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Transforming Termination by Self-Labelling

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ABSTRACT

We introduce a new technique for proving termination of term rewriting systems. The technique, a specialization of Zantema's semantic labelling technique, is especially useful for establishing the correctness of transformation methods that attempt to prove termination by transforming term rewriting systems into systems whose termination is easier to prove. We apply the technique to distribution elimination, dummy elimination, and currying, resulting in shorter correctness proofs, stronger results, and a positive solution to an open problem.

1. Introduction

Termination is an undecidable property of term rewriting systems. In the literature (Dershowitz [2] contains an early survey of termination techniques) several methods for proving termination are described that are quite successful in practice. We can distinguish roughly two kinds of termination methods:

(1) basic methods like recursive path order and polynomial interpretations that apply directly to a given term rewriting system, and

(2) methods that attempt to prove termination by transforming a given term rewriting system into a term rewriting system whose termination is easier to prove, e.g. by a method of the first kind, and implies termination of the given system.

Transformation orders (Bellegarde and Lescanne [1]) and distribution elimination (Zantema [8]) are examples of methods of the second kind. Semantic labelling (Zantema [7]) is a very powerful method of this kind. The starting point of the present paper is the observation that semantic labelling is in a sense too powerful. We show that any terminating term rewriting system can be transformed by semantic labelling into a system whose termination can be shown by the recursive path order. The proof of this result gives rise to a new termination method which we name *self-labelling*. We show that self-labelling is especially useful for proving the correctness of termination methods of the second kind:

- Using self-labelling we give a positive solution to an open problem in [8] concerning distribution elimination: right-linearity is not necessary for the correctness of distribution elimination in the absence of distribution rules. The proof reveals how to improve distribution elimination in the absence of distribution rules.
- Using self-labelling we give an alternative proof of the correctness of dummy elimination, a recent transformation method of Ferreira and Zantema [3]. From the proof we infer how to lift the restriction that the symbol to be eliminated may not occur in the left-hand sides of the rewrite rules.
- Using self-labelling we give a short proof of the main result of Kennaway, Klop, Sleep, and De Vries [4] stating the correctness of currying, which for the purpose of this paper we view as a termination method of the second kind.

The proofs of the above results are remarkably similar.

The remainder of this paper is organized as follows. In the next section we recapitulate semantic labelling. In Section 3 we show that every terminating term rewriting system can be transformed by semantic labelling into a term rewriting system whose termination is very easy to prove. This completeness result gives rise to the self-labelling technique. In Section 4 we use self-labelling to solve the conjecture concerning distribution elimination. The self-labelling proof gives rise to a stronger result, which we explain in Section 5. In Section 6 we observe how self-labelling can be used to show the correctness of an extension of dummy elimination. Our final illustration of the strength of self-labelling can be found in Section 7 where we present a short proof of the preservation of termination under currying.

2. Preliminaries

We assume the reader is familiar with the basics of term rewriting (as expounded in, e.g., Klop [5]). This paper deals with the termination property. A term rewriting system (TRS for short) $(\mathcal{F}, \mathcal{R})$ is said to be terminating if it doesn't admit infinite rewrite sequences. It is well-known that a TRS $(\mathcal{F}, \mathcal{R})$ is terminating if and only if there exists a reduction order—a well-founded order that is closed under contexts and substitutions—on $\mathcal{T}(\mathcal{F}, \mathcal{V})$ that orients the rewrite rules of $\mathcal{R}$ from left to right. Another well-known fact states that $(\rightarrow_{\mathcal{R}} \cup \rhd)^+$ is a well-founded order on $\mathcal{T}(\mathcal{F}, \mathcal{V})$ for any terminating TRS $(\mathcal{F}, \mathcal{R})$. Here $s \rhd t$ if and only if t is a proper subterm of s. Observe that $(\rightarrow_{\mathcal{R}} \cup \rhd)^+$ is in general not a reduction order as it lacks closure under contexts.

In this preliminary section we briefly recall the ingredients of semantic labelling (Zantema [7]). Actually we present a special case which is sufficient for our purposes. Let $(\mathcal{F}, \mathcal{R})$ be a TRS and $\mathcal{A} = (A, \{f_{\mathcal{A}}\}_{f \in \mathcal{F}})$ an $\mathcal{F}$-algebra. Let $\succ$ be a well-founded order on the carrier A of $\mathcal{A}$. We say that the pair $(\mathcal{A}, \succ)$ is a *quasi-model* for $(\mathcal{F}, \mathcal{R})$ if

- the interpretation $f_{\mathcal{A}}$ of every function symbol $f \in \mathcal{F}$ is weakly monotone (with respect to $\succ$) in all its coordinates, and
- $[\alpha](l) \succcurlyeq [\alpha](r)$ for every rewrite rule $l \rightarrow r \in \mathcal{R}$ and assignment $\alpha\colon \mathcal{V} \rightarrow A$. Here $[\alpha]$ denotes

the unique homomorphism from $\mathcal{T}(\mathcal{F},\mathcal{V})$ to $\mathcal{A}$ that extends α, i.e.,

$$[\alpha](t) = \begin{cases} \alpha(t) & \text{if } t \in \mathcal{V}, \\ f_{\mathcal{A}}([\alpha](t_1),\ldots,[\alpha](t_n)) & \text{if } t = f(t_1,\ldots,t_n). \end{cases}$$

The above takes care of the semantical content of semantic labelling. We now describe the labelling part. We label function symbols from $\mathcal{F}$ with elements of A. Formally, we consider the labelled signature $\mathcal{F}_{lab} = \{f_a \mid f \in \mathcal{F} \text{ and } a \in A\}$ where each f_a has the same arity as f. For every assignment α we inductively define a labelling function lab_α from $\mathcal{T}(\mathcal{F},\mathcal{V})$ to $\mathcal{T}(\mathcal{F}_{lab},\mathcal{V})$ as follows:

$$lab_\alpha(t) = \begin{cases} t & \text{if } t \in \mathcal{V}, \\ f_{[\alpha](t)}(lab_\alpha(t_1),\ldots,lab_\alpha(t_n)) & \text{if } t = f(t_1,\ldots,t_n). \end{cases}$$

So function symbols in t are simply labelled by the value (under the assignment α) of the corresponding subterms. We define the TRSs $\mathcal{R}_{lab}$ and $dec(\mathcal{F},\succ)$ over the signature $\mathcal{F}_{lab}$ as follows:

$$\begin{aligned} \mathcal{R}_{lab} &= \{lab_\alpha(l) \to lab_\alpha(r) \mid l \to r \in \mathcal{R} \text{ and } \alpha\colon \mathcal{V} \to A\}, \\ dec(\mathcal{F},\succ) &= \{f_a(x_1,\ldots,x_n) \to f_b(x_1,\ldots,x_n) \mid f \in \mathcal{F} \text{ and } a,b \in A \text{ with } a \succ b\}. \end{aligned}$$

The following theorem is a special case of the main result of Zantema [7].

THEOREM 2.1. *Let $(\mathcal{F},\mathcal{R})$ be a TRS, $\mathcal{A}$ an $\mathcal{F}$-algebra, and $\succ$ a well-founded order on the carrier of $\mathcal{A}$. If $(\mathcal{A},\succ)$ is a quasi-model then termination of $(\mathcal{F},\mathcal{R})$ is equivalent to termination of $(\mathcal{F}_{lab},\mathcal{R}_{lab} \cup dec(\mathcal{F},\succ))$.* □

Observe that in the above approach the labelling part of semantic labelling is completely determined by the semantics. This is not the case for semantic labelling as defined in [7]. The additional expressive power of [7] results in some quite impressive termination proofs. In this paper we hope to make clear that the special case of semantic labelling presented above is quite successful as well.

3. Self-Labelling

In this section we show that every terminating TRS can be transformed by semantic labelling into a TRS whose termination is very easily established. The proof of this result forms the basis of a powerful technique for proving the correctness of transformation techniques for establishing termination.

DEFINITION 3.1. A TRS $(\mathcal{F},\mathcal{R})$ is called *precedence terminating* if there exists a well-founded order $\sqsupset$ on $\mathcal{F}$ such that $root(l) \sqsupset f$ for every rewrite rule $l \to r \in \mathcal{R}$ and every function symbol $f \in Fun(r)$.

LEMMA 3.2. *Every precedence terminating TRS is terminating.*

PROOF. Let $(\mathcal{F}, \mathcal{R})$ be a precedence terminating TRS. So there exists a well-founded order $\sqsupset$ on $\mathcal{F}$ that satisfies the condition of Definition 3.1. An easy induction argument on the structure of r reveals that $l \sqsupset_{rpo} r$ for every $l \to r \in \mathcal{R}$. Since $\sqsupset_{rpo}$ is a reduction order, termination of $(\mathcal{F}, \mathcal{R})$ follows. □

The next result states any terminating TRS can be transformed by semantic labelling into a precedence terminating TRS.

THEOREM 3.3. *For every terminating TRS $(\mathcal{F}, \mathcal{R})$ there exists a quasi-model $(\mathcal{A}, \succ)$ such that $(\mathcal{F}_{lab}, \mathcal{R}_{lab} \cup dec(\mathcal{F}, \succ))$ is precedence terminating.*

PROOF. As $\mathcal{F}$-algebra $\mathcal{A}$ we take the term algebra $\mathcal{T}(\mathcal{F}, \mathcal{V})$. We equip $\mathcal{T}(\mathcal{F}, \mathcal{V})$ with the well-founded order $\succ = \to_{\mathcal{R}}^{+}$. (Well-foundedness is an immediate consequence of termination of $\mathcal{R}$.) Because rewriting is closed under contexts, all algebra operations are (strictly) monotone in all their coordinates. Because assignments in the term algebra $\mathcal{T}(\mathcal{F}, \mathcal{V})$ are substitutions and rewriting is closed under substitutions, $(\mathcal{A}, \succ)$ is a quasi-model for $(\mathcal{F}, \mathcal{R})$. It remains to show that $(\mathcal{F}_{lab}, \mathcal{R}_{lab} \cup dec(\mathcal{F}, \succ))$ is precedence terminating. To this end we define a well-founded order $\sqsupset$ on $\mathcal{F}_{lab}$ as follows: $f_s \sqsupset g_t$ if and only if $s\ (\to_{\mathcal{R}} \cup \rhd)^{+}\ t$. Let $l \to r$ be a rewrite rule of $\mathcal{R}_{lab} \cup dec(\mathcal{F}, \succ)$.

- If $l \to r \in \mathcal{R}_{lab}$ then there exist an assignment $\alpha: \mathcal{V} \to \mathcal{T}(\mathcal{F}, \mathcal{V})$ and a rewrite rule $l' \to r' \in \mathcal{R}$ such that $l = lab_\alpha(l')$ and $r = lab_\alpha(r')$. The label of $root(l)$ is $[\alpha](l') = l'\alpha$. Let ℓ be the label of a function symbol in r. By construction $\ell = [\alpha](t) = t\alpha$ for some subterm t of r'. Hence $l'\alpha \to_{\mathcal{R}} r'\alpha \unrhd \ell$. So $root(l) \sqsupset f$ for every $f \in Fun(r)$.
- If $l \to r \in dec(\mathcal{F}, \succ)$ then $l = f_s(x_1, \ldots, x_n)$ and $r = f_t(x_1, \ldots, x_n)$ with $s \to_{\mathcal{R}}^{+} t$. Clearly $root(l) = f_s \sqsupset f_t$.

□

The particular use of semantic labelling in the above proof (i.e., choosing the term algebra as semantics and thus labelling function symbols with terms) is what we will call *self-labelling.* One may argue that Theorem 3.3 is completely useless, since the construction of the quasi-model in the proof relies on the fact that $(\mathcal{F}, \mathcal{R})$ is terminating. Nevertheless, in the following sections we will see how self-labelling gives rise to many new results and significant simplifications of existing results on the correctness of transformation techniques for establishing termination. Below we sketch the general framework.

Let Φ be a transformation on TRSs, designed to make the task of proving termination easier. The question is how to prove correctness of the transformation, i.e., does termination of the transformed TRS $\Phi(\mathcal{F}, \mathcal{R})$ imply termination of the given TRS $(\mathcal{F}, \mathcal{R})$? Let $\Phi(\mathcal{F}, \mathcal{R})$ be the TRS $(\mathcal{F}', \mathcal{R}')$. The basic idea is to label the TRS $(\mathcal{F}, \mathcal{R})$ with terms of $(\mathcal{F}', \mathcal{R}')$. This is achieved by executing the following steps:

- turn the term algebra $\mathcal{T}(\mathcal{F}', \mathcal{V})$ into an $\mathcal{F}$-algebra $\mathcal{A}$ by choosing *suitable* interpretations for the function symbols in $\mathcal{F} \backslash \mathcal{F}'$ (and taking term construction as interpretation of the function symbols in $\mathcal{F} \cap \mathcal{F}'$),

- equip the $\mathcal{F}$-algebra $\mathcal{A}$ with the well-founded order $\succ\, = \,\rightarrow^+_{\mathcal{R}'}$, and
- define the well-founded order $\sqsupset$ on $\mathcal{F}_{lab}$ as follows: $f_s \sqsupset g_t$ if and only if $s\ (\rightarrow_{\mathcal{R}'} \cup \rhd)^+\ t$.

Now, if we can show that

- $(\mathcal{A}, \succ)$ is a quasi-model for $(\mathcal{F}, \mathcal{R})$, and
- the TRS $(\mathcal{F}_{lab}, \mathcal{R}_{lab} \cup dec(\mathcal{F}, \succ))$ is precedence terminating with respect to $\sqsupset$,

then termination of $(\mathcal{F}, \mathcal{R})$ and thus the correctness of the transformation Φ is a consequence of Theorem 2.1.

We would like to stress that the only creative step in this scheme is the choice of the interpretations for the function symbols that disappear during the transformation Φ. All our proofs will follow the above scheme, except that in Section 7 we have to consider a slight refinement of the well-founded order on the labelled signature in order to conclude precedence termination.

4. Distribution Elimination

Our first application of self-labelling is the proof of a conjecture of Zantema [8] concerning distribution elimination.

Let $(\mathcal{F}, \mathcal{R})$ be a TRS and let $e \in \mathcal{F}$ be a designated function symbol whose arity is at least one. A rewrite rule $l \rightarrow r \in \mathcal{R}$ is called a *distribution rule* for e if $l = C[e(x_1, \ldots, x_n)]$ and $r = e(C[x_1], \ldots, C[x_n])$ for some non-empty context C in which e doesn't occur and pairwise different variables $x_1, \ldots, x_n$. Distribution elimination is a technique that transforms $(\mathcal{F}, \mathcal{R})$ by eliminating all distribution rules for e and removing the symbol e from the right-hand sides of the other rules. First we inductively define a mapping E_{distr} that assigns to every term in $\mathcal{T}(\mathcal{F}, \mathcal{V})$ a non-empty subset of $\mathcal{T}(\mathcal{F} \backslash \{e\}, \mathcal{V})$, as follows:

$$E_{distr}(t) = \begin{cases} \{t\} & \text{if } t \in \mathcal{V}, \\ \displaystyle\bigcup_{i=1}^{n} E_{distr}(t_i) & \text{if } t = e(t_1, \ldots, t_n), \\ \{f(s_1, \ldots, s_n) \mid s_i \in E_{distr}(t_i)\} & \text{if } t = f(t_1, \ldots, t_n) \text{ with } f \neq e. \end{cases}$$

The mapping E_{distr} is illustrated in Figure 1, where we assume that the numbered contexts do not contain any occurrences of e. It is extended to rewrite systems as follows:

$$E_{distr}(\mathcal{R}) = \{l \rightarrow r' \mid l \rightarrow r \in \mathcal{R} \text{ is not a distribution rule for } e \text{ and } r' \in E_{distr}(r)\}.$$

Observe that e does not occur in $E_{distr}(\mathcal{R})$ if and only if e does not occur in the left-hand sides of rewrite rules of $\mathcal{R}$ that are not distribution rules for e.

One of the main result of Zantema [8] is stated below.

THEOREM 4.1. *Let $(\mathcal{F}, \mathcal{R})$ be a TRS and let $e \in \mathcal{F}$ be a non-constant symbol which does not occur in the left-hand sides of rewrite rules of $\mathcal{R}$ that are not distribution rules for e.*

(1) *If $E_{distr}(\mathcal{R})$ is terminating and right-linear then $\mathcal{R}$ is terminating.*
(2) *If $E_{distr}(\mathcal{R})$ is simply terminating and right-linear then $\mathcal{R}$ is simply terminating.*
(3) *If $E_{distr}(\mathcal{R})$ is totally terminating then $\mathcal{R}$ is totally terminating.*

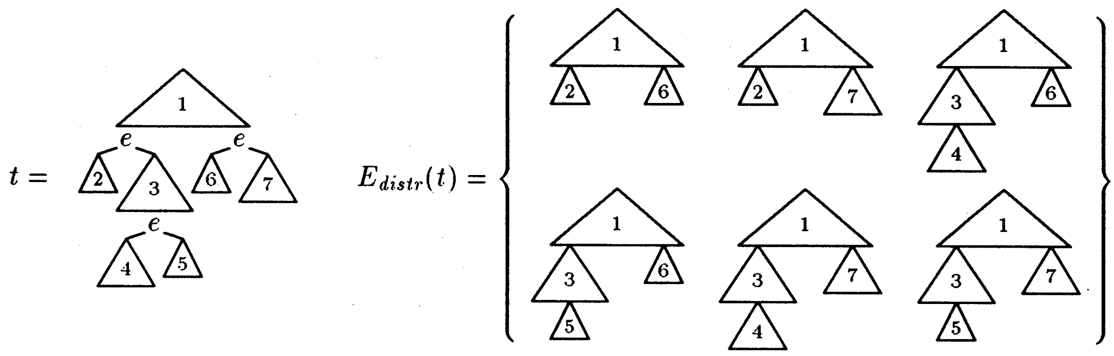

FIGURE 1.

□

The following example from [8] shows that right-linearity is essential in parts (1) and (2).

EXAMPLE 4.2. Consider the TRS

$$\mathcal{R} = \left\{ \begin{array}{rcl} f(a,b,x,x) & \to & f(x,x,e(a,b),e(a,b)) \\ f(e(x,y),z,v,w) & \to & e(f(x,z,v,w),f(y,z,v,w)) \\ f(x,e(y,z),v,w) & \to & e(f(x,y,v,w),f(x,z,v,w)) \end{array} \right\}.$$

The last two rules are distribution rules for e and e does not occur in the left-hand side of the first rule. The TRS

$$E_{distr}(\mathcal{R}) = \left\{ \begin{array}{rcl} f(a,b,x,x) & \to & f(x,x,a,a) \\ f(a,b,x,x) & \to & f(x,x,a,b) \\ f(a,b,x,x) & \to & f(x,x,b,a) \\ f(a,b,x,x) & \to & f(x,x,b,b) \end{array} \right\}.$$

is easily shown to be terminating. It is even simply terminating since for length-preserving TRSs termination and simple termination coincide. Nevertheless, the term $f(a,b,e(a,b),e(a,b))$ has an infinite reduction in $\mathcal{R}$.

In [8] it is conjectured that in the absence of distribution rules for e the right-linearity assumption in part (1) of Theorem 4.1 can be omitted. Before proving this conjecture with the technique of self-labelling, we show that a similar statement for simple termination doesn't hold, i.e., right-linearity is essential in part (2) of Theorem 4.1 even in the absence of distribution rules for e.

EXAMPLE 4.3. Let $\mathcal{R}'$ consist of the first rule of the TRS $\mathcal{R}$ of Example 4.2. Simple termination of $E_{distr}(\mathcal{R}') = E_{distr}(\mathcal{R})$ was established in Example 4.2, but $\mathcal{R}'$ fails to be simply terminating as $s = f(a,b,e(a,b),e(a,b)) \to_{\mathcal{R}'} f(e(a,b),e(a,b),e(a,b),e(a,b)) = t$ with s embedded in t. (Termination of $\mathcal{R}'$ follows from Theorem 4.4 below.)

THEOREM 4.4. *Let $(\mathcal{F},\mathcal{R})$ be a TRS and let $e \in \mathcal{F}$ be a non-constant symbol which does not occur in the left-hand sides of rewrite rules of $\mathcal{R}$. If $E_{distr}(\mathcal{R})$ is terminating then $\mathcal{R}$ is terminating.*

PROOF. We turn the term algebra $\mathcal{T}(\mathcal{F}\backslash\{e\},\mathcal{V})$ into an $\mathcal{F}$-algebra $\mathcal{A}$ by defining

$$e_{\mathcal{A}}(t_1,\ldots,t_n) = t_\pi$$

for all terms $t_1,\ldots,t_n \in \mathcal{T}(\mathcal{F}\backslash\{e\},\mathcal{V})$. Here π is an *arbitrary* but fixed element of $\{1,\ldots,n\}$. So $e_{\mathcal{A}}$ is simply projection onto the π-th coordinate. We equip $\mathcal{A}$ with the well-founded order $\succ\, = \,\rightarrow^+_{E_{distr}(\mathcal{R})}$ and we define a well-founded order $\sqsupset$ on $\mathcal{F}_{lab}$ as follows: $f_s \sqsupset g_t$ if and only if $s\ (\rightarrow_{E_{distr}(\mathcal{R})} \cup \rhd)^+\ t$. We have to show that $(\mathcal{A},\succ)$ is a quasi-model for $(\mathcal{F},\mathcal{R})$ and that the TRS $(\mathcal{F}_{lab},\mathcal{R}_{lab} \cup dec(\mathcal{F},\succ))$ is precedence terminating with respect to $\sqsupset$.

First we show that $(\mathcal{A},\succ)$ is a quasi-model for $(\mathcal{F},\mathcal{R})$. It is very easy to see that $e_{\mathcal{A}}$ is weakly monotone in all its coordinates. All other operations are strictly monotone in all their coordinates (as $\rightarrow_{E_{distr}(\mathcal{R})}$ is closed under contexts). Let ε be the identity assignment from $\mathcal{V}$ to $\mathcal{V}$. We denote $[\varepsilon](t)$ by $\langle t\rangle$. An easy induction proof shows that $[\alpha](t) = \langle t\rangle\alpha$ for all terms $t \in \mathcal{T}(\mathcal{F},\mathcal{V})$ and assignments $\alpha\colon \mathcal{V} \rightarrow \mathcal{T}(\mathcal{F}\backslash\{e\},\mathcal{V})$. Also the following two properties are easily shown by induction on the structure of $t \in \mathcal{T}(\mathcal{F},\mathcal{V})$: (1) $\langle t\rangle \in E_{distr}(t)$ and (2) if $s \trianglelefteq t$ then there exists a term $t' \in E_{distr}(t)$ such that $\langle s\rangle \trianglelefteq t'$.

(1) If $t \in \mathcal{V}$ then $\langle t\rangle = t$ and $E_{distr}(t) = \{t\}$. For the induction step we distinguish two cases. If $t = e(t_1,\ldots,t_n)$ then $\langle t\rangle = \langle t_\pi\rangle$ and $E_{distr}(t) = \bigcup_{i=1}^n E_{distr}(t_i)$. We have $\langle t_\pi\rangle \in E_{distr}(t_\pi)$ according to the induction hypothesis. Hence $\langle t\rangle \in E_{distr}(t)$. If $t = f(t_1,\ldots,t_n)$ with $f \neq e$ then $\langle t\rangle = f(\langle t_1\rangle,\ldots,\langle t_n\rangle)$ and $E_{distr}(t) = \{f(s_1,\ldots,s_n) \mid s_i \in E_{distr}(t_i)\}$. The induction hypothesis yields $\langle t_i\rangle \in E_{distr}(t_i)$ for all $i = 1,\ldots,n$. Hence also in this case we obtain the desired $\langle t\rangle \in E_{distr}(t)$.

(2) Observe that for $s = t$ the statement follows from property (1) because we can take $t' = \langle t\rangle$. This observation also takes care of the base of the induction. Suppose $t = f(t_1,\ldots,t_n)$ and let s be a proper subterm of t, so s is a subterm of t_k for some $k \in \{1,\ldots,n\}$. From the induction hypothesis we obtain a term $t'_k \in E_{distr}(t_k)$ such that $\langle s\rangle \trianglelefteq t'_k$. Again we distinguish two cases. If $f = e$ then $E_{distr}(t) = \bigcup_{i=1}^n E_{distr}(t_i)$ and thus we can take $t' = t'_k$. If $f \neq e$ then $E_{distr}(t) = \{f(s_1,\ldots,s_n) \mid s_i \in E_{distr}(t_i)\}$. Let $t' = f(\langle t_1\rangle,\ldots,t'_k,\ldots,\langle t_n\rangle)$. Using property (1) we infer that $t' \in E_{distr}(t)$. Clearly $\langle s\rangle \trianglelefteq t'$.

Now let $l \rightarrow r$ be an arbitrary rewrite rule of $\mathcal{R}$ and $\alpha\colon \mathcal{V} \rightarrow \mathcal{T}(\mathcal{F}\backslash\{e\},\mathcal{V})$ an arbitrary assignment. We have $[\alpha](l) = \langle l\rangle\alpha$ and $[\alpha](r) = \langle r\rangle\alpha$. Since e doesn't occur in l, $\langle l\rangle = l$ and hence $[\alpha](l) = l\alpha$. Because $\langle r\rangle \in E_{distr}(r)$, the rule $l \rightarrow \langle r\rangle$ belongs to $E_{distr}(\mathcal{R})$. Therefore $l\alpha \rightarrow_{E_{distr}(\mathcal{R})} \langle r\rangle\alpha$ and thus also $[\alpha](l) \succcurlyeq [\alpha](r)$.

It remains to show that $(\mathcal{F}_{lab},\mathcal{R}_{lab} \cup dec(\mathcal{F},\succ))$ is precedence terminating with respect to $\sqsupset$. Let $l \rightarrow r$ be a rewrite rule in $\mathcal{R}_{lab} \cup dec(\mathcal{F},\succ)$. We distinguish two cases. If $l \rightarrow r \in \mathcal{R}_{lab}$ then there exist an assignment $\alpha\colon \mathcal{V} \rightarrow \mathcal{T}(\mathcal{F}\backslash\{e\},\mathcal{V})$ and a rewrite rule $l' \rightarrow r' \in \mathcal{R}$ such that $l = lab_\alpha(l')$ and $r = lab_\alpha(r')$. The label of $root(l)$ is $[\alpha](l') = \langle l'\rangle\alpha = l'\alpha$. Let ℓ be the label of a function symbol in r. By construction $\ell = [\alpha](t) = \langle t\rangle\alpha$ for some subterm t of r'. According to property (2) above, $\langle t\rangle$ is a subterm of some $r'' \in E_{distr}(r)$. By construction $l' \rightarrow r'' \in E_{distr}(\mathcal{R})$. Hence $l'\alpha \rightarrow_{E_{distr}(\mathcal{R})} r''\alpha \trianglerighteq \ell$. So $root(l) \sqsupset f$ for every $f \in Fun(r)$. If $l \rightarrow r \in dec(\mathcal{F},\succ)$ then

$l = f_s(x_1, \ldots, x_n)$ and $r = f_t(x_1, \ldots, x_n)$ with $f \in \mathcal{F}$ and $s \succ t$. In this case we clearly have $root(l) = f_s \sqsupset f_t$. $\square$

The only creative element in the above proof is the choice of $e_{\mathcal{A}}$. The rest is a routine verification of the two proof obligations of self-labelling.

5. Distribution Elimination Revisited

In the proof of Theorem 4.4 we saw that we can take any projection function as semantics for e. This freedom makes it possible to improve distribution elimination (in the absence of distribution rules) by reducing the number of rewrite rules in $E_{distr}(\mathcal{R})$ while preserving correctness of the transformation.

What are the essential properties of E_{distr} that make the proof of Theorem 4.4 work? A careful inspection reveals, apart from the obvious termination requirement for $E_{distr}(\mathcal{R})$, the following two properties:
(1) $\langle t \rangle \in E_{distr}(t)$, and
(2) if $s \trianglelefteq t$ then there exists a term $t' \in E_{distr}(t)$ such that $\langle s \rangle \trianglelefteq t'$,
for every $t \in \mathcal{T}(\mathcal{F}, \mathcal{V})$. Below we define a new transformation E^{π}_{distr} that satisfies these two properties. The transformation is parameterized by the argument positions π of the function symbol e. The definition relies on the $\mathcal{F}$-algebra defined in the proof of Theorem 4.4 in that we use $\langle t \rangle$.

DEFINITION 5.1. Let $(\mathcal{F}, \mathcal{R})$ be a TRS and let $e \in \mathcal{F}$ be a function symbol whose arity is at least one. Fix $\pi \in \{1, \ldots, arity(e)\}$. We inductively define mappings ϕ and E^{π}_{distr} that assigns to every term in $\mathcal{T}(\mathcal{F}, \mathcal{V})$ a subset of $\mathcal{T}(\mathcal{F} \backslash \{e\}, \mathcal{V})$, as follows:

$$\phi(t) = \begin{cases} \varnothing & \text{if } t \in \mathcal{V}, \\ \phi(t_\pi) \cup \bigcup\limits_{i \neq \pi(f)} E^{\pi}_{distr}(t_i) & \text{if } t = e(t_1, \ldots, t_n), \\ \bigcup\limits_{i=1}^{n} \phi(t_i) & \text{if } t = f(t_1, \ldots, t_n) \text{ with } f \neq e, \end{cases}$$

$$E^{\pi}_{distr}(t) = \phi(t) \cup \{\langle t \rangle\}.$$

We extend the mapping E^{π}_{distr} to $\mathcal{R}$ as follows:

$$E^{\pi}_{distr}(\mathcal{R}) = \{l \to r' \mid l \to r \in \mathcal{R} \text{ is not a distribution rule for } e \text{ and } r' \in E^{\pi}_{distr}(r)\}.$$

Figure 2 shows the effect of E^{1}_{distr} and E^{2}_{distr} on the term t of Figure 1. Observe that each numbered context occurs exactly once in each set. The following lemma states that E^{π}_{distr} has the two required properties.

LEMMA 5.2. *Let $(\mathcal{F}, \mathcal{R})$ be a TRS and let e and π be as above. For every $t \in \mathcal{T}(\mathcal{F}, \mathcal{V})$ we have*

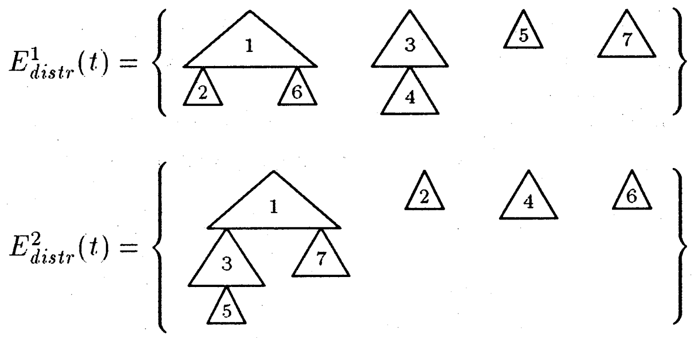

FIGURE 2.

(1) $\langle t \rangle \in E^{\pi}_{distr}(t)$, *and*
(2) *if* $s \trianglelefteq t$ *then there exists a term* $t' \in E^{\pi}_{distr}(t)$ *such that* $\langle s \rangle \trianglelefteq t'$.

PROOF. The first statement holds by definition. The second statement we prove by induction on the structure of $t \in \mathcal{T}(\mathcal{F}, \mathcal{V})$. If $s = t$ then the result follows from the first statement. Hence we may assume that $s \lhd t$. This is only possible if t is a non-variable term $f(t_1, \ldots, t_n)$. There exists a $k \in \{1, \ldots, n\}$ such that $s \trianglelefteq t_k$. The induction hypothesis yields a term $t'_k \in E^{\pi}_{distr}(t_k) = \phi(t_k) \cup \{\langle t_k \rangle\}$ such that $\langle s \rangle \trianglelefteq t'_k$. We distinguish two cases. Suppose $f = e$. In this case we have

$$E^{\pi}_{distr}(t) = \phi(t) \cup \{\langle t \rangle\} = \phi(t_\pi) \cup \{\langle t \rangle\} \cup \bigcup_{i \neq \pi} E^{\pi}_{distr}(t_i).$$

If $k = \pi$ then $t'_k \in \phi(t_\pi) \cup \{\langle t_k \rangle\} = \phi(t_\pi) \cup \{\langle t \rangle\} \subseteq E^{\pi}_{distr}(t)$. If $k \neq \pi$ then $t'_k \in E^{\pi}_{distr}(t_k) \subseteq E^{\pi}_{distr}(t)$. Hence in both cases we can take $t' = t'_k$. Suppose $f \neq e$. We have

$$E^{\pi}_{distr}(t) = \phi(t) \cup \{\langle t \rangle\} = \bigcup_{i=1}^{n} \phi(t_i) \cup \{f(\langle t_1 \rangle, \ldots, \langle t_n \rangle)\}.$$

If $t'_k \in \phi(t_k)$ then clearly $t'_k \in E^{\pi}_{distr}(t)$ and hence we can take $t' = t'_k$. If $t'_k = \langle t_k \rangle$ then we take $t' = f(\langle t_1 \rangle, \ldots, \langle t_n \rangle)$ which satisfies $\langle s \rangle \trianglelefteq t'_k \trianglelefteq t'$. □

Hence we obtain the following result along the lines of the proof of Theorem 4.4.

THEOREM 5.3. *Let* $(\mathcal{F}, \mathcal{R})$ *be a TRS and let* $e \in \mathcal{F}$ *be a non-constant symbol which does not occur in the left-hand sides of rewrite rules of* $\mathcal{R}$. *If* $E^{\pi}_{distr}(\mathcal{R})$ *is terminating for some* $\pi \in \{1, \ldots, arity(e)\}$ *then* $\mathcal{R}$ *is terminating.* □

EXAMPLE 5.4. Consider the TRS $\mathcal{R} = \{f(a) \to f(e(a, b))\}$. Distribution elimination results in the non-terminating TRS

$$E_{distr}(\mathcal{R}) = \left\{ \begin{array}{lcl} f(a) & \to & f(a) \\ f(a) & \to & f(b) \end{array} \right\}.$$

The termination of the TRS

$$E^2_{distr}(\mathcal{R}) = \left\{ \begin{array}{lcl} f(a) & \to & f(b) \\ f(a) & \to & a \end{array} \right\}$$

can be verified using, e.g., the recursive path order with precedence $a \succ b$. Hence termination of $\mathcal{R}$ follows from Theorem 5.3. Observe that $E^1_{distr}(\mathcal{R})$ fails to be terminating.

An obvious question is whether E^π_{distr} works in combination with distribution rules, i.e., does Theorem 4.1 hold for E^π_{distr}? The following example shows that the answer is negative.

EXAMPLE 5.5. Consider the non-terminating TRS

$$\mathcal{R} = \left\{ \begin{array}{rcl} f(a,b) & \to & f(e(a,b),e(a,b)) \\ f(e(x,y),z) & \to & e(f(x,z),f(y,z)) \\ f(x,e(y,z)) & \to & e(f(x,y),f(x,z)) \end{array} \right\}.$$

The TRS $E^\pi_{distr}(\mathcal{R})$ is right-linear and (simply and totally) terminating for both choices of π. For instance,

$$E^1_{distr}(\mathcal{R}) = \left\{ \begin{array}{lcl} f(a,b) & \to & f(a,a) \\ f(a,b) & \to & b \end{array} \right\}.$$

A natural question to ask is whether we need the assumption in Theorems 4.4 and 5.3 that e does not occur in the left-hand sides of the rewrite rules in $\mathcal{R}$. In the proof of Theorem 4.4 this assumption is only used to conclude that $\langle l \rangle = l$ (where l is the left-hand side of a rewrite rule in $\mathcal{R}$). We need this identity because the left-hand sides of rewrite rules in $E_{distr}(\mathcal{R})$ and $E^\pi_{distr}(\mathcal{R})$ are of the form l rather than $\langle l \rangle$. This implies that we can completely remove the restriction that e does not occur in the left-hand sides of rules in $\mathcal{R}$, provided we change $E^\pi_{distr}(\mathcal{R})$ accordingly:

$$E^\pi_{distr}(\mathcal{R}) = \{\langle l \rangle \to r' \mid l \to r \in \mathcal{R} \text{ and } r' \in E^\pi_{distr}(r)\}.$$

This extension is useful since it enables us to conclude the termination of a non-simply terminating TRS like $\mathcal{R} = \{f(e(a,b),a) \to f(e(a,b),e(a,b))\}$ by transforming it into the TRS

$$E^2_{distr}(\mathcal{R}) = \left\{ \begin{array}{lcl} f(b,a) & \to & f(b,b) \\ f(b,a) & \to & a \end{array} \right\}$$

whose termination can be verified using, e.g., the recursive path order with precedence $b \succ a$.

6. Dummy Elimination

In this section we show that the recent dummy elimination technique of Ferreira and Zantema [3] is also amenable to a self-labelling treatment. Let $(\mathcal{F},\mathcal{R})$ be a TRS and let $e \in \mathcal{F}$ be a designated function symbol. Dummy elimination transforms $(\mathcal{F},\mathcal{R})$ into a TRS $E_{dummy}(\mathcal{R})$ over the signature $\mathcal{F}_\diamond = (\mathcal{F} - \{e\}) \cup \{\diamond\}$. Here $\diamond$ is a fresh constant. First we inductively define a mapping *cap* that assigns to every term in $\mathcal{T}(\mathcal{F},\mathcal{V})$ a term in $\mathcal{T}(\mathcal{F}_\diamond,\mathcal{V})$, as follows:

$$cap(t) = \left\{ \begin{array}{ll} t & \text{if } t \in \mathcal{V}, \\ \diamond & \text{if } t = e(t_1,\ldots,t_n), \\ f(cap(t_1),\ldots,cap(t_n)) & \text{if } t = f(t_1,\ldots,t_n) \text{ with } f \neq e. \end{array} \right.$$

Next we associate with every term t in $\mathcal{T}(\mathcal{F},\mathcal{V})$ subsets $\psi(t)$ and $E_{dummy}(t)$ of $\mathcal{T}(\mathcal{F},\mathcal{V})$:

$$\psi(t) = \begin{cases} \varnothing & \text{if } t \in \mathcal{V}, \\ \bigcup_{i=1}^{n} E_{dummy}(t_i) & \text{if } t = e(t_1,\ldots,t_n), \\ \bigcup_{i=1}^{n} \psi(t_i) & \text{if } t = f(t_1,\ldots,t_n) \text{ with } f \neq e, \end{cases}$$

$$E_{dummy}(t) = \psi(t) \cup \{cap(t)\}.$$

The mapping E_{dummy} is extended to $\mathcal{R}$ by defining

$$E_{dummy}(\mathcal{R}) = \{l \to r' \mid l \to r \in \mathcal{R} \text{ and } r' \in E_{dummy}(r)\}.$$

Note the similarity between the mappings ϕ and E^{π}_{distr} of Definition 5.1 and the mappings ψ and E_{dummy}. Figure 3 shows the effect of E_{dummy} on the term t of Figure 1. Observe that $E_{dummy}(t)$ shares with $E^{\pi}_{distr}(t)$ the characteristic that each numbered contexts occurs exactly once.

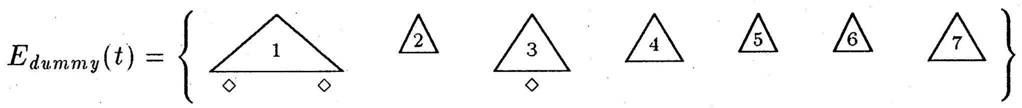

FIGURE 3.

The main result of Ferreira and Zantema [3] states that dummy elimination is a correct transformation technique for establishing termination.

THEOREM 6.1. *Let $(\mathcal{F},\mathcal{R})$ be a TRS and let $e \in \mathcal{F}$ be a non-constant symbol which does not occur in the left-hand sides of rewrite rules of $\mathcal{R}$. If $E_{dummy}(\mathcal{R})$ is terminating then $\mathcal{R}$ is terminating.* □

It is easy to prove this result along the lines of the proof of Theorem 4.4, because the two key properties identified earlier hold for E_{dummy} as well, i.e., for all $t \in \mathcal{T}(\mathcal{F},\mathcal{V})$:
(1) $cap(t) \in E_{dummy}(t)$, and
(2) if $s \trianglelefteq t$ then there exists a term $t' \in E_{dummy}(t)$ such that $cap(s) \trianglelefteq t'$.
The first property holds by definition and the second property is easily proved by induction. Observe that $cap(t) = \langle t \rangle$ in the term algebra $\mathcal{T}(\mathcal{F}\backslash\{e\},\mathcal{V})$ augmented with the operation

$$e_{\mathcal{A}}(t_1,\ldots,t_n) = \diamond$$

for all $t_1,\ldots,t_n \in \mathcal{T}(\mathcal{F}\backslash\{e\},\mathcal{V})$.

It is possible to strengthen Theorem 6.1 by dropping the restriction that e does not occur in the left-hand sides of rewrite rules in $\mathcal{R}$: simply replace every left-hand side l of a rule in $E_{dummy}(\mathcal{R})$ by $cap(l)$. This enables us to conclude the termination of a non-simply terminating TRS like $\mathcal{R} = \{f(e(a),b,x) \to f(e(x),e(x),e(x))\}$ by transforming it into the TRS

$$E_{dummy}(\mathcal{R}) = \left\{ \begin{array}{lcl} f(\diamond,b,x) & \to & f(\diamond,\diamond,\diamond) \\ f(\diamond,b,x) & \to & x \end{array} \right\}$$

whose termination can be verified using any standard technique.

A thorough investigation of the relative strength of (variants of) distribution elimination and dummy elimination will be detailed elsewhere. Here we only remark that for left-linear TRSs dummy elimination is to be preferred above distribution elimination.

7. Currying

In this final section we show that the main result of Kennaway, Klop, Sleep, and De Vries [4]—the preservation of termination under *currying*—is easily proved by self-labelling. Currying is the transformation on TRSs defined below.

DEFINITION 7.1. With every TRS $(\mathcal{F}, \mathcal{R})$ we associate a TRS $(\mathcal{F}_{@}, \mathcal{R}_{@})$ as follows: the signature $\mathcal{F}_{@}$ contains all function symbols of $\mathcal{F}$ together with

- function symbols f_i of arity i for every $f \in \mathcal{F}$ of arity n with $0 \leqslant i < n$,
- a binary function symbol @, called *application*,

and $\mathcal{R}_{@}$ is the extension of $\mathcal{R}$ with all rewrite rules

$$@(f_i(x_1, \ldots, x_i), y) \quad \to \quad f_{i+1}(x_1, \ldots, x_i, y)$$

with $f \in \mathcal{F}$ of arity $n \geqslant 1$ and $0 \leqslant i < n$. Here $x_1, \ldots, x_i, y$ are pairwise different variables and f_{i+1} denotes f if $i + 1 = n$.

Clearly termination of $\mathcal{R}_{@}$ implies termination of $\mathcal{R}$.

THEOREM 7.2 (Kennaway *et al.* [4]). *If $\mathcal{R}$ is a terminating TRS then $\mathcal{R}_{@}$ is terminating.* $\square$

The proof in [4] is rather involved. We present a self-labelling proof.

PROOF. Let $\mathcal{F}' = \mathcal{F}_{@} \backslash \{@\}$. Using the well-known fact that termination is preserved under signature extension—this follows e.g. from modularity considerations, see [6]—we infer the termination of the TRS $(\mathcal{F}', \mathcal{R})$. So the question is how termination of $(\mathcal{F}_{@}, \mathcal{R}_{@})$ follows from termination of $(\mathcal{F}', \mathcal{R})$. We turn $\mathcal{T}(\mathcal{F}', \mathcal{V})$ into an $\mathcal{F}_{@}$-algebra $\mathcal{A}$ by defining $@_{\mathcal{A}}(s,t)$ by induction on the structure of s, as follows:

$$@_{\mathcal{A}}(s,t) = \begin{cases} t & \text{if } s \in \mathcal{V}, \\ f_{i+1}(s_1, \ldots, s_i, t) & \text{if } s = f_i(s_1, \ldots, s_i) \text{ with } i < \mathit{arity}(f), \\ f(@_{\mathcal{A}}(s_1,t), \ldots, @_{\mathcal{A}}(s_n,t)) & \text{if } s = f(s_1, \ldots, s_n). \end{cases}$$

As well-founded order on $\mathcal{T}(\mathcal{F}', \mathcal{V})$ we take $\to_{\mathcal{R}}^{+}$. We equip the labelled signature $(\mathcal{F}_{@})_{lab}$ with the well-founded order $\sqsupset$ defined as follows: $f_s \sqsupset g_t$ if and only if

- $s\ (\to_{\mathcal{R}} \cup \rhd)^{+}\ t$ and either $f, g \neq @$ or $f, g = @$, or
- $f = @$ and $g \neq @$.

It is easy to see that $\sqsupset$ is indeed a well-founded order. We have to show that $(\mathcal{A}, \rightarrow_{\mathcal{R}}^{+})$ is a quasi-model for $(\mathcal{F}_{@}, \mathcal{R}_{@})$ and that the TRS $(\mathcal{R}_{@})_{lab} \cup dec(\mathcal{F}_{@}, \rightarrow_{\mathcal{R}}^{+})$ is precedence terminating with respect to $\sqsupset$.

First we show that $(\mathcal{A}, \rightarrow_{\mathcal{R}}^{+})$ is a quasi-model for $(\mathcal{F}_{@}, \mathcal{R}_{@})$. We claim that every algebra operation is strictly monotone in all its coordinates. Here we consider only the first coordinate of $@_{\mathcal{A}}$, which is the most interesting case. Before proceeding we mention the following fact, which is easily proved by induction on the structure of s:

$$\text{if } s \in \mathcal{T}(\mathcal{F}, \mathcal{V}),\ t \in \mathcal{T}(\mathcal{F}', \mathcal{V}), \text{ and } \sigma \in \Sigma(\mathcal{F}', \mathcal{V}) \text{ then } @_{\mathcal{A}}(s\sigma, t) = s\, @_{\mathcal{A}}(\sigma, t).$$

Here the substitution $@_{\mathcal{A}}(\sigma,t)$ is defined as the mapping that assigns to every variable x the term $@_{\mathcal{A}}(x\sigma,t)$. We show that $@_{\mathcal{A}}(s,t) \rightarrow_{\mathcal{R}} @_{\mathcal{A}}(u,t)$ whenever $s,t,u \in \mathcal{T}(\mathcal{F}',\mathcal{V})$ with $s \rightarrow_{\mathcal{R}} u$ by induction on the structure of s. Strict monotonicity of $@_{\mathcal{A}}$ in its first coordinate follows from this by an obvious induction argument. Since s cannot be a variable, we have either $s = f_i(s_1,\dots,s_i)$ with $i < arity(f)$ or $s = f(s_1,\dots,s_n)$. In the former case we have $@_{\mathcal{A}}(s,t) = f_{i+1}(s_1,\dots,s_i,t)$. Moreover, as s is root-stable, u must be of the form $f_i(s_1,\dots,u_j,\dots,s_i)$ with $s_j \rightarrow_{\mathcal{R}} u_j$. Hence $@_{\mathcal{A}}(s,t) \rightarrow_{\mathcal{R}} f_{i+1}(s_1,\dots,u_j,\dots,s_i,t) = @_{\mathcal{A}}(u,t)$. Suppose $s = f(s_1,\dots,s_n)$. If the rewrite step from s to u takes place in one of the arguments of s then $v = f(s_1,\dots,u_j,\dots,s_n)$ with $s_j \rightarrow_{\mathcal{R}} u_j$. From the induction hypothesis we obtain $@_{\mathcal{A}}(s_j,t) \rightarrow_{\mathcal{R}} @_{\mathcal{A}}(u_j,t)$ and therefore $@_{\mathcal{A}}(s,t) = f(@_{\mathcal{A}}(s_1,t),\dots,@_{\mathcal{A}}(s_j,t),\dots,@_{\mathcal{A}}(s_n,t)) \rightarrow_{\mathcal{R}} f(@_{\mathcal{A}}(s_1,t),\dots,@_{\mathcal{A}}(u_j,t),\dots,@_{\mathcal{A}}(s_n,t)) = @_{\mathcal{A}}(u,t)$. If the rewrite step from s to u takes place at the root of s then $s = l\sigma$ and $u = r\sigma$ for some rewrite rule $l \rightarrow r \in \mathcal{R}$ and substitution $\sigma \in \Sigma(\mathcal{F}',\mathcal{V})$. Because l and r do not contain function symbols from $\mathcal{F}' \backslash \mathcal{F}$, we obtain $@_{\mathcal{A}}(s,t) = l\, @_{\mathcal{A}}(\sigma,t)$ and $@_{\mathcal{A}}(u,t) = r\, @_{\mathcal{A}}(\sigma,t)$ from the above fact. Therefore also in this case we have $@_{\mathcal{A}}(s,t) \rightarrow_{\mathcal{R}} @_{\mathcal{A}}(u,t)$. In order to conclude that $(\mathcal{A}, \rightarrow_{\mathcal{R}}^{+})$ is a quasi-model for $(\mathcal{F}_{@}, \mathcal{R}_{@})$, it remains to show that $[\alpha](l) \rightarrow_{\mathcal{R}}^{*} [\alpha](r)$ for every rewrite rule $l \rightarrow r \in \mathcal{R}_{@}$ and assignment α from $\mathcal{V}$ to $\mathcal{T}(\mathcal{F}',\mathcal{V})$. If $l \rightarrow r \in \mathcal{R}$ then $[\alpha](l) = l\alpha \rightarrow_{\mathcal{R}} r\alpha = [\alpha](r)$. Otherwise $l = @(f_i(x_1,\dots,x_i),y)$ and $r = f_{i+1}(x_1,\dots,x_i,y)$ for some $f \in \mathcal{F}$ and $i < arity(f)$, in which case we have $[\alpha](l) = f_{i+1}(x_1,\dots,x_i,y)\alpha = [\alpha](r)$ by definition.

To conclude our proof we show that $(\mathcal{R}_{@})_{lab} \cup dec(\mathcal{F}_{@}, \rightarrow_{\mathcal{R}}^{+})$ is precedence terminating with respect to $\sqsupset$. Clearly $(\mathcal{R}_{@})_{lab} = \mathcal{R}_{lab} \cup (\mathcal{R}_{@} \backslash \mathcal{R})_{lab}$. The rewrite rules in $\mathcal{R}_{lab} \cup dec(\mathcal{F}_{@}, \rightarrow_{\mathcal{R}}^{+})$ are taken care of by the first clause of the definition of $\sqsupset$, just as in the proof of Theorem 3.3. For the rules in $(\mathcal{R}_{@} \backslash \mathcal{R})_{lab}$ we use the second clause. $\square$

The reader is invited to compare our proof with the one of Kennaway *et al.* [4].

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