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A group generated by the Lévy Laplacian

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1. INTRODUCTION

The Lévy Laplacian Δ_L is one of infinite dimensional Laplacians introduced by P. Lévy in his book [Lé 22]. In his book, he mentioned that Δ_L comes from the singular part f_s'' of the second derivative f'', i.e.,

$$\Delta_L f(x) = \int_0^1 f_s''(x;u)du.$$

This Laplacian has been studied by many authors. In 1975, T.Hida introduced Δ_L into the theory of generalized white noise functionals in [Hi 75]. H.-H. Kuo [Ku 83,89,92a,92b] defined the Fourier-Mehler transform on the space $(\mathcal{S})^*$ of generalized white noise functionals and gave a relation between its transform and Δ_L. An interesting characterization of Δ_L in terms of rotation groups was obtained by N. Obata [Ob 90]. Recently, T. Hida [Hi 92b] applied Δ_L to S. Tomonaga's many time theory in quantum physics.

The purpose of this paper is to construct a group generated by Δ_L.

In §2, we will explain a construction of the space of generalized white noise functionals and define the Lévy Laplacian Δ_L^T for a finite interval T in $\mathbf{R}$ in that space. Moreover, we introduce an operator Δ and prove that Δ coincides with $2\Delta_L^T$ on a domain D_L^T in $(\mathcal{S})^*$. In §3, we will construct a (C_0)-group $\{G_t\}_{t\in\mathbf{R}}$ generated by Δ_L^T. In the last section, we will give a relation between the adjoint operator of Kuo's Fourier-Mehler transform and a group $\{G_{it}\}_{t\in\mathbf{R}}$.

2. THE LÉVY LAPLACIAN IN THE WHITE NOISE CALCULUS

In this section, we introduce a space of Hida distributions following [Hi 80], [KT 80-82] and [PS 91] (See also, [HKPS 93], [HOS 92] and [Ob 92]) and the Lévy Laplacian defined on a domain in this space.

1) Let $L^2(\mathbf{R})$ be the Hilbert space of real square-integrable functions on $\mathbf{R}$ with norm $|\cdot|_0$. Consider a Gel'fand triple

$$\mathcal{S} = \mathcal{S}(\mathbf{R}) \subset L^2(\mathbf{R}) \subset \mathcal{S}^* = \mathcal{S}^*(\mathbf{R}),$$

where $\mathcal{S}(\mathbf{R})$ is the Schwartz space consisting of rapidly decreasing functions on $\mathbf{R}$ and $\mathcal{S}^*(\mathbf{R})$ is the dual space of $\mathcal{S}(\mathbf{R})$.

Let A be the following operator

$$A = -(d/dx)^2 + x^2 + 1.$$

For each $p \in \mathbf{Z}$, we define $|f|_p = |A^p f|_0$ and let $\mathcal{S}_p$ be the completion of $\mathcal{S}$ with respect to the norm $|\cdot|_p$. Then the dual space of $\mathcal{S}_p'$ of $\mathcal{S}_p$ is the same as $\mathcal{S}_{-p}$.

2) Let μ be a probability measure on $\mathcal{S}^*$ with the characteristic functional given by

$$C(\xi) \equiv \int_{\mathcal{S}^*} \exp\{i < x, \xi >\} \, d\mu(x) = \exp\{-\frac{1}{2}|\xi|_0^2\}, \ \xi \in \mathcal{S}.$$

Let $(L^2) = L^2(\mathcal{S}^*, \mu)$ be the space of complex-valued square-integrable functionals defined on $\mathcal{S}^*$ and define the *S-transform* by

$$S\varphi(\xi) = C(\xi) \int_{\mathcal{S}^*} \exp\{< x, \xi >\}\varphi(x) \, d\mu(x), \ \varphi \in (L^2).$$

The Hilbert space admits the well-known Wiener-Itô decomposition:

$$(L^2) = \oplus_{n=0}^{\infty} H_n,$$

where H_n is the space of multiple Wiener integrals of order $n \in \mathbf{N}$ and $H_0 = \mathbf{C}$. From this decomposition theorem, each $\varphi \in (L^2)$ is uniquely represented as

$$\varphi = \sum_{n=0}^{\infty} \mathbf{I}_n(f_n), \ f_n \in L^2_{\mathbf{C}}(\mathbf{R})^{\hat{\otimes} n},$$

where $\mathbf{I}_n \in H_n$ and $L^2_{\mathbf{C}}(\mathbf{R})^{\hat{\otimes} n}$ denotes the n-th symmetric tensor product of the complexification of $L^2(\mathbf{R})$.

For each $p \in \mathbf{Z}, p \geq 0$, we define the norm $\|\varphi\|_p$ of $\varphi = \sum_{n=0}^{\infty} \mathbf{I}_n(f_n)$, by

$$\|\varphi\|_p = \left(\sum_{n=0}^{\infty} n! |f_n|_{p,n} \right)^{1/2},$$

where $|\cdot|_{p,n}$ is the norm of $\mathcal{S}_{\mathbf{C},p}^{\hat{\otimes} n}$(the n-th symmetric tensor product of the complexification of $\mathcal{S}_p$). The norm $\|\cdot\|_0$ is nothing but the (L^2)-norm. We put

$$(\mathcal{S})_p = \{\varphi \in (L^2); \|\varphi\|_p < \infty\}$$

for $p \in \mathbf{Z}, p \geq 0$. Let $(\mathcal{S})_p^*$ be the dual space of $(\mathcal{S})_p$. Then $(\mathcal{S})_p$ and $(\mathcal{S})_p^*$ are Hilbert spaces with the norm $\|\cdot\|_p$ and the dual norm of $\|\cdot\|_p$, respectively.

Denote the projective limit space of the $(\mathcal{S})_p, p \in \mathbf{Z}, p \geq 0$, and the inductive limit space of the $(\mathcal{S})_p^*, p \in \mathbf{Z}, p \geq 0$, by $(\mathcal{S})$ and $(\mathcal{S})^*$, respectively. Then $(\mathcal{S})$ is a nuclear space and $(\mathcal{S})^*$ is nothing but the dual space of $(\mathcal{S})$. The space $(\mathcal{S})^*$ is called the space of *Hida distributions* (or *generalized white noise functionals*).

Since $\exp < \cdot, \xi > \in (\mathcal{S})$, the S-transform is extended to an operator U defined on $(\mathcal{S})^*$:

$$U\Phi(\xi) = C(\xi) \ll \Phi, \exp < \cdot, \xi > \gg, \xi \in \mathcal{S},$$

where $\ll \cdot, \cdot \gg$ is the canonical pairing of $(\mathcal{S})$ and $(\mathcal{S})^*$. We call $U\Phi$ the *U-functional* of Φ.

3) We next introduce the definition of the Lévy Laplacian following Kuo [Ku 92] (see also [HKPS 93]). Let U be a *Fréchet differentiable* function defined on $\mathcal{S}$, i.e. we assume that there exists a map U' from $\mathcal{S}$ to $\mathcal{S}^*$ such that

$$U(\xi + \eta) = U(\xi) + U'(\xi)(\eta) + o(\eta), \eta \in \mathcal{S},$$

where $o(\eta)$ means that there exists $p \in \mathbf{Z}, p \geq 0$, depending on ξ such that $o(\eta)/|\eta|_p \to 0$ as $|\eta|_p \to 0$. Then the first variation

$$\delta U(\xi; \eta) = dU(\xi + \lambda\eta)/d\lambda|_{\lambda=0}$$

is expressed in the form

$$\delta U(\xi; \eta) = \int_{\mathbf{R}} U'(\xi; u)\eta(u)\, du$$

for every $\eta \in \mathcal{S}$ by using the generalized function $U'(\xi; \cdot)$. We define the *Hida derivative* $\partial_t\Phi$ of Φ to be the generalized white noise functional whose U-functional is given by $U'(\xi; t)$.

Definition. (I) A Hida distribution Φ is called an *L-functional* if fo r each $\xi \in \mathcal{S}$, there exist $(U\Phi)'(\xi; \cdot) \in L^1_{loc}(\mathbf{R}), (U\Phi)''_s(\xi; \cdot) \in L^1_{loc}(\mathbf{R})$ and $(U\Phi)''_r(\xi; \cdot, \cdot) \in L^1_{loc}(\mathbf{R}^2)$ such that the first and second variations are uniquely expressed in the forms:

$$(U\Phi)'(\xi)(\eta) = \int_{\mathbf{R}} (U\Phi)'(\xi; u)\eta(u)\, du,$$

and

$$(U\Phi)''(\xi)(\eta, \zeta) = \int_{\mathbf{R}} (U\Phi)''_s(\xi; u)\eta(u)\zeta(u)\, du + \int_{\mathbf{R}^2} (U\Phi)''_r(\xi; u, v)\eta(u)\zeta(v)\, dudv, \tag{2.1}$$

for each $\eta, \zeta \in \mathcal{S}$, respectively and for any finite interval T, $\int_T (U\Phi)''_s(\cdot; u)\, du$ is a U-functional.

(II) Let D_L denote the set of all L-functionals. For $\Phi \in D_L$ and any finite interval T in $\mathbf{R}$, the *Lévy Laplacian* Δ_L^T is defined by

$$\Delta_L^T \Phi = U^{-1}\left[\frac{1}{|T|}\int_T (U\Phi)_s''(\cdot;u)\,du\right].$$

Remark. Explicit conditions for the uniqueness of the above decomposition (2.1) is given in [HKPS 93, chapter 6].

Let T be a finite interval in $\mathbf{R}$. Take a smooth function e defined on $\mathbf{R}$ satisfying $0 \leq e(u) \leq 1$ for all $u \in \mathbf{R}$, $e(u) = 1$ for $|u| \leq 1/2$ and $e(u) = 0$ for $|u| \geq 1$. Let $\rho_n *$ be the Friedrichs mollifier. Put $e_n(u) = e(u/n)$ and $\theta_n^T = \sqrt{2}|\rho_n|_0^{-1}|T|^{-1/2}$, $n = 1, 2, \ldots$. We define an operator Δ for a Hida distribution Φ by

$$U[\Delta\Phi](\xi) = \lim_{n\to\infty}\int_{\mathcal{S}^*} U\Phi''(\xi)(\theta_n^T e_n(\rho_n * x), \theta_n^T e_n(\rho_n * x))\,d\mu(x),$$

if the limit exists in $U[(\mathcal{S})^*]$. From now on, we denote $e_n(\rho_n * x)$ by $j_n(x)$. Let D_L^T denote the set of all L-functionals Φ satisfying $U\Phi(\eta) = 0$ for η with $\mathrm{supp}(\eta) \subset T^c$. In [Sa 94], we obtained the following result. (For the proof, see [Sa 94].)

THEOREM 1. *Let T be a finite interval in $\mathbf{R}$ and Φ an L-functional in D_L^T. Then, we have $\Delta\Phi = 2\Delta_L^T\Phi$.*

3. The Lévy Laplacian as the infinitesimal generator

A generalized functional Φ is called a *normal functional* if its U- functional $U\Phi$ is given by a finite linear combination of

$$\int_{A^k} f(u_1, \ldots, u_k)\xi(u_1)^{p_1}\cdots\xi(u_k)^{p_k}\,du_1\cdots du_k, \tag{3.1}$$

where $f \in L^1(A^k), p_1, \ldots, p_k \in \mathrm{N} \cup \{0\}, k \in \mathbf{N}$, and A : a finite interval in $\mathbf{R}$. This functional Φ is in D_L. Let $\mathcal{N}_T$ denote the set of all normal functionals in D_L^T. For $p > 1$ and $\Phi \in D_L^T$, we define a $-p$-norm $|||\cdot|||$ by

$$|||\Phi|||_{-p}^2 = \sum_{k=0}^{\infty} \|(\Delta_L^T)^k\Phi\|_{-p}^2 (\in [0, \infty])$$

and denote the completion of $\mathcal{N}_T$ with respect to the norm $|||\cdot|||_{-p}$ by $D_L^{(-p)}$. Then $D_L^{(-p)}$ is the Hilbert space with the norm $|||\cdot|||_{-p}$ and Δ_L^T is a bounded linear operator on $D_L^{(-p)}$. Hence a (C_0)-group $\{G_t, t \in \mathbf{R}\}$ is given by

$$G_t = \lim_{n \to \infty} \sum_{k=0}^{n} \frac{t^k}{k!} (\Delta_L^T)^k, \tag{3.2}$$

in the sence of the operator norm. It is easily checked that $\| G_t \| \leq e^{|t|}$, for any $t \in \mathbf{R}$.

Define an operator g_t on $\mathcal{N}_T$ for $t \geq 0$ by

$$U[g_t \Phi](\xi) = \lim_{n \to \infty} \int_{\mathcal{S}^*} U\Phi(\xi + \sqrt{t}\theta_n^T j_n(x))\, d\mu(x), \quad \Phi \in \mathcal{N}_T.$$

For a normal functional Φ which $U\Phi$ is given as in (3.1) with the domain $A^k \subset T^k$, it is easily checked that

$$U[g_t \Phi](\xi) = \sum_{\nu_1=0}^{[p_1/2]} \cdots \sum_{\nu_k=0}^{[p_k/2]} \frac{p_1! \cdots p_k!}{(2\nu_1)!!(p_1 - 2\nu_1)! \cdots (2\nu_k)!!(p_k - 2\nu_k)!} \cdot$$

$$\left(\frac{2t}{|T|} \right)^{\nu_1 + \cdots + \nu_k} \int_{A^k} f(u_1, \ldots, u_k) \xi(u_1)^{p_1 - 2\nu_1} \cdots \xi(u_k)^{p_k - 2\nu_k}\, du_1 \cdots du_k.$$

Therefore, g_t is a linear operator from $\mathcal{N}_T$ to itself. By Theorem 1, it can be checked that $G_t = g_t$ on $\mathcal{N}_T$. Since $\mathcal{N}_T$ is dense in $D_L^{(-p)}$, we have the following

THEOREM 2. *For any $t \geq 0$, g_t is extended to the operator G_t.*

4. The Fourier-Mehler transform and the Lévy Laplacian

An characterization of Hida distributions was obtained by J. Potthoff and L. Streit [PS 91]. From [PS 91], we see that for any U-functional F, and ξ, η in $\mathcal{S}$, the function $F(\lambda\xi + \eta)$, $\lambda \in \mathbf{R}$, extends to an entire function $F(z\xi + \eta)$, $z \in \mathbf{C}$. Then we can define an operator g_{it}, $t \in \mathbf{R}$, by

$$U[g_{it}\Phi](\xi) = \lim_{n \to \infty} \int_{\mathcal{S}^*} U\Phi(\xi + \sqrt{it}\theta_n^T j_n(x)) d\mu(x),$$

if the limit exists. Since μ is symmetric, the integral is defined independent of choices of the branch of $\sqrt{it}$. As in (3.2), we can naturally define G_{it}, $t \in \mathbf{R}$, by

$$G_{it} = \lim_{n \to \infty} \sum_{k=0}^{n} \frac{(it)^k}{k!} (\Delta_L^T)^k,$$

on $D_L^{(-p)}$.

An infinite dimensional Fourier-Mehler transform $\mathbf{F}_\theta$, $\theta \in \mathbf{R}$, on $(\mathcal{S})^*$ was defined by H.-H. Kuo [Ku 91] as follows. The transform $\mathbf{F}_\theta \Phi$, $\theta \in \mathbf{R}$ of $\Phi \in (\mathcal{S})^*$ is defined by the unique generalized white noise functional with the U-functional

$$U[\mathbf{F}_\theta \Phi](\xi) = U\Phi(e^{i\theta}\xi) \exp\left[\frac{i}{2} e^{i\theta} \sin\theta |\xi|_0^2\right], \ \xi \in \mathcal{S}.$$

Moreover, the adjoint operator $\mathbf{F}_\theta^*$ of $\mathbf{F}_\theta$ is given by

$$\mathbf{F}_\theta^* \Phi = \textstyle\sum_{n=0}^\infty \mathbf{I}_n(h_n(\Phi;\theta)) \text{ for } \Phi = \sum_{n=0}^\infty \mathbf{I}_n(f_n) \in (\mathcal{S}),$$

where

$$h_n(\Phi;\theta) = \sum_{m=0}^\infty \frac{(n+2m)!}{n!m!} (\frac{i}{2}\sin\theta)^m e^{i(m+n)\theta} \tau^{\otimes m} * f_{n+2m};$$

$$\tau^{\otimes m} = \int_{\mathbf{R}^m} \delta_{t_1} \otimes \delta_{t_1} \otimes \cdots \otimes \delta_{t_m} \otimes \delta_{t_m} dt_1 \cdots dt_m.$$

This operator $\mathbf{F}_\theta^*$ is a continuous linear operator on $(\mathcal{S})$. (For details, see [Ku 91] and also [HKO 90]) On $(\mathcal{S})$, the Gross Laplacian Δ_G (See [Gr 65, 67]) and the number operator N is given by

$$\Delta_G \Phi = \int_{\mathbf{R}} \partial_t^2 \Phi dt$$

and

$$N\Phi = \int_{\mathbf{R}} \partial_t^* \partial_t \Phi dt,$$

respectively (see [Ku 86]). The operator $e^{i\theta N}$ is called the *Fourier- Wiener transform* (see [HKO 90]). Now, we introduce an operator $e^{\frac{i}{2}\theta\Delta_G}$ from $(\mathcal{S})$ into itself given by

$$e^{\frac{i}{2}\theta\Delta_G}\Phi = \sum_{n=0}^\infty \mathbf{I}_n(\ell_n(\Phi;\theta)); \tag{4.1}$$

$$\ell_n(\Phi;\theta) = \sum_{m=0}^\infty \frac{(n+2m)!}{n!m!} (\frac{i}{2}\theta)^m \tau^{\otimes m} * f_{n+2m},$$

for $\Phi = \sum_{n=0}^\infty \mathbf{I}_n(f_n) \in (\mathcal{S})$. Then we have the followings.

LEMMA 1.

$$\mathbf{F}_\theta^* = e^{i\theta N} \circ e^{\frac{i}{2}(e^{i\theta}\sin\theta)\Delta_G}. \tag{4.2}$$

PROOF: Take $\Phi = \sum_{n=0}^\infty \mathbf{I}_n(f_n) \in (\mathcal{S})$. From (4.1), we see that

$$e^{\frac{i}{2}(e^{i\theta}\sin\theta)\Delta_G}\Phi = \sum_{n=0}^\infty \mathbf{I}_n(\ell_n(\Phi; e^{i\theta}\sin\theta)).$$

Hence,

$$e^{i\theta N}(e^{\frac{i}{2}(e^{i\theta}\sin\theta)\Delta_G}\Phi) = \sum_{n=0}^\infty \mathbf{I}_n(e^{in\theta}\ell_n(\Phi; e^{i\theta}\sin\theta)).$$

Since $e^{in\theta}\ell_n(\Phi; e^{i\theta}\sin\theta) = h_n(\Phi;\theta)$, we obtain (4.2). ∎

LEMMA 2. *For any* $\Phi \in (\mathcal{S})$, *we have*

$$U[e^{\frac{i}{2}\theta\Delta_G}\Phi](\xi) = \int_{\mathcal{S}^*} U\Phi(\xi + \sqrt{i\theta}y)\, d\mu(y). \tag{4.3}$$

Remark. For any $\Phi \in (\mathcal{S})$, $\xi \in \mathcal{S}$ and $z_1, z_2 \in \mathbf{C}$, the functional $U\Phi(z_1\xi + z_2\eta)$, $\eta \in \mathcal{S}$, can be extended to a functional $\widetilde{U\Phi}(z_1\xi + z_2y)$, same symbol $U\Phi(z_1\xi + z_2y)$.

PROOF: For $\Phi = \sum_{n=0}^{\infty} \mathbf{I}_n(f_n) \in (\mathcal{S})$, the right-hand side of (4.3) has the following expansion:

$$\sum_{n=0}^{\infty} \int_{\mathbf{R}^n} f_n(\mathbf{u}) \int_{\mathcal{S}^*} \{\xi(u_1) + \sqrt{i\theta}x(u_1)\} \cdots \{\xi(u_n) + \sqrt{i\theta}x(u_n)\} d\mu(x) d\mathbf{u}$$

$$= \sum_{n=0}^{\infty} \sum_{\nu=0}^{[n/2]} \frac{n!}{(2\nu)!!(n-2\nu)!}(i\theta)^{\nu} < \xi^{\otimes(n-2\nu)}, \tau^{\nu} * f_n >= \sum_{m=0}^{\infty} < \xi^{\otimes m}, \ell_m(\Phi;\theta) > .$$

From (4.1), we see that the last series is equal to $U[e^{\frac{i}{2}\theta\Delta_G}\Phi](\xi)$. ∎

Define an operator J_n by

$$U[J_n\Phi](\xi) = U\Phi \circ j_n(\xi), \ \Phi \in D_L^{(-p)}, \ \xi \in \mathcal{S}.$$

For all $n \in \mathbf{N}$ and $\Phi \in D_L^{(-p)}$, we can easily check $J_n\Phi \in (\mathcal{S})$. Then we have the following.

THEOREM 3. *Let* $\Phi \in D_L^{(-p)}$ *be a generalized white noise functional with the U-functional given by* $\psi(F_1, \ldots, F_n)$, *where* ψ *is an entire function on* $\mathbf{C}$ *and* $F_1, \ldots, F_n \in U[\mathcal{N}_T]$. *We assume the condition*

$$\sum_{k_1,\ldots,k_n=0}^{\infty} \frac{1}{k_1!\cdots k_n!} |\partial_{u_1}^{k_1} \cdots \partial_{u_n}^{k_n} \psi(0,\ldots,0)| \cdot$$

$$\sup_N \int_{\mathcal{S}^*} \left|((F_1 \circ j_N)^{k_1} \cdots (F_n \circ j_N)^{k_n})(ie^{i\alpha_N(t)}\xi + \sqrt{ie^{i\alpha_N(t)}\sin\alpha_N(t)}x)\right| \, d\mu(x) < \infty$$

holds for all $t > 0$ *and* $\xi \in \mathcal{S}$, *where* $\alpha_N(t) = t(\theta_N^T)^2$. *Then*

$$\lim_{N\to\infty} U[\mathbf{F}^*_{\alpha_N(t)} J_N\Phi](\xi) = U[G_{it}\Phi](\xi), \ \xi \in \mathcal{S}. \tag{4.4}$$

PROOF: From Lemma 2, we have

$$U[e^{\frac{i}{2}e^{i\alpha_N(t)}\sin\alpha_N(t)\Delta_G} J_N\Phi](\xi) = \int_{\mathcal{S}^*} U[J_N\Phi](\xi + \sqrt{ie^{i\alpha_N(t)}\sin\alpha_N(t)}y) d\mu(y).$$

This functional is expressed in the form given by

$$\sum_{\ell=0}^{\infty}\langle \xi^{\otimes \ell}, f_{N,\ell}\rangle,$$

where $f_{N,\ell} \in \mathcal{S}_{\mathbf{C}}^{\hat{\otimes}\ell}$. Hence, from Lemma 1, we get

$$U[\mathbf{F}^*_{\alpha_N(t)}J_N\Phi](\xi) = \sum_{\ell=0}^{\infty} e^{i\alpha_N(t)\ell}\langle \xi^{\otimes \ell}, f_{N,\ell}\rangle.$$

From the condition of this theorem and the Lebesgue convergence theorem, we can calculate as follows:

$$\lim_{N\to\infty} U[\mathbf{F}^*_{\alpha_N(t)}J_N\Phi](\xi) = \lim_{N\to\infty} U[e^{\frac{i}{2}e^{i\alpha_N(t)}\sin\alpha_N(t)\Delta_G}J_N\Phi](e^{i\alpha_N(t)}\xi)$$

$$= \lim_{N\to\infty}\int_{\mathcal{S}^*} U[J_N\Phi](ie^{i\alpha_N(t)}\xi + \sqrt{ie^{i\alpha_N(t)}\sin\alpha_N(t)}y)d\mu(y)$$

$$= \sum_{k_1,\dots,k_n=0}^{\infty} \frac{1}{k_1!\cdots k_n!}\partial_{u_1}^{k_1}\cdots\partial_{u_n}^{k_n}\psi(0,\dots,0)\cdot$$

$$\lim_{N\to\infty}\int_{\mathcal{S}^*}((F_1\circ j_N)^{k_1}\cdots(F_n\circ j_N)^{k_n})(ie^{i\alpha_N(t)}\xi + \sqrt{ie^{i\alpha_N(t)}\sin\alpha_N(t)}x)d\mu(x).$$

By the direct calculations, it is easily checked that

$$\lim_{N\to\infty}\int_{\mathcal{S}^*}((F_1\circ j_N)^{k_1}\cdots(F_n\circ j_N)^{k_n})(ie^{i\alpha_N(t)}\xi + \sqrt{ie^{i\alpha_N(t)}\sin\alpha_N(t)}x)d\mu(x)$$

$$= U[g_{it}U^{-1}(F_1^{k_1}\cdots F_n^{k_n})](\xi) = U[g_{it}U^{-1}F_1](\xi)^{k_1}\cdots U[g_{it}U^{-1}F_n](\xi)^{k_n}.$$

Consequently, we obtain (4.4). ∎

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