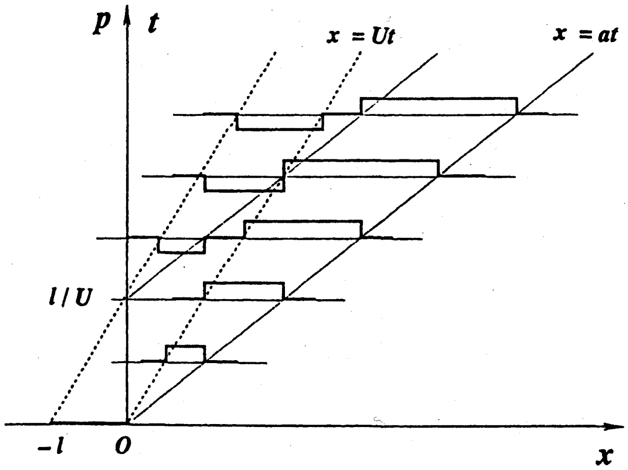

Figure 2: Pressure field due to the entry motion of the pair of the monopoles into the semi-infinitely long tunnel in $x \geq 0$ where the bold solid line shows the total pressure profile and the dotted lines are the paths of the train's head and tail.

obtained as

$$p_{+m} = \frac{mU}{1-M^2}\left\{-h[M(x-at)] + h(x-Ut)\right\}. \tag{10}$$

Combining p_{+m} and p_{-m} due to the negative one with t shifted to $t - l/U$ this time, the sound field is shown in Fig.2. The positive square pulse of width al/U, lengthened eventually by the factor a/U $(= 1/M)$ due to a delay in the entry time of the negative source, is propagated with the sound speed and the negative pulse of the same magnitude but of width l is moving with the train. The magnitude of the positive pulse Δp_+ relative to p_0 is given by

$$\frac{\Delta p_+}{p_0} = \frac{mU}{(1-M^2)p_0} = \frac{\gamma\chi M^2}{1-M^2}. \tag{11}$$

If the limit $l \to 0$ is taken again with ml $(= \mu)$ fixed for the dipole, p is given by

$$p = \frac{\mu U}{1-M^2}\left\{\delta\left[M(x-at)\right] - \delta(x-Ut)\right\}. \tag{12}$$

Here the argument of the first delta function implies that the pulse width is lengthened. Because $\delta[M(x-at)] = M^{-1}\delta(x-at)$, its integral is increased by the factor M^{-1}.

4. Three-dimensional analysis

The one-dimensional analysis rests crucially on the assumption that the source strength can be averaged over the tunnel's cross-section. In order to justify this assumption, we now solve a three-dimensional sound field produced by the impulsive motion of the dipole along the axis of the infinitely long tunnel of circular cross-section. With the radial coordinate r, Eq.(1) becomes on assuming the axisymmetry and taking $\mu = \rho_0 SUl$

$$\frac{\partial^2 p}{\partial t^2} - a^2\left(\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial x^2}\right) = -a^2\frac{\partial}{\partial t}\left[\mu h(t)\delta(r)\frac{\partial}{\partial x}\delta(x - Ut)\right], \tag{13}$$

with the boundary condition on the tunnel wall at $r = R$:

$$\frac{\partial p}{\partial r} = 0 \quad \text{on} \quad r = R. \tag{14}$$

Introducing ξ and τ defined as before and effecting the Laplace and the Fourier transforms, it follows that

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r} - \beta^2\right)\tilde{\bar{p}} = -\frac{\mu}{(2\pi)^{1/2}}\frac{(\sigma + iUk)ik}{\sigma}\,\delta(r), \tag{15}$$

with $\beta = [(\sigma + iUk)^2 + a^2k^2]^{1/2}/a$. For $0 < r < R$, $\tilde{\bar{p}}$ can easily be solved as

$$\tilde{\bar{p}} = c_1 I_0(\beta r) + c_2 K_0(\beta r), \tag{16}$$

with unknown constants c_1 and c_2 where I_n and K_n (with $n = 0$) stand, respectively, for the modified n-th order Bessel function of the first and the second kind. As will be used later, in passing, J_n denotes, of course, the n-th order Bessel function of the first kind. Imposing the boundary condition (14), we derive $c_1 = c_2 K_1(\beta R)/I_1(\beta R)$. The inhomogeneous equation (15) is satisfied by integrating it over a small circular disk centered at $r = 0$ and making use of the divergence theorem applied to the disk of unit axial thickness. Noting the asymptotic form of $K_0(\beta r) \sim -\log r$ as $r \to 0$, c_2 is found to be $(2\pi)^{-3/2}\mu(\sigma + iUk)ik/\sigma$. With c_1 and c_2 thus determined, p is obtained by evaluating the following inverse transforms:

$$p = \frac{\mu}{4\pi^2}\int_{-\infty}^{\infty} \mathrm{d}k\ \mathrm{e}^{-ik\xi}\left\{\frac{1}{2\pi i}\int_{Br}\frac{(\sigma + iUk)ik}{\sigma}\left[\frac{K_1(\beta R)}{I_1(\beta R)}I_0(\beta r) + K_0(\beta r)\right]\mathrm{e}^{\sigma\tau}\mathrm{d}\sigma\right\}, \tag{17}$$

where Br stands for the Bromwich path.

The inverse Laplace transform is facilitated by introducing auxiliary paths along circular arcs of infinite radius and invoking the Cauchy's integral theorem. Because the integrand has the branch points at $\beta = 0$, i.e., at $\sigma = i(a - U)k$ and $-i(a + U)k$, and $\beta = \infty$, the branch cuts are introduced in the σ-plane as shown in Fig.3 where the cuts extend from the branch points to infinity in parallel with the nagative axis. Defining β so as to take a positive value along the imaginary axis between the branch points, β takes asymptotically σ/a on the arcs C_0, C_1 and C_3 and takes $-\sigma/a$ on C_2. Examining an asymptotic form of the integrand over the infinite arcs (see e.g., Watson, 1958), it is found that for $0 < \tau < r/a$, the Bromwich path is closed by C_0 so that p vanishes. For $r/a < \tau$, on the other hand, the path is closed by the arcs on the left-hand half plane together with the detours along the cuts.

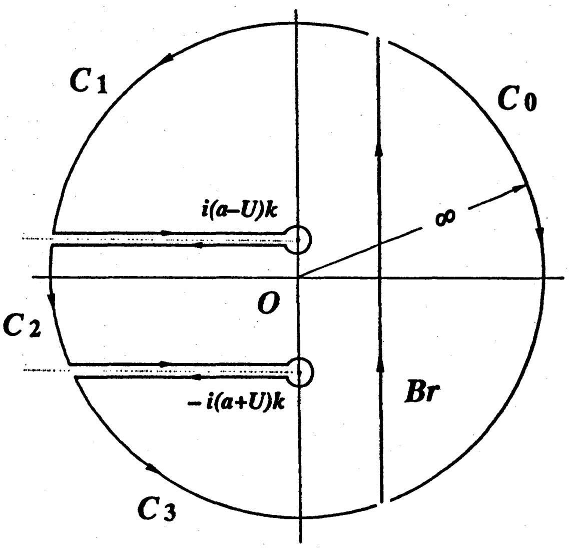

Figure 3: Complex σ-plane with the branch cuts extending from the branch points at $\sigma = i(a-U)k$ and $-i(a+U)k$ to infinity in parallel with the negative axis; C_0 is the semi-circle of infinite radius on the right-hand half plane while C_1, C_2 and C_3 are the circular arcs of infinite radius on the left-hand half plane.

Because the integral along the arcs vanish, we calculate the contributions from the poles located at $\sigma = 0$, $\beta = 0$ and the roots of the equation $I_1(\beta R)$ ($\beta \neq 0$). First, the contribution from the poles at $\beta = 0$, i.e., $\sigma = i(a-U)k$ and $-i(a+U)k$, is nothing but the one-dimensional solution (8) propagating with the sound speed and is given for $t > 0$ by

$$p_0 = \frac{\mu a}{2\pi R^2}\left[\frac{1}{1-M}\delta(x-at) - \frac{1}{1+M}\delta(x+at)\right]. \tag{18}$$

Next, the contribution from the pole at $\sigma = 0$ gives the steady pressure field p_1 moving with the train as

$$p_1 = -\frac{\mu U}{4\pi^2}\int_{-\infty}^{\infty} k^2 \left[\frac{K_1(\alpha R|k|)}{I_1(\alpha R|k|)} I_0(\alpha r|k|) + K_0(\alpha r|k|)\right] \mathrm{e}^{-ik\xi}\mathrm{d}k, \tag{19}$$

where $\alpha^2 = 1 - M^2$. Since the second term in the square bracket can analytically be integrated, (19) is also expressed as follows:

$$p_1 = \frac{\mu U}{4\pi\alpha}\frac{\partial^2}{\partial\xi^2}\left(r^2 + \frac{\xi^2}{\alpha^2}\right)^{-1/2} - \frac{\mu U}{2\pi^2(\alpha R)^3}\int_0^{\infty}\frac{z^2 K_1(z)}{I_1(z)} I_0\left(\frac{r}{R}z\right)\cos\left(\frac{\xi}{\alpha R}z\right)\mathrm{d}z. \tag{20}$$

The first term represents the steady dipole field moving with the subsonic speed in unbounded space, while the second term represents the effect of the tunnel wall. This integral can be evaluated with the aid of infinite circular arcs and the imaginary z axis with small circular detours around $z = \pm i\zeta_n$ where ζ_n ($n = 1, 2, 3, ...$) designate the positive roots of $J_1(\zeta) = 0$ numbered from the smallest one as ($0 < \zeta_1 < \zeta_2 < ...$). Then p_1 is given in terms of the infinite sum as follows:

$$p_1 = \frac{\mu U}{2\pi(\alpha R)^3}\sum_{n=1}^{\infty}\frac{\zeta_n}{J_0^2(\zeta_n)} J_0\left(\zeta_n\frac{r}{R}\right)\exp\left(-\zeta_n\frac{|\xi|}{\alpha R}\right). \tag{21}$$

The third contribution results from the roots of $I_1(\beta R) = 0$. The roots $\beta R = \pm i\zeta_n$ ($n = 1, 2, 3, \ldots$) correspond to $\sigma = \sigma_n^\pm = -iUk \pm i\omega_n$ with $\omega_n = a(k^2 + \zeta_n^2/R^2)^{1/2}$ where the sign $\pm$ is understood to be vertically ordered hereafter. Then the residue of each pole in the inverse Laplace transform is given by

$$\pm \frac{a^2}{R^2} \frac{(\sigma_n^\pm + iUk)k}{\sigma_n^\pm J_0^2(\zeta_n)\omega_n} J_0\left(\zeta_n \frac{r}{R}\right) \exp(\sigma_n^\pm \tau). \tag{22}$$

Noting that the factor $\sigma_n^\pm + iUk$ implies the differentiation with respect to t, the third contribution p_2 is evaluated as follows:

$$p_2 = \frac{\mu}{4\pi^2 R^2} \frac{\partial}{\partial t} \left[\sum_{n=1}^{\infty} \frac{1}{J_0^2(\zeta_n)} J_0\left(\zeta_n \frac{r}{R}\right) \Phi_n(\xi, \tau) \right], \tag{23}$$

with $\Phi_n = \Phi_n^+ - \Phi_n^-$ where $\Phi_n^\pm$ are defined by

$$\Phi_n^\pm = \int_{-\infty}^{\infty} \frac{a^2 k}{\sigma_n^\pm \omega_n} \exp(\sigma_n^\pm \tau - ik\xi) \mathrm{d}k. \tag{24}$$

Using the definitions of $\sigma_n^\pm$, Φ_n are rewritten in the following form:

$$\Phi_n = \int_{-\infty}^{\infty} \frac{2aMk^2}{\alpha^2 \kappa_n^2 \omega_n} \sin(\omega_n t) \exp(-ikx) \mathrm{d}k - \int_{-\infty}^{\infty} \frac{2ik}{\alpha^2 \kappa_n^2} \cos(\omega_n t) \exp(-ikx) \mathrm{d}k, \tag{25}$$

with $\kappa_n^2 = k^2 + \zeta_n^2/\alpha^2 R^2$. For $at < |x|$, these integrals can be evaluated analytically (Oberhettinger, 1957). It follows that

$$\Phi_n = -\frac{2\pi}{\alpha^2} (\mathrm{sgn}\ x) \exp\left[-\zeta_n \frac{(x - Ut)\mathrm{sgn}\ x}{\alpha R} \right], \tag{26}$$

so that

$$p_2 = -\frac{\mu U}{2\pi(\alpha R)^3} \sum_{n=1}^{\infty} \frac{\zeta_n}{J_0^2(\zeta_n)} J_0\left(\zeta_n \frac{r}{R}\right) \exp\left(-\zeta_n \frac{|\xi|}{\alpha R}\right). \tag{27}$$

For $|x| < at$, on the other hand, Φ_n is alternatively expressed as

$$\Phi_n = \frac{2\pi M}{\alpha^2} J_0(\eta) - \frac{\pi M \zeta_n}{\alpha^3 R} \int_{-at}^{at} J_0(\eta') \exp(-\zeta_n \frac{|x - x'|}{\alpha R}) \mathrm{d}x'$$

$$- \frac{\pi}{\alpha^2 a} \frac{\partial}{\partial t} \int_{-at}^{at} J_0(\eta') \mathrm{sgn}(x - x') \exp(-\zeta_n \frac{|x - x'|}{\alpha R}) \mathrm{d}x', \tag{28}$$

where η and η' are defined, respectively, as $\zeta_n(a^2t^2 - x^2)^{1/2}/R$ and $\zeta_n(a^2t^2 - x'^2)^{1/2}/R$.

Finally the contribution from the integral along the branch cuts vanishes because the jump across the cut cancels out as a whole of the square bracket in (17). Thus the integral (17) is evalutated to be the sum

$$p = p_0 + p_1 + p_2. \tag{29}$$

It is immediately seen that for $at < |x|$, p_1 and p_2 cancel out so that p vanishes, namely, an undisturbed region prevails. At the wavefront $|x| = at$, p is given by the one-dimensional solution p_0. As the factor $\mu/\pi R^2$ indicates $\rho_0 \chi U l$, it is justified that the strength can be averaged over the tunnel's cross-section. For $|x| < at$, there appear the dispersive disturbances p_2 in addition to the steady pressure field p_1 expressed in terms of the infinite sum of the discrete higher modes. In this paper, we pause here to present only the solution p_1 and p_2 in the closed form.

5. Discussions

When a train starts suddenly with constant speed in a long tunnel or when a train rushes into a tunnel, the rapid change gives rise to the radiation of the pressure pulse forward. Its magnitude in the near field has been evaluated by solving the linear wave equation with the source term of the pair of monopoles or of the dipole. For the impulsive motion, the magnitude (6) is proportional to the Mach number M rather than M^2 except for the Doppler factor $1 - M$. For the entry motion, by contrast, the magnitude (11) is proportional to M^2. The magnitude for the impulsive motion is found to be greater than that for the entry motion. In either case, the magnitude is proportional to χ. But it should be noted that this result is valid only for a small cross-sectional ratio χ.

According to these results, we now estimate a pressure level in a plausible case with $M = 0.44$ ($U = 150$ m/s (540 km/h)) and $\chi = 0.1$ for the magnetically levitated trains. For the impulsive motion, (6) gives 0.055 (169 dB in SPL) and for the entry motion, (11) gives 0.034 (165 dB). While no experiments have been performed in such a situation, many data are available for the entry motion of the present high-speed trains. For $M = 0.18$ ($U = 62.5$ m/s (225 km/h)) and $\chi = 0.216$, (11) gives 0.011 (155 dB), which is to be compared with 0.013 measured at the wavefront (Ozawa *et al.*). This shows a fairly good agreement.

In this connection, we refer to another estimate derived by Hara in the private report of the then Japan National Railways. Since this report is not easily available, the outline of the derivation is introduced. Assuming a train of infinite length to enter a tunnel, the field is divided into three regions, i.e., an undisturbed region ahead of the wavefront, a disturbed region between the wavefront and a point just behind the train's nose and a region between this point and the tunnel exist. Across the wavefront, the continuity of mass, momentum and energy is required just as in derivation of the shock relations. In the second region, the tunnel's cross-sectional change displaced by the train is essential and is taken into account through the continuity of mass together with the Bernoulli equation and the adiabatic relation. In the third region, if the friction loss due to the tunnel wall and the train's surface is neglected, it is shown that the magnitude across the wavefront is given by

$$\frac{\Delta p_+}{p_0} = \frac{\gamma M^2}{2} \frac{1-(1-\chi)^2}{(1-M)[M+(1-\chi)^2]}. \tag{30}$$

Here χ appears to be unrestricted but it is assumed small implicitly. If χ is so small that only its first-order term is retained, (30) agrees with (11). Although both approaches are totally different, we have the same result for a small train's cross-section.

For the magnetically levitated trains, the pressure level in the near field is estimated to become several times greater than the one due to the present high-speed trains even if the cross-sectional ratio χ is reduced from 0.216 to 0.1. As the pressure level increases, the far field comes close to the train so that a shock wave tends to appear even in a shorter tunnel. Of course, the emerging shock wave becomes strong. In view of the pressure level estimated, however, its evolution will still be well covered by the theory of nonlinear acoustics. For the impulsive motion in a long tunnel, the negative pulse is also radiated backward. But since this is the expansion wave, no shock waves emerge behind the train.

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