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A Collision Avoidance Control Problem for Moving Bodies in the Plane

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1 Introduction

There appeared several works related with the applicability of direct method of Liapunov to the find-path problems lies in the qualitive theory of differential games and the avoidance control strategies. We refer to, e.g., Vincent and Skowronski[4], Skowronski and Vincent[5], Stonier[7], and Skowronski and Stonier[6] for differential game aspects, and Leitman[3], Stonier[8] and JHN[9] for avoidance strategy aspects. In these works the generation of a suitable Liapunov function is the key in the view of the Liapunov stability theory. Recently, L.T. Grujic[1,2] has established that an asymptotically stable nonlinear system permits the construction of a Liapunov function to guarantee the asymptotic stability. That is, there is no Liapunov function which makes a given system be asymptotically stable if the given system is not asymptotically stable. But, it is very difficult or impossible to determine a suitable Liapunov function for the given complexed nonlinear system, because we have not to integrate the dini-derivative, see[1,2]. Therefore, in many cases we need to construct a Liapunov function, which implies that a system may be stable, and often we can obtain asymptotic stability under some restricted conditions.

Approaching the findpath problem to a collision avoidance strategy of robot arms, Stonier[8] adopts the Liapunov theory from the control and differential game literature for capture within targets, and for avoidance of antitargets. It may be the first good proposal in [8] to solve the collision avoidance control problem in the basis of Liapunov theory. The essential feature of his approach is to construct Liapunov functions for the approaching targets and collision avoiding of antitargets and to determine control variables according to the time derivatives of Liapunov functions. However, in the determination of feedback control variables, he used assumption called "right-of way", which is reasonable in numerical simulations but not meaningful in mathematical sense, and unfortunately the generalized Liapunov functions do not satisfy the sufficient condition of Liapunov stability theory; see[8]. In our previous work JHN[9], we can remove assumption such as "right-of way", and we introduce the elliptic Liapunov function to obtain good paths of orbit of moving objects. But, in [9] we have failed to treat control parameters which may make the path change freely, and futher the Liapunov function does not satisfy the sufficient condition of the Liapunov stability.

In this paper, we introduce a new Liapunov function which satisfies the Liapunov stability sufficient conditions, and by using the Liapunov function we may easily change the paths freely via the control parameters. Finally, we note that almost all are "regular cases" in that we are getting in nice, smooth paths for the collision avoidance in numerical simulations. These are illustrated by several examples, and the comparisions of our numerical results with the cases of [8] and [9] are given.

2 Control plan for m numbers of moving objects

Let us consider a system, containing m numbers of moving objects and m numbers of fixed targets in a plane workspace, for the trajectories of the moving objects being controlled to obtain collision avoidance and to reach the targets. The collsion avoidance control problem is to control the movement of the i-th moving object to reach the center of the i-th target, while ensuring the i-th entire moving object to avoid the j-th target and the j-th entire moving object which is regarded as an antitarget with respect to the i-th one, where $i \neq j,\ 1 \leq i, j \leq m$. We will use the Liapunov technique as known as a powerful mathematical method to accomplish the plan for solving the collision avodience control problem. Therefore, to utilize the Liapunov technique, it is necessary to introduce the Liapunov function which can be applied to the given system, and we give it below.

2.1 The Liapunov techique

Let $\mathbb{R}^+$ be the set of positive real numbers. We will denote by A_i the i-th moving object and by T_j the j-th target respectively, where $1 \le i, j \le m$. Let us regard the centers of moving objects A_i as the points (x_i, y_i) on the plane. When each moving object A_i moves continuously depending on $t \in \mathbb{R}^+$, we can consider (x_i, y_i) as a continuous function for $t \in \mathbb{R}^+$. In the paper, as studied in Stonier[8] and J-H-N[9], we suppose that the dynamics of m point objects $(x_i, y_i), i = 1, 2, \cdots, m$, are described by the system of the controlled ordinary differential equations,

$$\begin{cases} \dot{x}_i = z_i \\ \dot{z}_i = u_i \\ \dot{y}_i = w_i \\ \dot{w}_i = v_i, \quad i = 1, 2, \cdots, m. \end{cases} \tag{2.1}$$

Here in (2.1), $(z_i, w_i) = (\dot{x}_i, \dot{y}_i)$ denotes the time derivatives of the i-th point object and (u_i, v_i) denotes the i-th control variables pair. We remark that the special case where $m = 2$ is considered in Stonier[8] and J-H-N[9]. By the Liapunov technique, the controlls $(u_i, v_i), 1 \le i \le m$ will be determined as feedback controls which are obtained by the result of differentiating the Liapunov function associated with the system equation (2.1). Let us define the target set TS_i of the i-th target T_i with center $(p_i c_1, p_i c_2)$ and radius rp_i and the moving object set AS_j of the j-th moving object A_j with center (x_j, y_j) and length rap_j of the j-th moving object A_j as follows:

$$TS_i = \{(x, y) : (x - p_i c_1)^2 + (y - p_i c_2)^2 \le rp_i^2\}, \quad 1 \le i \le m,$$

$$AS_j = \{(x, y) : (x - x_j)^2 + (y - y_j)^2 \le rap_j^2\}, \quad 1 \le j \le m.$$

In order to determine the controls which give the trajectories to avoid collision, we need to define the Liapunov functions such as approaching to the targets and avoiding the antitargets. Therefore, let us define such functions on the plane as follows. Let us define the following (sub)-Lapunov functions:

V_i the Liapunov function to make the i-th moving object A_i approach to the i-th target T_i;

$$V_i = \frac{1}{2}\{(x_i - p_i c_1)^2 + (y_i - p_i c_2)^2 + z_i^2 + w_i^2\}, \ 1 \le i \le m,$$

W_{ij} the Liapunov function to make the i-th moving object A_i avoid the j-th target $T_j, i \neq j$;

$$W_{ij} = \frac{1}{2}\{(x_i - p_j c_1)^2 + (y_i - p_j c_2)^2 - rp_j^2\},\ 1 \leq i, j \leq m,$$

V_{ij} the Liapunov function to avoid the i-th moving object A_i and the j-th moving object $A_j, i \neq j$ each other;

$$V_{ij} = \frac{1}{2}\{(x_i - x_j)^2 + (y_i - y_j)^2 - \max\{rap_i^2, rap_j^2\}\}\ , 1 \leq i, j \leq m,$$

G_i the function which denotes the distance between centers of the i-th moving object and the i-th target;

$$G_i = \frac{1}{2}\{(x_i - p_i c_1)^2 + (y_i - p_i c_2)^2\},\ 1 \leq i \leq m.$$

Using the above Liapunov functions V_i, W_{ij}, V_{ij} and G_i, we can now define the total Liapunov function $\mathcal{L}$ on $\mathcal{D}(\mathcal{L}) = \{(\mathbf{x}, \mathbf{z}) \in \mathbb{R}^{2m} \times \mathbb{R}^{2m} : V_{ij}(x_i, y_i, x_j, y_j) > 0, W_{ij}(x_i, y_i) > 0, 1 \leq i, j \leq m\}$ for the system (2.1) as follows,

$$\mathcal{L}((\mathbf{x}, \mathbf{z})) = \sum_{i=1}^{m} V_i(x_i, y_i, z_i, w_i) + \sum_{i=1}^{m}\sum_{j=1}^{m} \frac{\alpha_{ij} G_i(x_i, y_i)}{W_{ij}(x_i, y_i)} + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} \frac{\beta_{ij} G_i(x_i, y_i) G_j(x_j, y_j)}{V_{ij}(x_i, y_i, x_j, y_j)},$$

where $\mathbf{x} = (x_1, y_1, \cdots, x_m, y_m) \in \mathbb{R}^{2m}, \mathbf{z} = (z_1, w_1, \cdots, z_m, w_m) \in \mathbb{R}^{2m}$ and for all j, $\alpha_{ii} = \beta_{ii} = 0$, $\alpha_{ij}, \beta_{ij} > 0$ and $\beta_{ij} = \beta_{ji}$ for $1 \leq i, j \leq m$. Then it is verified by the direct calculations of the time derivative $\dot{\mathcal{L}}_{(2.1)}((\mathbf{x}, \mathbf{z}))$ along the equation (2.1) is given by

$$\dot{\mathcal{L}}_{(2.1)}((\mathbf{x}, \mathbf{z})) = -\sum_{i=1}^{m} \left(\gamma_i z_i^2 + \mu_i w_i^2\right)$$

provided that the feedback control variables (u_k, v_k) are given by

$$\begin{aligned}
u_k &= -(x_k - p_k c_1)\left(1 + \sum_{i=1}^{m} \frac{\alpha_{ki}}{W_{ki}} + \sum_{i=1}^{m} \frac{\beta_{ki} G_i}{V_{ki}}\right) \\
&+ \sum_{i=1}^{m} \frac{\alpha_{ki} G_k}{W_{ki}^2}(x_k - p_i c_1) - \sum_{i=1}^{m} \frac{\beta_{ik} G_i G_k}{G_{ik}^2}(x_i - x_k) - \gamma_k z_k, \\
v_k &= -(y_k - p_k c_2)\left(1 + \sum_{i=1}^{m} \frac{\alpha_{ki}}{W_{ki}} + \sum_{i=1}^{m} \frac{\beta_{ki} G_i}{V_{ki}}\right) \\
&+ \sum_{i=1}^{m} \frac{\alpha_{ki} G_k}{W_{ki}^2}(y_k - p_i c_2) - \sum_{i=1}^{m} \frac{\beta_{ik} G_i G_k}{V_{ik}^2}(y_i - y_k) - \mu_k w_k, \qquad (2.2)
\end{aligned}$$

where $k = 1, 2, \cdots, m$. From now on we will call α_{ij}, β_{ij} the control parameters and $\gamma_i > 0, \mu_i > 0$ the convergence parameters. The role of the numerators G_i and $G_i G_j$

appeared in second and third trems of $\mathcal{L}$ is to wipe out the unnecessary effect of W_{ij} and V_{ij} when A_i approach to T_i or A_j approach to T_j, where $1 \le i, j \le m$. Then we can easily know that $\mathcal{L}((\mathbf{x},\mathbf{z})) > 0$ and for $\mathbf{z} \neq \mathbf{0}$, $\dot{\mathcal{L}}_{(2.1)}((\mathbf{x},\mathbf{z})) \le 0$ for the solution $(\mathbf{x},\mathbf{z}) \in \mathcal{D}(\mathcal{L})$ associated with (2.1) and (2.2). Also, the Liapunov function $\mathcal{L}$ satisfies $\mathcal{L}(\mathbf{P}) = 0$ which becomes a sufficient condition for the stability, and simultaneously, which guarantees that $\mathcal{L}((\mathbf{x},\mathbf{z})) \to 0$ as $t \to \infty$ implies $(\mathbf{x},\mathbf{z}) \to \mathbf{P}$, i.e., each moving object goes to each target, where $\mathbf{P} \equiv ((p_1c_1, p_1c_2, p_2c_1, p_2c_2), \mathbf{0})$ is an equilibrium point for the dynamics equation (2.1) with (2.2). But in Stonier[8] and JHN[9], for $m = 2$ they required some restricted conditions that the control parameters $\alpha_{ij}, i, j = 1,2$ and $\beta_{ij}, i, j = 1,2$ are sufiiciently small in order that $\mathbf{V}((\mathbf{x},\mathbf{z})) \to 0$ as $t \to \infty$, where $\mathbf{V}$ is the Liapunov functions introduced by Stonier[8]

$$\mathbf{V}_{\mathbf{Stonier}} = \left(V_1 + \frac{\alpha_{12}}{V_{12}} + \frac{\beta_{12}}{W_{12}}, V_2 + \frac{\alpha_{21}}{V_{21}} + \frac{\beta_{21}}{W_{21}}\right),$$

and by JHN[9]

$$\mathbf{V}_{\mathbf{JHN}} = V_1 + V_2 + \frac{\alpha_{12}}{V_{12}} + \frac{\alpha_{21}}{V_{21}} + \frac{\beta_{12}}{W_{12}} + \frac{\beta_{21}}{W_{21}} + \text{Elliptic Liapnov Function}.$$

As the result, since they have to demand the control parameters $\alpha_{12}, \alpha_{21}, \beta_{12}$ and β_{21}, sufficiently small, it is difficult or impossible to control the trajectories for the system (2.1) with the controlls which they determined under the Liapunov functions, V_{Stonier} and V_{Jito}. That is to say, they failed to give their's control parameters intrinsic means owing to some constraints for all control parameters to be small. Beside, we can not expect the avoidance of collsion between moving objects or moving objects and targets in the case where the control parameters are very small, because the effect of V_{12}, V_{21}, W_{12} and W_{21} disappeares for such the cases. For the new Liapunov function, it is easily verified that β_{12} plays the role of adjusting the distance between moving objects, A_1, A_2 and the α_{12}(resp. α_{21}) plays the part of modulating the distance between moving object A_1(resp. A_2) and target T_2(resp. T_1). Therefore, we have the advantage point of turnning a trajectory for the system (2.1) with (2.3) into the best trajectory by artficial. We will survey such the points from some examples.

2.2 Analysis of trajectories for $m = 2$

For the case of $m = 2$, where it becomes an original problem introduced by Stonier[8], the forms of new Liapunov function and controlls are given as follows:

Forms of Liapunov function and controlls for $m = 2$

$$\mathcal{L} = V_1 + V_2 + \frac{\alpha_{12}G_1}{W_{12}} + \frac{\alpha_{21}G_2}{W_{21}} + \beta_{12}\frac{G_1G_2}{V_{12}}.$$

$$\left.\begin{aligned}
u_1 &= -A(x_1 - p_1c_1) + \frac{\alpha_{12}G_1}{W_{12}^2}(x_1 - p_2c_1) + \frac{\beta_{12}}{V_{12}^2}(x_1 - x_2)G_1G_2 - \gamma_1 z_1,\\
v_1 &= -A(y_1 - p_1c_2) + \frac{\alpha_{12}G_1}{W_{12}^2}(y_1 - p_2c_2) + \frac{\beta_{12}}{V_{12}^2}(y_1 - y_2)G_1G_2 - \mu_1 w_1,\\
u_2 &= -B(x_2 - p_2c_1) + \frac{\alpha_{21}G_2}{W_{21}^2}(x_2 - p_1c_1) + \frac{\beta_{12}}{V_{12}^2}(x_2 - x_1)G_1G_2 - \gamma_2 z_2,\\
v_2 &= -B(y_2 - p_2c_2) + \frac{\alpha_{21}G_2}{W_{21}^2}(y_2 - p_1c_2) + \frac{\beta_{12}}{V_{12}^2}(y_2 - y_1)G_1G_2 - \mu_2 w_2,
\end{aligned}\right\} \qquad (2.3)$$

where $A = 1 + \frac{\alpha_{12}}{W12} + \frac{\beta_{12}G_2}{V_{12}}$ and $B = 1 + \frac{\alpha_{21}}{W21} + \frac{\beta_{12}G_1}{V_{12}}$. Since the asymptotic stability of the system (2.1) with (2.2) was not expected in general, there exists a possibility such as $E = \{\mathbf{x} \in \mathbb{R}^4 : \dot{\mathcal{L}}_{(2.1)}((\mathbf{x}, \mathbf{z})) = 0,\ \mathbf{x} \neq (p_1c_1, p_1c_2, p_2c_1, p_2c_2)\}$ is not empty. When the solution $x(t) \equiv (x_1(t), y_1(t), x_2(t), y_2(t))$ satisfies $\mathbf{x}(t) \in E$ for all $t \geq 0$, we shall call such one the singular solution. It is difficult to find the conditions for $E = \emptyset$ because of the complexity of controlls in (2.3), but we may search for the cases where the singular solutions exist under some initial conditions. In particular, one may guess that the trajectories caused by the symetricity of initial conditions belong to the set E. Indeed, firstly, let $\mathbf{x}(t)$ satisfying

$$x_1(t) - p_1c_1 = -(x_2(t) - p_2c_1),\ y_1(t) = y_2(t),\ p_2c_1 = p_2c_2,\ \forall t \geq 0 \qquad (2.4)$$

be the solution of (2.1) with (2.3) under initial conditions satisfying $z_1(0) + z_2(0) = 0$ and $w_1(0) = w_2(0)$. Then either $\gamma_1 = \gamma_2$ or $\mu_1 = \mu_2$ implies that $\alpha_{12} = \alpha_{21}, \gamma_1 = \gamma_2, \mu_1 = \mu_2$ and $rp_1 = rp_2$. Thus, either $\alpha_{12} \neq \alpha_{21}$ or $rp_1 \neq rp_2$ implies the fact that there is the time t_1 when trajectories satisfying above initial conditions don't hold the equation (2.4), moreover the trajectories at t_f when $x(t_f) \in E$ can't satisfy the equation (2.4), where t_f denotes the final time when all trajectories are stopped. Secondly, for given $m, n \in \mathbb{R}$, let $p_ic_2 = mp_ic_1 + n, i = 1, 2$ and let initial conditions satisfy $z_1(0) + z_2(0) = 0$ and $w_i(0) = mz_i(0)$. Then we can easily see that for each

$t \geq 0$, the solutions $y_i(t) = mx_i(t) + n, i = 1,2$ satisfies (2.1) with (2.3) if $\gamma_1 = \mu_1$ and $\gamma_2 = \mu_2$. Therefore, the case where the i-th target or trajectory is between j-th target and trajectory indicates $x(t) \in E$. Similar to the first case, one can know when $\gamma_i \neq \mu_i, i = 1$ or $i = 2$, for the trajectories to escape from the line $y_i(t) = mx_i(t) + n$ and never to return to a parallel line with $y_i(t) = mx_i(t) + n$, because of considering the first case after rotating it proper.

EXAMPLE 2.1 We start to compare with three results through this example. This example shows that an absolute value of controls is very small than two results, and the same time, reaching time to targets is to be shorten largely.

i) initial condition

x_1	z_1	y_1	w_1	x_2	z_2	y_2	w_2
−20	1	5	5	20	−1	2	2

ii) position of target and size of moving object, target and RK4(Runge-Kutta 4th)

p_1c_1	p_1c_2	p_2c_1	p_2c_2	rp_1	rp_2	rap_1	rap_2	$RK4$
12	0	−12	0	6	6	6	6	0.01

iii) control and convergence parameter

	β_{12}	α_{12}	α_{21}	γ_1	μ_1	γ_2	μ_2
case 1	1	1	1	1	1	1	1

iv)maximum and minimum value of controls

max		u_1	v_1	u_2	v_2
	Stonier	21.99	38.62	1287.61	513.86
	JHN	17.93	17.19	443.08	148.12
	New	25.71	7.60	10.16	3.60
min	Stonier	−69.14	−54.99	−86.48	−51.99
	JHN	−46.02	−74.95	−37.27	−15.98
	New	−4.90	−28.41	−16.84	−10.75

v)reaching time to targets

	Stonier	JHN	New
$A_1 \to T_1$	48.31	69.13	26.40
$A_2 \to T_2$	26.07	38.10	11.59

Trajectories for three results in example 2.1 are illustrated in picture 2.1.

EXAMPLE 2.2 In this example, we consider the case where initial condition and center of target are placed on the graph $\{(x,y) : y = mx + n,\ m,n \in \mathbb{R}\}$. The case 1 where targets and initial points are put on two parallel lines is that the trajectories don't go to the targets, but we can make the trajectories move to the targets by changing the value of α_{12} different to α_{21}. The case 2 where all datum are put on the line $y = 2x + 6$ can become asyptotically stable by virtue of varying the values of convergent parameters, for example, $\mu_1 = 4$.

i) initial condition

	x_1	z_1	y_1	w_1	x_2	z_2	y_2	w_2
Case 1	-20	1	10	1	20	-1	10	1
Case 2	-10	1	-14	2	6	-1	18	-2

ii) position of target and size of moving object, target and RK4

	p_1c_1	p_1c_2	p_2c_1	p_2c_2	rp_1	rp_2	rap_1	rap_2	$RK4$
Case 1	10	5	-10	5	5	5	5	5	0.05
Case 2	2	10	-5	-4	3	4	4	4	0.05

iii) control and convergence parameter

	β_{12}	α_{12}	α_{21}	γ_1	μ_1	γ_2	μ_2
case 1	1	1(2)	1	3	3	3	3
case 2	1	2	3	3	3(4)	3	3

iv)maximum and minimum value of controls

		u_1	v_1	u_2	v_2
max	case 1	28.32	17.95	22.88	28.23
	case 2	9.61	17.22	7.63	15.49
min	case 1	−14.83	−9.29	−29.57	−11.02
	case 2	−5.78	−13.79	−7.95	−15.91

v)reaching time to targets

	case 1	case 2
$A_1 \to T_1$	23.6	29.6
$A_2 \to T_2$	20.6	24.0

Trajectories for the case 1 and 2 in example 2.2 are illustrated in picture 2.2 and picture 2.3, respectively.

2.3 Analysis of the trajectories for $m \geq 3$

In order to verify that even for $m \geq 3$ the new Liapunov function has no obstacle finding a path for the collision avoidance control problem, we will give some examples for the cases of $m = 3, m = 4$ and $m = 5$. It may occure the case, similar to the case $m = 2$, that the solution of the system (2.1) with (2.2) belongs to an invariant set or becomes asymptotically stable according to varying the values of parameters and initial conditions. Here we can get an information about the positions where the trajectories stop on the way, which is occurred when the trajectories fall into a dead alley. Therefore, it is necessary to block up the entrance of a dead alley for the trajectories not to enter into a dead alley, which can be obtained by taking the control parameter α_{ij} around a target where a dead alley arises sufficiently large.

2.3.1 An example for $m = 3$

Form of Liapunov function and controllers for $m = 3$

$$\mathcal{L} = V_1 + V_2 + V_3 + G_1 \left(\frac{\beta_{12} G_2}{V_{12}} + \frac{\alpha_{12}}{W_{12}} + \frac{\alpha_{13}}{W_{13}} \right)$$

$$+ \quad G_2\left(\frac{\beta_{23}G_3}{V_{23}}+\frac{\alpha_{21}}{W_{21}}+\frac{\alpha_{23}}{W_{23}}\right)+G_3\left(\frac{\beta_{13}G_1}{V_{13}}+\frac{\alpha_{31}}{W_{31}}+\frac{\alpha_{32}}{W_{32}}\right)$$

$$\begin{aligned} u_1 &= -(x_1-p_1c_1)\left(1+\frac{\alpha_{12}}{W_{12}}+\frac{\alpha_{13}}{W_{13}}+\frac{G_2\beta_{12}}{V_{12}}+\frac{G_3\beta_{13}}{V_{13}}\right)-\gamma_1 z_1 \\ &+ G_1\left[\frac{\beta_{12}G_2}{V_{12}^2}(x_1-x_2)-\frac{\beta_{13}G_3}{V_{13}^2}(x_3-x_1)+\frac{\alpha_{12}}{W_{12}^2}(x_1-p_2c_1)+\frac{\alpha_{13}}{W_{13}^2}(x_1-p_3c_1)\right] \end{aligned}$$

$$\begin{aligned} v_1 &= -(y_1-p_1c_2)\left(1+\frac{\alpha_{12}}{W_{12}}+\frac{\alpha_{13}}{W_{13}}+\frac{G_2\beta_{12}}{V_{12}}+\frac{G_3\beta_{13}}{V_{13}}\right)-\mu_1 w_1 \\ &+ G_1\left[\frac{\beta_{12}G_2}{V_{12}^2}(y_1-y_2)-\frac{\beta_{13}G_3}{V_{13}^2}(y_3-y_1)+\frac{\alpha_{12}}{W_{12}^2}(y_1-p_2c_2)+\frac{\alpha_{13}}{W_{13}^2}(y_1-p_3c_3)\right] \end{aligned}$$

$$\begin{aligned} u_2 &= -(x_2-p_2c_1)\left(1+\frac{\alpha_{21}}{W_{21}}+\frac{\alpha_{23}}{W_{23}}+\frac{G_1\beta_{12}}{V_{12}}+\frac{G_3\beta_{23}}{V_{23}}\right)-\gamma_2 z_2 \\ &+ G_2\left[\frac{\beta_{23}G_3}{V_{23}^2}(x_2-x_3)-\frac{\beta_{12}G_1}{V_{12}^2}(x_1-x_2)+\frac{\beta_{21}}{W_{21}^2}(x_2-p_1c_1)+\frac{\beta_{23}}{W_{23}^2}(x_2-p_3c_1)\right] \end{aligned}$$

$$\begin{aligned} v_2 &= -(y_2-p_2c_2)\left(1+\frac{\alpha_{21}}{W_{21}}+\frac{\alpha_{23}}{W_{23}}+\frac{G_1\beta_{12}}{V_{12}}+\frac{G_3\beta_{23}}{V_{23}}\right)-\mu_2 w_2 \\ &+ G_2\left[\frac{\beta_{23}G_3}{V_{23}^2}(y_2-y_3)-\frac{\beta_{12}G_1}{V_{12}^2}(y_1-y_2)+\frac{\beta_{21}}{W_{21}^2}(y_2-p_1c_2)+\frac{\beta_{23}}{W_{23}^2}(y_2-p_3c_2)\right] \end{aligned}$$

$$\begin{aligned} u_3 &= -(x_3-p_3c_1)\left(1+\frac{\alpha_{31}}{W_{31}}+\frac{\alpha_{32}}{W_{32}}+\frac{G_1\beta_{13}}{V_{13}}+\frac{G_2\beta_{23}}{V_{23}}\right)-\gamma_3 z_3 \\ &+ G_3\left[\frac{\beta_{13}G_1}{V_{13}^2}(x_3-x_1)-\frac{\beta_{23}G_2}{V_{23}^2}(x_2-x_3)+\frac{\alpha_{31}}{W_{31}^2}(x_3-p_1c_1)+\frac{\alpha_{32}}{W_{32}^2}(x_3-p_2c_1)\right] \end{aligned}$$

$$\begin{aligned} u_3 &= -(y_3-p_3c_2)-\mu_3 w_3 \\ &+ G_3\left[\frac{\beta_{13}G_1}{V_{13}^2}(y_3-y_1)-\frac{\beta_{23}G_2}{V_{23}^2}(y_2-y_3)+\frac{\alpha_{31}}{W_{31}^2}(y_3-p_1c_2)+\frac{\alpha_{32}}{W_{32}^2}(y_3-p_2c_2)\right] \end{aligned}$$

EXAMPLE 2.3 In this example, we consider the case where targets and initial points are concentrated around the origin, which are considered as a difficult situation to control the trajectory. In the case 1, the trajectories do not go to the targets in the desired time, and asymptotically stable under the case 2 where we change the control parameters α_{12}, α_{23} and α_{31}, which play a role of making A_i travel T_3 in the direction, A_2 to T_1 and A_3 to T_2 and removing of the entrance into three dead alleys simultaneously.

i) initial condition

x_1	z_1	y_1	w_1	x_2	z_2	y_2	w_2	x_3	z_3	y_3	w_3
-7	1	6	1	7	1	6	1	0	1	-5	1

ii) position of target and size of moving object, target and RK4

p_1c_1	p_1c_2	p_2c_1	p_2c_2	p_3c_1	p_3c_2
3.5	0	-3.5	0	0	6

$$rp_i = 3.5,\ rap_i = 3,\ i = 1,2,3, \text{ and } RK4 = 0.05$$

iii) control and convergence parameter

	β_{12}	β_{13}	β_{23}	α_{12}	α_{13}	α_{21}	α_{23}	α_{31}	α_{32}	γ_1	μ_1	γ_2	μ_2	γ_3	μ_3
case 1	5	5	5	1	1	1	1	1	1	5	5	5	5	5	5
case 2	5	5	5	15	1	1	15	15	1	5	5	5	5	5	5

iv) reaching time to targets

	$A_1 \to T_1$	$A_2 \to T_2$	$A_3 \to T_3$
case 2	20.0	20.0	20.5

Trajectories for the case 1 and 2 in example 2.3 are illustrated in picture 2.4.

2.3.2 An example for $m = 4$

EXAMPLE 2.4 This example may not occure in a realistic system, but it is a very interesting case. Since the moving objects are closed up, they may get out of a workspace, otherwise they may collide each other or a moving object may collide with a target. Thus, it is necessary to adjust the strength between the moving objects to weak, which means making the control parameters β_{ij} be small enough.

i) initial condition

x_1	z_1	y_1	w_1	x_2	z_2	y_2	w_2	x_3	z_3	y_3	w_3	x_4	z_4	y_4	w_4
-16	1	0	-1	-13	1	0	1	-10	1	0	-1	-7	1	0	1

ii) position of target and size of moving object, target and RK4

p_1c_1	p_1c_2	p_2c_1	p_2c_2	p_3c_1	p_3c_2	p_4c_1	p_4c_2
0	0	7	0	12	0	15	0

$$rp_1 = 4,\ \ rp_2 = 3\ \ rp_3 = 2\ \ rp_4 = 1$$

$$rap_i = 2,\ i = 1, 2, 3, 4.$$

iii) control and convergence parameter

1. $\gamma_i = \mu_i = 5,\ 1 \le i \le 4,\ \beta_{ij} = 0.05,\ 1 \le i < j \le 4$ and $\alpha_{ij} = 1,\ 1 \le i, j \le 4,\ i \ne j, RK4 = 0.005$.

2. $\gamma_i = \mu_i = 5,\ 1 \le i \le 4,\ \beta_{ij} = 0.01,\ 1 \le i < j \le 4$ and $\alpha_{ij} = 1,\ 1 \le i, j \le 4,\ i \ne j, RK4 = 0.01$.

Trajectories for the case 1 and 2 in example 2.4 are illustrated in picture 2.5(a) and picture 2.5(b).

2.3.3 Some examples for $m = 5$

We present two interesting examples where the shape of located targets has four dead alleys and where all moving objects are concentrated on the very small workspace.

EXAMPLE 2.5 When we regard the T_1 as a big pillar placed on the plane and the other targets as some small bodies which are attached on the T_1, there exist four dead alleys which swallow all moving objects $A_i, i = 2, 3, 4, 5$.

i) initial condition

x_1	z_1	y_1	w_1	x_2	z_2	y_2	w_2	x_3	z_3	y_3	w_3
0	0	0	0	−15	1	0	−1	0	1	−15	1

x_4	z_4	y_4	w_4	x_5	z_5	y_5	w_5
15	−1	0	1	0	−1	15	−1

ii) position of target and size of moving object, target and RK4

p_1c_1	p_1c_2	p_2c_1	p_2c_2	p_3c_1	p_3c_2	p_4c_1	p_4c_2	p_5c_1	P_5c_2
0	0	8	0	0	8	−8	0	0	−8

$$rp_1 = 5,\ \ rp_i = 3,\ 2 \le i \le 5,$$

$$rap_i = 2,\ 1 \le i \le 5 \text{ and } RK4 = 0.05.$$

iii) control and convergence parameter

$$\gamma_i = \mu_i = 3,\ 1 \le i \le 5,\ \beta_{ij} = 1,\ 1 \le i < j \le 4,$$

$$\alpha_{21} = \alpha_{31} = \alpha_{41} = \alpha_{51} = 20,\ \alpha_{25} = \alpha_{32} = \alpha_{43} = \alpha_{54} = 0.5$$

and other than then $\alpha_{ij} = 1$.

Trajectories for m=5 in example 2.5 are illustrated in picture 2.6.

EXAMPLE 2.6 Since all moving objects are closed up in the very small workspace, we have to make the control parameters $\beta_{ii+1}, i = 1,2,3,4$ be small to prevent moving object and moving object or moving object and target from colliding each other, and then it is necessary to arrange the control parameters α_{ij} to obtain the smooth of the trajectories. The results are below.

i) initial condition

x_1	z_1	y_1	w_1	x_2	z_2	y_2	w_2	x_3	z_3	y_3	w_3
0.0	1.0	−5.0	1.0	4.75	−1.0	−1.55	1.0	2.95	1.0	3.0	1.0

x_4	z_4	y_4	w_4	x_5	z_5	y_5	w_5
−2.95	1.0	4.05	−1.0	−4.75	1.0	−1.55	1.0

ii) position of target and size of moving object, target and RK4

p_1c_1	p_1c_2	p_2c_1	p_2c_2	p_3c_1	p_3c_2	p_4c_1	p_4c_2	p_5c_1	P_5c_2
0	10	−9.5	3.1	−5.9	−8.1	5.9	−8.1	9.5	3.1

$$rp_i = 4,\ 1 \le i \le 5,$$

$$rap_i = 2,\ 1 \le i \le 5 \text{ and } RK4 = 0.05.$$

iii) control and convergence parameter

1. $\gamma_i = \mu_i = 5,\ 1 \le i \le 5,\ \beta_{ij} = 0.05,\ 1 \le i < j \le 5$ and $\alpha_{ij} = 1,\ 1 \le i,j \le 4,\ i \ne j$.

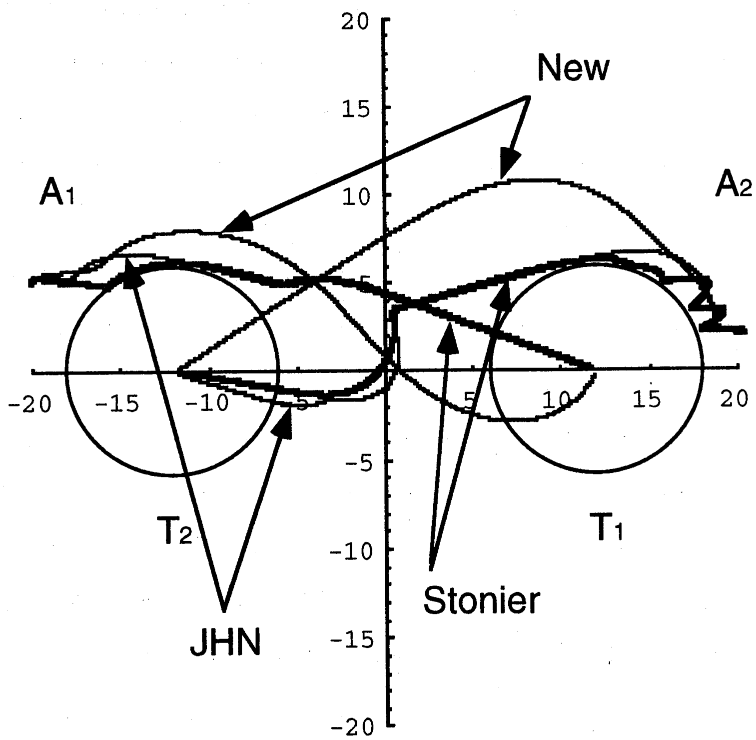
2. $\gamma_i = \mu_i = 5,\ 1 \le i \le 5,\ \beta_{ij} = 0.05,\ 1 \le i < j \le 5,$
   $\alpha_{13} = \alpha_{24} = \alpha_{35} = \alpha_{41} = \alpha_{52} = 10$ and other than them $\alpha_{ij} = 1$.

Trajectories for the case 1 and 2 in example 2.6 are illustrated in picture 2.7.

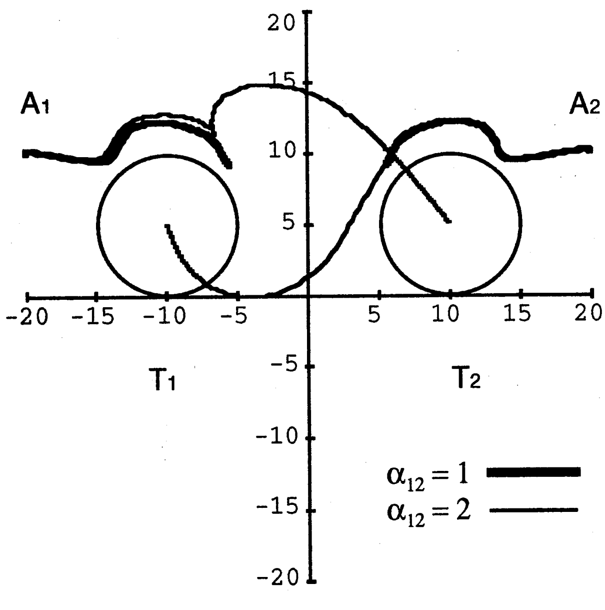
3 Conclusion

The most important feature of this paper is that the Liapunov function for the system (2.1) is setted up skilfully, so that the cotrolls and convergence parameters play their proper roles such as altering the trajectory of the system (2.1) into the desired one. It is obvious that the system (2.1) with (2.2) is stable. However, under the new Liapunov function, the asymptotic stability for the system (2.1) with (2.2) is not verified in general. In fact, for $m = 2$ there were many examples of the trajectories being stopped on the way, but we could avoid it by means of manipulating every condition to break out a symmetrical condition. We have hardly a stopping situation halfway, because the new Liapunov function have many parameters which are not necessary symmetry. When $m \geq 3$, we were confronted with many situations that the trajectories belong to the invariant set, but most situations were solved by adjusting the control parameters. If one want to make the state which is not asymptotically stable be asymptotically stable, we have to compose another Liapunov function with relation to a neural system, but it is a problem in the future.

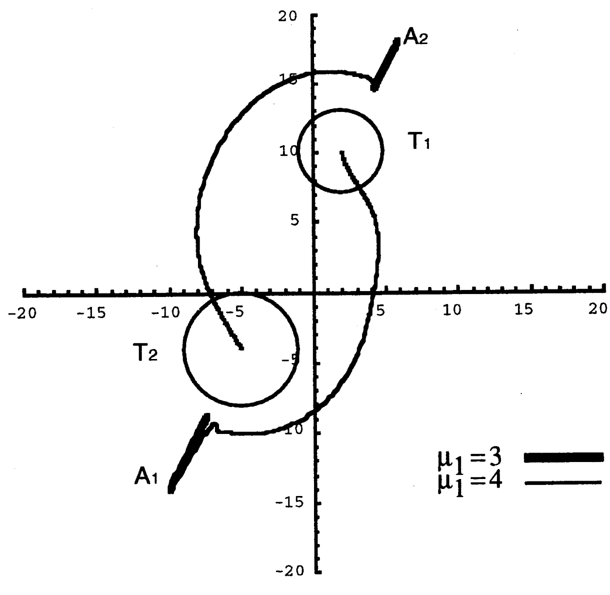
PICTURES



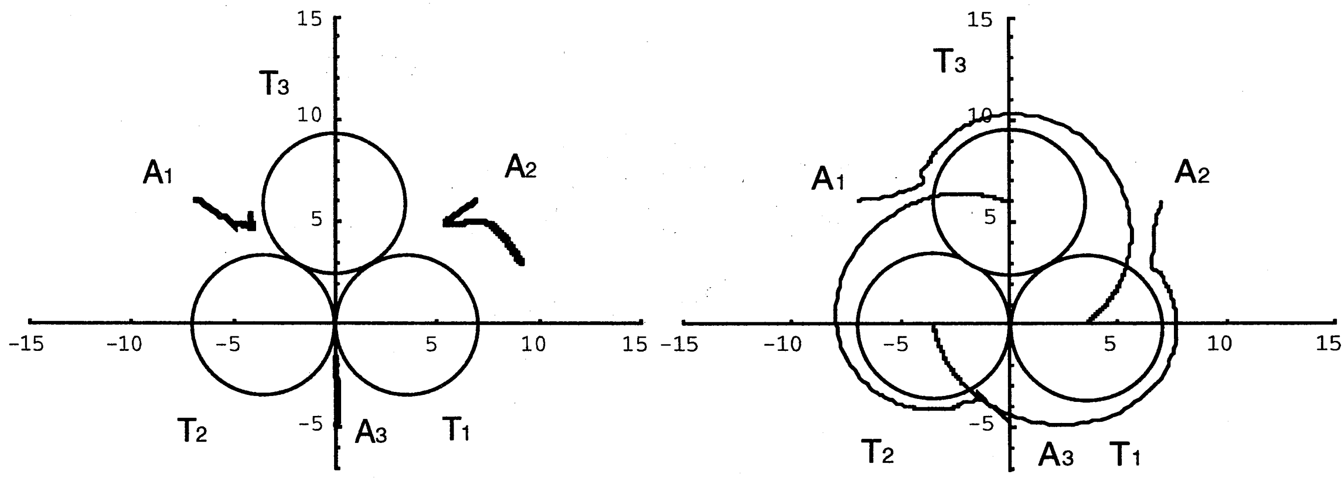
Picture 2.1. Trojectories for three results.



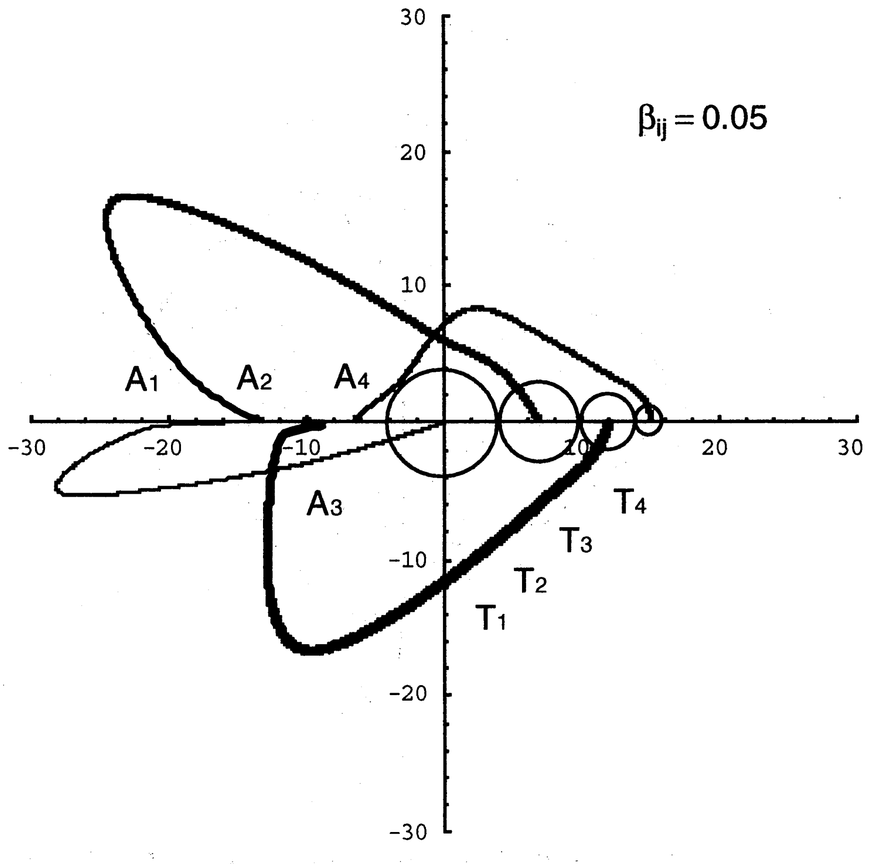
Picture 2.2. Trajectories for the case 1.



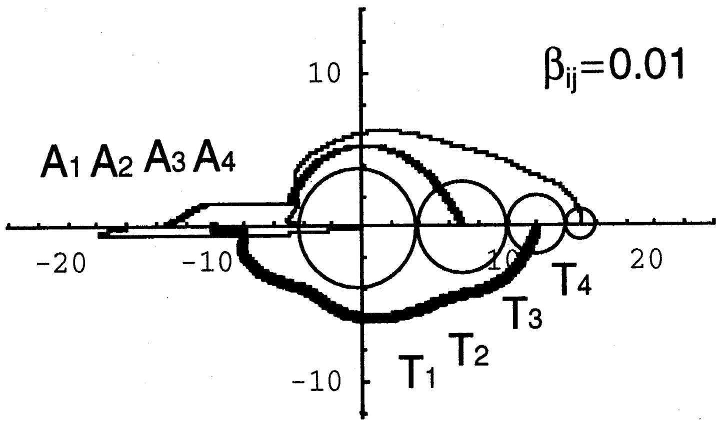
Picture 2.3. Trajectories for the case 2.



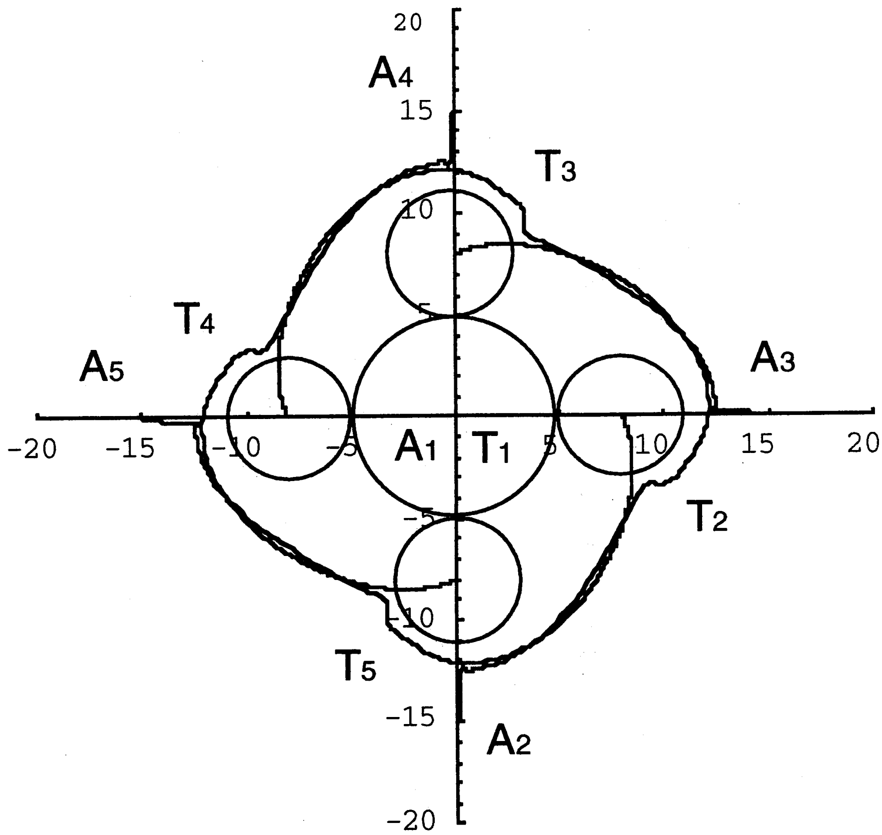
Picture 2.4. Trajectories for the case 1 and 2.



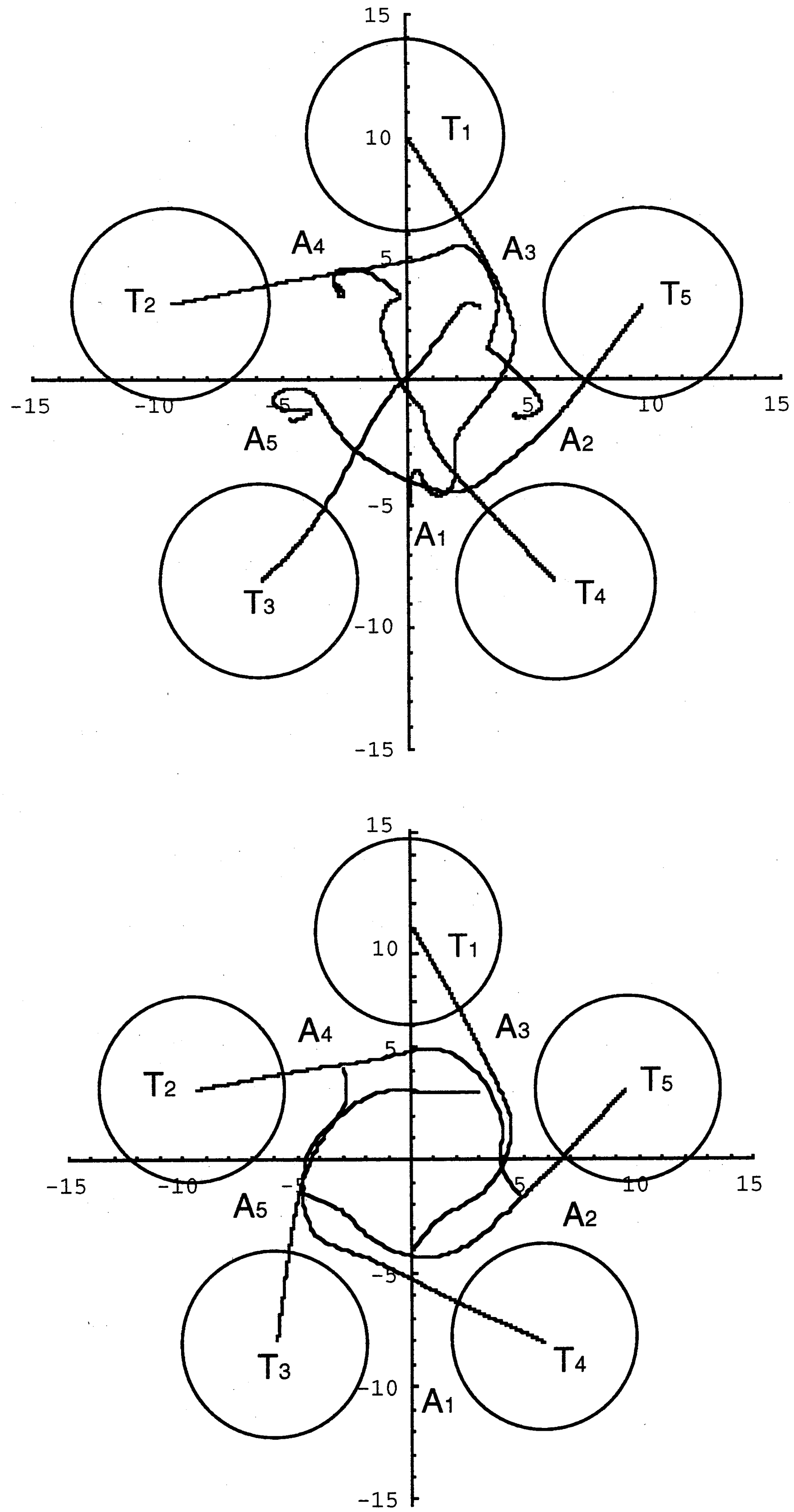
Picture 2.5. (a) Trojectories for the case 1.



Picture 2.5. (b) Trajectories for the case 2.



Picture 2.6. Trajectories for m=5.



Picture 2.7. Trajectories for the case 1 and 2.

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