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Interaction of an algebraic soliton with uneven bottom topography in a two-layer fluid system

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Abstract

The propagation of long interfacial waves of finite amplitude is investigated in a two-layer fluid system where a layer of a light fluid overlies a layer of heavier one resting on an uneven bottom. The upstream condition is imposed such that the constant flow is in the same direction in both layers. Under appropriate balance among nonlinearity, dispersion and topographic effect, the long term evolution of the interfacial elevation is shown to be governed by a forced Benjamin-Ono (fBO) equation. A direct soliton perturbation theory is applied to the fBO equation to study the interaction of an algebraic soliton with a bottom topography when the Froude number is nearly equal to unity. A system of ordinary differential equations describing the change of the amplitude and position of the soliton is derived for a simple bottom profile. The solutions of these equations exhibit a variety of phenomena like the capture and repulsion of the soliton by topography and the occurrence of soliton-like phase shifts due to the interaction of the soliton with topography. We also examine the effect of small dissipation on the dynamics of the soliton. Special emphasis is given to the appearance of new branch of stationary states of the soliton which has never been observed in the absence of dissipation.

I. INTRODUCTION

There has been much interest in studying the effect of external forces on the propagation of nonlinear waves. In the context of fluid dynamics, the forcing is mainly due to a moving pressure distribution and/or a bottom topography. Many authors have been concerned with a variety of phenomena arising from the forced systems such as the generation of solitary waves by a moving pressure force [1-4], or by uniform flow over a localized bottom topography [5-7]. These problems have been elucidated extensively from both analytical and numerical points of view on the basis of the Korteweg-de Vries (KdV) equation with forcing terms which is called the forced KdV (fKdV) equation. It is now well-known that the fKdV equation is derived from the underlying system of equations on the assumptions of shallow water and weak nonlinearity together with weak topographic effect. At this stage, a natural question arises: what kind of nonlinear evolution equations are obtained when we introduce deep-water approximation in place of shallow-water approximation. This is a motivation of the present study.

To answer this question, we shall consider a simple two-layer fluid system where a layer of a light fluid overlies a layer of heavier one resting on an uneven bottom. The depth of the upper layer is taken to be infinite corresponding to a deep-water approximation. The upstream condition is imposed such that the constant flow is in the same direction in both layers. We consider the two-dimensional irrotational flow of an incompressible and inviscid fluid. Applying a singular perturbation method [8-10] developed recently to this system, we show that the long term evolution of the interfacial elevation is governed by a forced Benjamin-Ono (fBO) equation. In the present case, the forcing term comes from an uneven bottom topography. We assume that the effect of topography is weak so that it can be regarded as a small perturbation to the BO equation. It is noteworthy that a fKdV equation is derived similarly from the same system if we make the assumption of shallow water in the upper layer [7].

While a number of problems to be resolved by the fBO equation are at hand, we shall be concerned here with an analytical study of the interaction process of an algebraic soliton with the bottom topography. For the purpose, we employ a direct soliton perturbation theory [11,12] developed quite recently to examine the dynamics of interacting algebraic solitons under small dispersive or dissipative perturbations. The present analysis predicts the phenomena like the capture and repulsion of a soliton by topography and the occurrence of soliton-like phase shifts. We also discuss the effect of small dissipation on the propagation of the soliton. Although similar problems have been studied extensively using fKdV equations with various types of forcing terms [13-16], the analysis based on a fBO equation has not been performed as yet.

In Sec. II, we first derive a finite-depth analog of the fBO equation starting from the basic system of hydrodynamic equations. The fBO equation is then obtained simply by taking the depth of the upper layer infinite. In Sec. III, the interaction of an algebraic soliton with topography is investigated on the basis of the fBO equation. We focus our attention on the situation when the Froude number is nearly equal to unity. In this problem, the forcing term is regarded as a small perturbation to the BO equation so that the amplitude and initial position of the soliton are no more constant and become slowly varying functions of time. A system of ordinary differential equations describing the change of these soliton parameters is derived by using the soliton perturbation theory. For a simple bottom profile, the system is shown to be completely integrable. The analytical solutions thus obtained are investigated in detail in phase plane to show that they exhibit the interesting phenomena already mentioned above. Also we briefly discuss the effect of small dissipation on the dynamics of the soliton. To simplify the analysis, we add the Burgers term to the fBO equation which is proportional to the second derivative of the wave profile. It is found that the characteristics of the stationary states of the soliton are changed drastically owing to the dissipation. Section IV is devoted to summary and outlook.

II. DERIVATION OF FORCED BENJAMIN-ONO EQUATION

A. Basic hydrodynamic equations

In this section, we shall derive the fBO equation starting from the hydrodynamic equations subject to appropriate boundary conditions. The configuration of the two-layer fluid system under consideration is shown in Fig. 1.

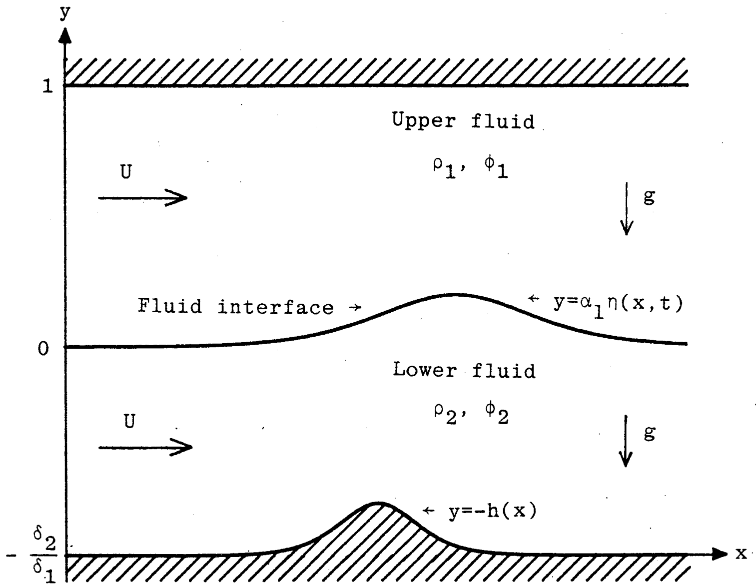


FIG.1. Configuration of a two-layer fluid system

All the physical quantities are written in terms of the dimensionless variables. The dimensional quantities, with tildes, are related to the corresponding dimensionless ones by the relations

$$\tilde{x} = lx,\ \tilde{y} = h_1 y,\ \tilde{t} = (l/c_0)t,\ \tilde{\phi}_j = (gla/c_0)\phi_j\ (j = 1,2),\ \tilde{\eta} = a\eta,\ \tilde{h} = h_1 h, \tag{2.1}$$

and the dimensionless parameters are defined by

$$\epsilon = \frac{a}{l},\ \alpha_j = \frac{a}{h_j}\ (j = 1,2),\ \delta_j = \frac{h_j}{l}\ (j = 1,2),\ F = \frac{U}{c_0},\ \Delta = \frac{\rho_1}{\rho_2}, \tag{2.2}$$

where l, a and c_0 are characteristic scales of length, amplitude and velocity of the wave, respectively, h_1 is the thickness of the upper layer. The parameters α_j and δ_j measure the magnitude of nonlinearity and dispersion of the wave, respectively and $\epsilon(= \alpha_j\delta_j)$ is the steepness parameter. We assume that ϵ, $\alpha_j (j = 1,2)$ and δ_2 are small compared to unity while δ_1 is very large, which is equivalent to introducing shallow-water approximation in the lower fluid and deep-water approximation in the upper fluid in addition to weak nonlinearity of the wave. The lower boundary is represented by the equation

$$y = -h(x) = -\frac{\delta_2}{\delta_1}\left[1 - \gamma\alpha_2^2 B(x)\right], \tag{2.3}$$

where $B(x)$ characterizes the unevenness of the bottom topography and γ is a positive constant. In what follows, we assume that its profile is sufficiently smooth, localized and hence it vanishes at infinity. When $\mid x \mid \rightarrow \infty$, h tends to δ_2/δ_1, implying that h_2 is the thickness of the lower layer at infinity because it has been scaled by the thickness of the upper layer h_1. The coefficient $\gamma\alpha_2^2$ measures the magnitude of $B(x)$. While this can be taken arbitrary, we must impose for it appropriate smallness to lead correctly to the fBO equation, as will be shown below. The basic flow is specified at $x = -\infty$ and it is supposed to be in the positive x-direction with a constant velocity U in both layers. The external force is only due to the gravitational acceleration g and the effect of surface tension is neglected. A characteristic velocity of the linear wave is given by $c_0 = \sqrt{gl/\kappa}$ where κ is a parameter depending on Δ and δ_j $(j = 1, 2)$. Here Δ is the density ratio which is assumed to be less than unity to assure a stable equilibrium state of fluids. The Froude number F is a very important parameter characterizing the property of the flow. Its magnitude will be specified in the analysis developed in Sec. III.

Now, the dimensionless governing equations for inviscid incompressible and irrotational flow can be written as follows:
(i) *continuity equations*

$$\delta_1^2\phi_{1,xx} + \phi_{1,yy} = 0 \ (-\infty < x < \infty, \alpha_1\eta < y < 1), \tag{2.4a}$$

$$\delta_1^2\phi_{2,xx} + \phi_{2,yy} = 0 \ (-\infty < x < \infty, -h(x) < y < \alpha_1\eta). \tag{2.4b}$$

(ii) *kinematic boundary conditions at the fluid interface*

$$\eta_t + F\eta_x + \kappa\epsilon\phi_{1,x}\eta_x = \frac{\kappa}{\delta_1}\phi_{1,y} \ (y = \alpha_1\eta), \tag{2.5a}$$

$$\eta_t + F\eta_x + \kappa\epsilon\phi_{2,x}\eta_x = \frac{\kappa}{\delta_1}\phi_{2,y} \ (y = \alpha_1\eta). \tag{2.5b}$$

(iii) *dynamic boundary condition at the fluid interface*

$$\Delta\left[\phi_{1,t} + \frac{\kappa\epsilon}{2\delta_1^2}\{\delta_1^2(\frac{F}{\kappa\epsilon} + \phi_{1,x})^2 + \phi_{1,y}^2\} + \eta - \eta_0\right]$$

$$= \phi_{2,t} + \frac{\kappa\epsilon}{2\delta_1^2}\{\delta_1^2(\frac{F}{\kappa\epsilon} + \phi_{2,x})^2 + \phi_{2,y}^2\} + \eta - \eta_0 \ (y = \alpha_1\eta). \tag{2.6}$$

(iv) *upper and lower boundary conditions*

$$\phi_{1,y} = 0 \ (y = 1), \tag{2.7a}$$

$$\phi_{2,y} = -\delta_1^2(\frac{F}{\kappa\epsilon} + \phi_{2,x})h_x \ (y = -h(x)). \tag{2.7b}$$

Here, $\phi_1 = \phi_1(x, y, t)$ and $\phi_2 = \phi_2(x, y, t)$ are the velocity potentials for the upper and lower fluids, respectively about the perturbations of the upstream constant flow, $\eta = \eta(x, t)$ is the interfacial elevation and η_0 is a constant determined by the undisturbed uniform state at infinity. The subscripts t, x and y appended to ϕ, η and h denote partial differentiations. This notation is used throughout the paper.

B. Solutions for the velocity potentials

The solutions of the Laplace equations (2.4) for the velocity potentials which satisfy the boundary conditions (2.7) are constructed by following the procedure developed recently [8-10]. For the upper fluid, it can be written in a closed form as

$$\phi_1(x,y,t) = -i[f_1^+(x+i\delta_1 y,t) - f_1^-(x-i\delta_1 y,t)], \tag{2.8a}$$

with the complex functions $f_1^{\pm}$ given by

$$f_1^{\pm}(z,t) = \pm\frac{1}{4i\delta_1}P\int_{-\infty}^{\infty}\coth[\pi(y-z)/2\delta_1]f_1(y,t)dy \tag{2.8b}$$

where f_1 is a real function defined appropriately on the real axis and the symbol P denotes the Cauchy principal value integral.

For the lower fluid, on the other hand, the solution is expressed in a form of infinite series as

$$\phi_2(x,y,t) = \sum_{n=0}^{\infty}[y+h(x)]^n\phi_2^{(n)}(x,t), \tag{2.9a}$$

where the functions $\phi_2^{(n)}$ $(n=0,1,...)$ are determined by the recurrence formula

$$\delta_1^2\left[(n+1)h_{xx}\phi_2^{(n+1)} + (n+1)(n+2)h_x^2\phi_2^{(n+2)} + 2(n+1)h_x\phi_{2,x}^{(n+1)} + \phi_{2,xx}^{(n)}\right]$$

$$+(n+1)(n+2)\phi_2^{(n+2)} = 0, \ (n=0,1,...), \tag{2.9b}$$

subject to the boundary condition (2.7b). This can be solved easily and the first two of which are written in the form

$$\phi_2^{(1)} = -\frac{\delta_1^2\left(\frac{F}{\kappa\epsilon} + \phi_{2,x}^{(0)}\right)h_x}{1+\delta_1^2h_x^2}, \tag{2.10a}$$

$$\phi_2^{(2)} = -\frac{\delta_1^2\left(h_{xx}\phi_2^{(1)} + 2h_x\phi_{2,x}^{(1)} + \phi_{2,xx}^{(0)}\right)}{2(1+\delta_1^2h_x^2)}. \tag{2.10b}$$

Here $\phi_2^{(0)}$ is an unknown function to be determined later.

C. Perturbation analysis

To perform the perturbation analysis, we must specify the magnitude of the parameters α_j and δ_j. We assume the weak nonlinearity of the wave, i.e. $\alpha_j << 1$ $(j=1,2)$ as well as the shallow-water approximation for the lower layer, i.e. $\delta_2 << 1$. For the upper layer, however, we tentatively suppose $\delta_1 = O(1)$ for the sake of generality. Furthermore, it is necessary to balance the magnitude of nonlinearity, dispersion and topography appropriately to obtain forced soliton equations. In the present case, this can be accomplished by introducing the scalings $\alpha_2 = O(\delta_2)$ and $\gamma = O(1)$.

These assumptions enable us to expand all the physical quantities in powers of the small parameters α_2 and δ_2. In particular, the derivatives of the velocity potentials are easily evaluated at the fluid interface. To that end, it is convenient to introduce the horizontal components of the interfacial velocities by

$$u_j = \phi_{j,x}\,|_{y=\alpha_1\eta}, \quad (j=1,2). \tag{2.11}$$

In addition, we take $\kappa = [(1-\Delta)\delta_2]^{-1}$ to normalize the phase velocity of the linear wave to unity since the linear dispersion relation relevant to the present problem is given by $\omega^2 = \kappa(1-\Delta)\delta_2 k^2$. If we retain the first two terms of the expansions, we obtain for the upper fluid

$$\phi_{1,y}\,|_{y=\alpha_1\eta} = -\delta_1\left[\tilde{T}_1 u_1 + \epsilon\{\eta u_{1,x} + \tilde{T}_1(\eta \tilde{T}_1 u_{1,x})\} + O(\epsilon^2)\right], \tag{2.12a}$$

$$(\phi_{1,t}\,|_{y=\alpha_1\eta})_x = u_{1,t} + \epsilon(\eta_t \tilde{T}_1 u_{1,x} - \eta_x \tilde{T}_1 u_{1,t}) + O(\epsilon^2), \tag{2.12b}$$

where $\tilde{T}_1$ is an integral operator defined by

$$\tilde{T}_1 f(x,t) = -\frac{1}{2\delta_1} P\int_{-\infty}^{\infty} \frac{f(y,t)}{\sinh[\pi(y-x)/2\delta_1]}dy. \tag{2.12c}$$

For the lower fluid, on the other hand, the corresponding expressions take the form

$$\phi_{2,y}\,|_{y=\alpha_1\eta} = -\delta_1\delta_2\left[u_{2,x} + \alpha_2\{\eta u_{2,x} - \gamma(1-\Delta)FB_x\} + O(\alpha_2^2)\right], \tag{2.13a}$$

$$(\phi_{2,t}\,|_{y=\alpha_1\eta})_x = u_{2,t} + O(\alpha_2^2). \tag{2.13b}$$

Finally, substituting (2.12) and (2.13) into (2.5) and (2.6) and neglecting the terms of the order of $(\alpha_2^2, \alpha_2\delta_2, \delta_2^2)$, we obtain a closed system of equations for η, u_1 and u_2. We quote only the final result:

$$\eta_t + F\eta_x + (1-\Delta)^{-1}u_{2,x} + (1-\Delta)^{-1}\alpha_2(\eta u_2)_x = \gamma\alpha_2 FB_x, \tag{2.14}$$

$$\eta_t + F\eta_x + \frac{1}{(1-\Delta)\delta_2}\tilde{T}_1 u_1 + \frac{\alpha_2}{(1-\Delta)}\left[(u_1\eta)_x + \tilde{T}_1(\eta\tilde{T}_1 u_{1,x})\right] = 0, \tag{2.15}$$

$$\begin{aligned}\Delta\Big[u_{1,t} + Fu_{1,x} + \eta_x + \frac{\alpha_2}{2(1-\Delta)}[u_1^2 + (\tilde{T}_1 u_1)^2]_x\Big]\\ = u_{2,t} + Fu_{2,x} + \eta_x + (1-\Delta)^{-1}\alpha_2 u_2 u_{2,x}.\end{aligned} \tag{2.16}$$

In the above equations, the forcing term appears only on the right-hand side of (2.14) because of the present scaling hypothesis. Although these equations incorporate the lowest-order effect of nonlinearity and dispersion, we can proceed the perturbation analysis to include higher-order terms. However, the resulting equations become too complicated to extract useful informations within the framework of analytical treatment. Nevertheless, it is nothing to say that higher-order equations are necessary particularly in dealing with large amplitude waves.

It is also possible to derive a single equation for η. This can be performed by eliminating u_1 and u_2 from (2.14)-(2.16). In the same approximation used above, the final equation thus obtained can be written as follows:

$$\eta_{tt} - (1 - F^2)\eta_{xx} + 2F\eta_{xt} - \Delta\delta_2 T_1 \eta_{xxx}$$

$$+\alpha_2[\eta_t S\eta_t - (1 + 2F^2)\eta\eta_x - 2F\eta\eta_t + F\eta_x S\eta_t]_x = \gamma\alpha_2 F^2 B_{xx}, \tag{2.17a}$$

where the integral operators T_1 and S are defined by

$$T_1 u(x,t) = \tilde{T}_1^{-1} u(x,t) = \frac{1}{2\delta_1} P \int_{-\infty}^{\infty} \coth[\pi(y-x)/2\delta_1] u(y,t) dy, \tag{2.17b}$$

$$Su(x,t) = \int_{-\infty}^{\infty} \mathrm{sgn}(y-x) u(y,t) dy, \tag{2.17c}$$

While Eq. (2.17) describes interfacial waves propagating to both right and left directions, we can separate it into two equations, each describing a unidirectional motion. Following the standard procedure [17], we find the evolution equations

$$\eta_t + (F-1)\eta_x - \frac{3\alpha_2}{2}\eta\eta_x - \frac{\Delta\delta_2}{2} T_1 \eta_{xx} = \frac{\gamma\alpha_2 F^2}{2} B_x, \tag{2.18}$$

$$\eta_t + (F+1)\eta_x + \frac{3\alpha_2}{2}\eta\eta_x + \frac{\Delta\delta_2}{2} T_1 \eta_{xx} = -\frac{\gamma\alpha_2 F^2}{2} B_x. \tag{2.19}$$

In the absence of the forcing term, the above equations reduce to the intermediate long wave (ILW) equations [10,18-20]. Therefore, they may be termed the forced ILW equations. The fBO equations are simply obtained from (2.18) and (2.19) by taking the deep-water limit $\delta_1 \to \infty$. Since in this limit the operator T_1 reduces to the following Hilbert transform

$$Hu(x,t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{u(y,t)}{y-x} dy, \tag{2.20}$$

Eqs. (2.18) and (2.19) become

$$\eta_t + (F-1)\eta_x - \frac{3\alpha_2}{2}\eta\eta_x - \frac{\Delta\delta_2}{2} H\eta_{xx} = \frac{\gamma\alpha_2 F^2}{2} B_x, \tag{2.21}$$

$$\eta_t + (F+1)\eta_x + \frac{3\alpha_2}{2}\eta\eta_x + \frac{\Delta\delta_2}{2} H\eta_{xx} = -\frac{\gamma\alpha_2 F^2}{2} B_x. \tag{2.22}$$

It is obvious that Eq. (2.22) describes unidirectional motion of the wave moving to the right. However, the situation is quite different for Eq. (2.21). In fact, the direction of propagation will change according to the magnitude of F as well as the amplitude of the wave. Furthermore, the forcing term may affect the propagation characteristics even if it is small when compared to other terms. In view of this observation, Eq. (2.21) is supposed to have a richer structure than Eq. (2.22) has in describing underlying physical phenomena. This is a reason why we use Eq. (2.21) extensively in the next section.

III. INTERACTION OF ALGEBRAIC SOLITON WITH TOPOGRAPHY

A. Statement of the problem

While the fBO equation (2.21) has wide applications in real physical phenomena, we shall study the interaction of an algebraic soliton with topography in the presence of nearly critical flow, namely

$$F = 1 + \alpha_2 \Gamma, \tag{3.1}$$

where Γ is a parameter which measures the departure of the Froude number from unity. Furthermore, we assume that the effect of topography is weak in comparison with other terms. More specifically, we introduce the scaling $\alpha_2 << \gamma << 1$. Under these conditions, the second to fourth terms on the left-hand side of Eq. (2.21) enter at the same order whereas the forcing term becomes higher order and hence it can be treated as a small perturbation to the BO equation. It should be noted that the inequality $\alpha_2 << \gamma$ ensures that the forcing term is still larger than neglected terms of order $(\alpha_2^2, \alpha_2\delta_2, \delta_2^2)$ in deriving Eq. (2.21), which is consistent with the perturbation analysis developed in Sec. II.

To proceed the analysis further, it is convenient to rescale the variables according to $t \to (\Delta\delta_2/\alpha_2^2)t, x \to (\delta_2/\alpha_2)x, \eta \to (8/3)u$ and $B \to (16/3F^2)B$. Then, Eq. (2.21) takes the following form

$$u_t + \Gamma u_x - 4uu_x - Hu_{xx} = \gamma B_x. \tag{3.2}$$

It is well-known that if the forcing term is absent, Eq. (3.2) exhibits soliton solutions of algebraic type [21-23]. The main concern here is to investigate the effect of small perturbation which originates from localized topography on the dynamics of an algebraic soliton. For the purpose, we shall consider a symmetric isolated bottom topography of the form

$$B(x) = \frac{\lambda b}{(bx)^2 + 1}, \ (\lambda = \pm 1), \tag{3.3}$$

where b^{-1} is a typical length scale of topography and λ characterizes the nature of forcing. Although the functional form of B adopted here is rather specific, an advantage is that the resulting motion of a soliton is described by an integrable Hamiltonian system, as will be seen below. The problem under consideration can now be dealt with by means of the soliton perturbation theory, which we shall describe briefly.

B. Soliton perturbation theory

A direct multisoliton perturbation theory for the BO equation has been developed quite recently using multiple time scale expansion [11,12]. We shall summarize the main ingredient of the theory.

We write the perturbed BO equation in the form

$$u_t + 4uu_x + Hu_{xx} = \epsilon R[u], \tag{3.4}$$

where $\epsilon R[u]$ represents a perturbation. We first introduce the different time scales t_j by $t_j = \epsilon^j t$ $(j = 0, 1, ...)$ so that the time derivative is replaced by the relation $\partial/\partial t = \sum_{j=0}^{\infty} \epsilon^j \partial/\partial t_j$. If we expand u into an asymptotic series of the form $u = \sum_{j=0}^{\infty} \epsilon^j u_j$, $u_j = u_j(x, t_0, t_1, ...)$, substitute these expressions into (3.4) and equate the coefficient of like powers of ϵ, we obtain the hierarchy of equations for u_j, the first two members of which read in the form

$$u_{0,t_0} + 4u_0 u_{0,x} + H u_{0,xx} = 0, \tag{3.5}$$

$$u_{1,t_0} + 4(u_0 u_1)_x + H u_{1,xx} = R[u_0] - u_{0,t_1}. \tag{3.6}$$

Equation (3.5) is just the BO equation. We solve the above system of equations successively starting with a specific solution of Eq. (3.5). Since we are interested in the soliton problem, we take it the N-soliton solution which includes the N amplitude parameters a_j and the N position parameters ξ_j $(j = 1, 2, ...)$. In the absence of the perturbation, a_j are constant independent of t_0 and $\xi_j = a_j t_0 + \xi_{j0}$ with ξ_{j0} being the initial position of the jth soliton. Under the action of the perturbation, these parameters would be modulated slowly on the time scale of order ϵ^{-1}, so we may assume their time dependence as $a_j = a_j(t_1, t_2, ...)$ and $\xi_{j0} = \xi_{j0}(t_1, t_2, ...)$, $(j = 1, 2, ..., N)$.

The time evolution of a_j and ξ_{j0} is determined by requiring that the correction term u_1 is no more singular than the leading-order solution u_0. This turns out to be equivalent to imposing the following compatibility conditions

$$(g_j, R[u_0] - u_{0,t_1}) \equiv \int_{-\infty}^{\infty} g_j (R[u_0] - u_{0,t_1}) dx = 0, \ (j = 1, 2, \ldots). \tag{3.7}$$

Here g_j are solutions of the adjoint equation for the homogeneous part of (3.6)

$$g_{j,t_0} + 4u_0 g_{j,x} + H g_{j,xx} = 0, \ (j = 1, 2, ..). \tag{3.8}$$

The $2N$ independent bounded solutions for (3.8) are found to be as

$$g_j = \int_{-\infty}^{x} \frac{\partial u_0}{\partial a_j} dx, \ g_{j+N} = \int_{-\infty}^{x} \frac{\partial u_0}{\partial \xi_{j0}} dx, \ (j = 1, 2, \ldots, N). \tag{3.9a, b}$$

The final step of our perturbation scheme is to calculate the inner products in (3.7) with the aid of certain orthogonality relations which hold between g_j and $\partial u_0/\partial p_j$ where p_j stands for a_j or ξ_{j0} while noting the relation $u_{0,t_1} = \sum_{j=1}^{N} (a_{j,t_1} u_{0,a_j} + \xi_{j,t_1} u_{0,\xi_j})$. These orthogonality relations are derived directly from (3.5) and (3.9) and they are expressed as $(g_i, u_{0,\xi_{j0}}) = -(g_{i+N}, u_{0,a_j}) = (\pi/4)\delta_{ij}, (g_{i+N}, u_{0,\xi_{j0}}) = (g_i, u_{0,a_j}) = 0$ $(i, j = 1, 2, ..., N)$ where δ_{ij} is Kronecker's delta. The evolution equations for the soliton parameters thus obtained are written in terms of the original time variables as follows:

$$\frac{da_j}{dt} = -\frac{4\epsilon}{\pi}(g_{j+N}, \ R[u_0]), \ \frac{d\xi_j}{dt} = a_j + \frac{4\epsilon}{\pi}(g_j, \ R[u_0]), \ (j = 1, 2, \ldots, N), \tag{3.10a, b}$$

where the position parameter ξ_j of the jth soliton has been used instead of ξ_{j0} for practical convenience. It should be emphasized that for the purpose of calculating the inner products in (3.10) we only need the information of the N-soliton solution which can be usually obtained without recourse to inverse scattering transform method [21-23]. In this respect, it is worth remarking that the soliton perturbation theory based on the inverse scattering transform method has not been applied as yet to a certain class of integrodifferential evolution equations like the BO and ILW equations.

C. Perturbation equations for soliton parameters

Now let us derive the perturbation equations for the soliton parameters on the basis of Eq. (3.10). In the present one-soliton problem, one can take an algebraic soliton

$$u_0 = \frac{a}{a^2(x-\xi)^2+1}, \tag{3.11}$$

as a lowest-order solution of Eq. (3.2) where a and ξ are the amplitude and position of the soliton, respectively. Under the action of small perturbation, a and ξ evolve according to the equations

$$\frac{da}{dt} = -\frac{4\gamma}{\pi}(g_2, B_x), \quad \frac{d\xi}{dt} = \Gamma - a + \frac{4\gamma}{\pi}(g_1, B_x), \tag{3.12a,b}$$

where

$$g_1 = \int_{-\infty}^{x} \frac{\partial u_0}{\partial a} dx = \frac{x-\xi}{a^2(x-\xi)^2+1}, \quad g_2 = \int_{-\infty}^{x} \frac{\partial u_0}{\partial \xi} dx = -\frac{a}{a^2(x-\xi)^2+1}. \tag{3.13a,b}$$

It is pointed out that the amplitude equation (3.12a) coincides with the equation derived from the energy balance equation $\partial(u,u)/\partial t = 2\gamma(u, B_x)$ with u given by (3.11). However, the momentum balance equation $\partial(1,u)/\partial t = \gamma(1, B_x) = 0$ gives rise to a trivial result $0 = 0$ since $\int_{-\infty}^{\infty} u dx = \pi$ in the leading order of the expansion. As for the latter equation, the situation is quite different from the corresponding one for the fKdV equation where the integral $\int_{-\infty}^{\infty} u_0 dx$ depends on a soliton amplitude.

The inner products in (3.12) are now evaluated by using the formulas

$$\int_{-\infty}^{\infty} \frac{x}{[a^2(x-\xi)^2+1](b^2x^2+1)^2} dx = \frac{\pi}{ab^3} \frac{(\frac{1}{a}+\frac{1}{b})\xi}{[\xi^2+(\frac{1}{a}+\frac{1}{b})^2]^2}, \tag{3.14a}$$

$$\int_{-\infty}^{\infty} \frac{x^2}{[a^2(x-\xi)^2+1](b^2x^2+1)^2} dx = \frac{\pi}{2ab^3} \frac{(\frac{1}{a}+\frac{2}{b})\xi^2+\frac{1}{a}(\frac{1}{a}+\frac{1}{b})^2}{[\xi^2+(\frac{1}{a}+\frac{1}{b})^2]^2}. \tag{3.14b}$$

After some algebras, we obtain the final evolution equations for the soliton parameters a and ξ as follows:

$$\frac{da}{dt} = -\frac{8\lambda\gamma(\frac{1}{a}+\frac{1}{b})\xi}{[\xi^2+(\frac{1}{a}+\frac{1}{b})^2]^2}, \quad \frac{d\xi}{dt} = \Gamma - a + \frac{4\lambda\gamma}{a^2} \frac{\xi^2-(\frac{1}{a}+\frac{1}{b})^2}{[\xi^2+(\frac{1}{a}+\frac{1}{b})^2]^2}. \tag{3.15a,b}$$

In the absence of the perturbation, i.e. $\gamma = 0$, a remains its initial value while $\xi = (\Gamma - a)t + \xi_0$ with ξ_0 being the initial position. Furthermore, if we introduce a new dependent variable θ by $\theta = (\frac{1}{a} + \frac{1}{b})^{-1}\xi$, we can transform Eqs. (3.15) into the more transparent forms as

$$\frac{da}{dt} = -\frac{8\lambda\gamma}{(\frac{1}{a}+\frac{1}{b})^2}\frac{\theta}{(\theta^2+1)^2}, \quad \frac{d\theta}{dt} = \frac{\Gamma - a}{\frac{1}{a}+\frac{1}{b}} - \frac{4\lambda\gamma}{a^2(\frac{1}{a}+\frac{1}{b})^3}\frac{1}{\theta^2+1}. \tag{3.16a, b}$$

Remarkably, we find that the above system of equations is completely integrable. To show this, we define a new variable A by $A = a/b + \ln a$ and a Hamiltonian H given by

$$H = H(A,\theta) = \Gamma a - \frac{a^2}{2} - \frac{1}{\frac{1}{a}+\frac{1}{b}}\frac{4\lambda\gamma}{\theta^2+1}, \quad (a = a(A)). \tag{3.17}$$

Note that a is a single-valued function of A. Then, Eqs. (3.16) are seen to be equivalent to Hamilton's equations

$$\frac{dA}{dt} = -\frac{\partial H}{\partial \theta}, \quad \frac{d\theta}{dt} = \frac{\partial H}{\partial A}. \tag{3.18a, b}$$

Since the Hamiltonian (3.17) does not depend on time explicitly, it becomes a constant of motion. For later use, we write it in the form

$$[(a-\Gamma)^2 + p](\frac{1}{a} + \frac{1}{b}) = -\frac{8\lambda\gamma}{\theta^2+1}, \tag{3.19}$$

where p is an integration constant which can be determined by the initial conditions for a and θ.

Lastly, we shall comment on an analogous work based on a fKdV equation. Using a second-order perturbation theory, Grimshaw et al [16] obtained the equations for the amplitude and position of a soliton that correspond to Eqs. (3.15), where they introduced a simple *sech*-type forcing. However, they restricted their analysis to the two extreme cases when the external force has a lengthscale much greater than the soliton (broad forcing) and when it is much less (narrow forcing). They found that the second-order perturbation equations turn out to be integrable only for the broad forcing where the soliton behaves like a delta function. On the other hand, the present second-order equations (3.15) become to be integrable without any assumption on the lengthscale of the forcing as just proved. It is not clear, however, whether the structural difference between the two dynamical systems mentioned above should be attributed to the perturbation schemes employed or to any other reason such as the structure of the underlying forced soliton equations.

D. Properties of solutions

1. Stationary solutions and their linear stability

Before discussing the properties of solutions which evolve from various initial conditions, we look for stationary solutions of Eqs. (3.15) and consider their stability characteristics. In the following analysis, we put $b = 1$ without loss of generality. If we denote the stationary solutions by a_s and ξ_s, we immediately find from the equations $da_s/dt = d\xi_s/dt = 0$ that these are determined by the relations

$$\Gamma - a_s = \frac{4\lambda\gamma}{(a_s+1)^2}, \quad \xi_s = 0. \tag{3.20a, b}$$

Figure 2 is a plot of the stationary amplitude a_s as a function of the parameter Γ for $\gamma = 0.05$ and $\lambda = \pm 1$.

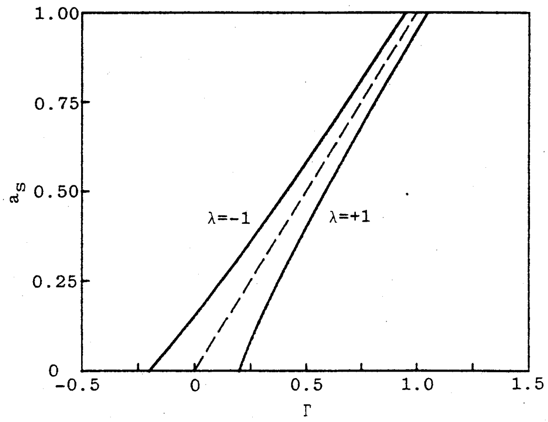


FIG. 2. Plot of the stationary amplitude a_s as a function of Γ.

In the case of $\lambda = -1$, we can see that for any value of Γ within the range $\Gamma > -4\gamma$, there exists one stationary amplitude. On the other hand, in the case of $\lambda = +1$, there exists one stationary amplitude only if $\Gamma > 4\gamma$. To discuss the linear stability of the solution (3.20), we linearize Eqs. (3.15) around the solution stationary solution as

$$a = a_s + \hat{a}, \quad \xi = \xi_s + \hat{\xi}. \tag{3.21a, b}$$

Then we obtain a system of linearized equations for $\hat{a}$ and $\hat{\xi}$

$$\frac{d\hat{a}}{dt} = -\frac{8\lambda\gamma a_s^3\hat{\xi}}{(a_s+1)^3}, \quad \frac{d\hat{\xi}}{dt} = -[1 - \frac{8\lambda\gamma}{(a_s+1)^3}]\hat{a}. \tag{3.22a, b}$$

Elimination of the variable $\hat{\xi}$ from the above system of equations yields a second-order differential equation for $\hat{a}$

$$\frac{d^2\hat{a}}{dt^2} = \frac{8\lambda\gamma a_s^3}{(a_s+1)^3}[1 - \frac{8\lambda\gamma}{(a_s+1)^3}]\hat{a}. \tag{3.23}$$

Since $a_s > 0$ and $0 < \gamma << 1$, the coefficient of $\hat{a}$ on the right-hand side of (3.23) becomes negative (positive) for $\lambda = -1$ ($\lambda = +1$). Substituting the solution of the form $\hat{a} \propto \exp(\sigma t)$ and examining the sign of the eigenvalue σ, we find that the solution corresponding to $\lambda = -1$ ($\lambda = +1$) is stable (unstable) against infinitesimal perturbations. In the case of $\lambda = -1$, the stable points represent centers since the corresponding eigenvalues are pure imaginary while for $\lambda = +1$, the unstable points are seen to be unstable nodes with one of the two eigenvalues being real and positive.

As an important remark, we point out that Eq. (3.2) exhibits an exact stationary solution for a special value of Γ. Indeed, careful investigation of the stationary form of Eq. (3.2) shows that an exact solution which satisfies the boundary condition $u \to 0$ as $|x| \to \infty$ exists when $\Gamma = 1 + \lambda\gamma$. Explicitly, it reads in the form

$$u_s = \frac{1}{x^2+1}. \tag{3.24}$$

It turns out that this solution corresponds to the stationary solutions $a_s = 1$ and $\xi_s = 0$ of Eq. (3.15) as seen from (3.20).

2. Analysis of solutions in phase plane

Let us now investigate the properties of the general solution. As already demonstrated, the system of equations (3.16) exhibits a first integral given by (3.19). Using this fact, the explicit time dependence of the variables a and ξ can be obtained in principle by integrating the equations of the form $da/dt = f(a)$ and $d\xi/dt = g(\xi)$ where f and g are known functions. These integrals are, however, found to be difficult to perform analytically. Here we shall analyze the solutions in phase plane (a, θ) for the following four different combinations of the parameters λ and Γ:

a. $\lambda = -1,\ \Gamma > -4\gamma$

This case represents a bottom topography which is concave downward. Figure 3 shows a typical phase portrait of the solution (3.19) for various values of the constant p where the parameters Γ and γ have been chosen as 1.0 and 0.05, respectively. The arrow in each trajectory indicates the direction of the motion of a soliton. Note in this case that there exists only one linearly stable stationary amplitude for a given Γ as seen from Fig. 2.

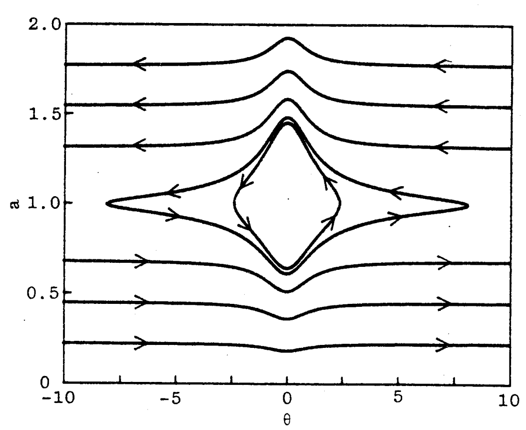


FIG. 3. Phase portrait of the solution (3.19) with $\lambda = -1, \Gamma = 1.0$ and $\gamma = 0.05$ for various values of p.

If the initial amplitude of the soliton is either much larger or much less than unity, the soliton interacts with the bottom topography whereby it suffers a small change of the amplitude near the origin $\theta = 0$. After passing through topography, the soliton gradually recovers its initial amplitude. The orbit of the solution is then described by the equation

$$(a-a_\infty)(a+a_\infty-2)(\frac{1}{a}+1)=\frac{0.4}{\theta^2+1}, \tag{3.25}$$

in the present example, where a_∞ is the amplitude of the soliton when $\theta \to \pm\infty$. While there is no net change in the amplitude after interaction, the soliton exhibits a phase shift which may be given by the relation

$$\Delta\xi = \int_{-\infty}^{\infty}\left[\frac{d\xi}{dt}-(\Gamma-a_0)\right]dt, \tag{3.26}$$

where a_0 is the initial amplitude of the soliton. The second term $\Gamma - a_0$ in the brackets represents the soliton velocity in the absence of the interaction and hence (3.26) yields a net phase shift due to the interaction. Substituting (3.15b) into (3.26), we can write it in the form

$$\Delta\xi = \int_{-\infty}^{\infty}\left[a-a_0+\frac{4\lambda\gamma}{a^2}\frac{\xi^2-(\frac{1}{a}+1)^2}{[\xi^2+(\frac{1}{a}+1)^2]^2}\right]dt. \tag{3.27}$$

If we use (3.15a) and (3.19), we can transform (3.27) into an integral with respect to a. The resulting expression is, however, too complicated to calculate analytically. Instead of performing numerical integration, we shall evaluate (3.27) by a successive approximation. We readily find from (3.15) that in the approximation up to order γ

$$a = a_0+\frac{4\lambda\gamma}{\Gamma-a_0}\frac{\frac{1}{a_0}+1}{[(\Gamma-a_0)t+\xi_0]^2+(\frac{1}{a_0}+1)^2}+O(\gamma^2), \tag{3.28a}$$

$$\xi = (\Gamma-a_0)t+\xi_0+O(\gamma), \tag{3.28b}$$

where ξ_0 is the initial position of the soliton. Introducing (3.28) into (3.27) and performing the integral with respect to t, we obtain the expression of $\Delta\xi$ correct up to order γ as

$$\Delta\xi = -\frac{4\pi\lambda\gamma}{(\Gamma-a_0)^2}\,\mathrm{sgn}(\Gamma-a_0). \tag{3.29}$$

Note that this quantity is proportional to the cross-sectional area of the bottom topography because $\int_{-\infty}^{\infty}B(x)dx=\pi\lambda$ from (3.3). The phase shift of the soliton predicted by (3.29) is quite interesting since algebraic solitons usually exhibit no phase shift after their collisions [21-23].

Figure 3 also indicates another fascinating aspect of the solutions. To be more specific, if the soliton enters from the right with an amplitude a bit larger than unity, it interacts with topography and then propagates to the left direction while decreasing its amplitude. After a lapse of time, the soliton changes its direction and goes back to the right. The motion is repeated in the same way. One observes that the orbit of the soliton becomes periodic around a stationary state $a_s = 1.048$ and $\xi_s = 0$ which turns out to be a center in the present example. This phenomenon implies that the soliton has been captured by topography while oscillating around the stationary state indicated above. We find from (3.19) that the capture occurs inside the orbit

$$(a-1)^2(\frac{1}{a}+1)=\frac{0.4}{\theta^2+1}. \tag{3.30}$$

Note that Eq. (3.30) represents two curves coalescing at $\theta=\pm\infty$ since it always possesses two positive amplitude solutions for a given θ.

b. $\lambda=-1,\ \Gamma<-4\gamma$

Different from the case a, there exist no stationary amplitudes in the parameter range $\Gamma<-4\gamma$. The figure corresponding to Fig. 3 is drawn in Fig. 4 where the parameters are given by $\Gamma=-1.0$ and $\gamma=0.05$.

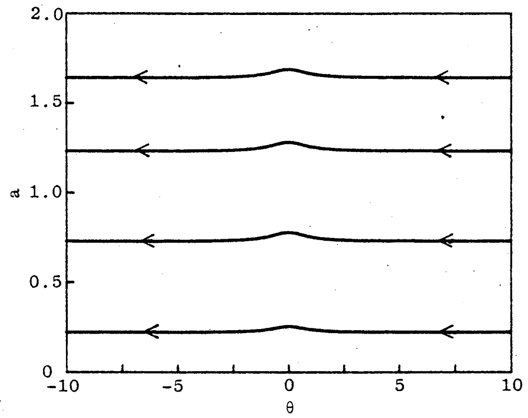


FIG. 4. As in FIG. 3 except $\Gamma=-1.0$.

This case is not so interesting because solitons propagate to the left direction irrespective of their initial amplitudes and positions. In fact, one can see that the velocity of the soliton given by (3.15b) always becomes negative provided that $\gamma<<1$. The only effect due to the interaction with topography is a phase shift given by (3.29).

c. $\lambda=+1,\ \Gamma>4\gamma$

This corresponds to a bottom topography which is convex upward. In the parameter range $\Gamma>4\gamma$, there is only one linearly unstable stationary amplitude. A typical phase portrait is depicted in Fig. 5 for various values of p where we have put $\Gamma=1.0$ and $\gamma=0.05$.

As seen from the figure, solitons with larger or smaller amplitudes compared to unity pass through topography while suffering small phase shifts. On the other hand, solitons with amplitudes a bit larger than unity decrease gradually their amplitudes in the course of the propagation to the left direction and eventually they are repelled by topography so that they propagate back to the right direction with amplitudes smaller than unity. The two types of the propagation characteristics of the solutions described above are separated by the orbit

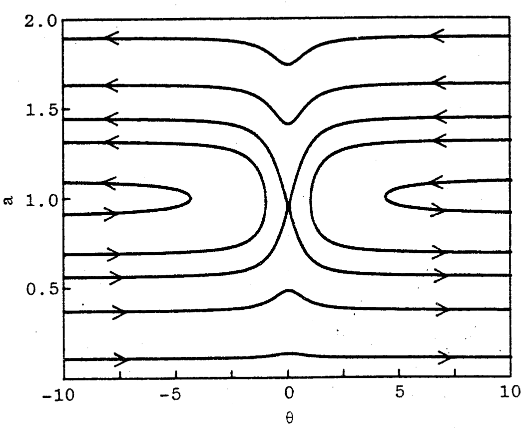
$$\left[(a-1)^2-0.197\right](\frac{1}{a}+1)=-\frac{0.4}{\theta^2+1}, \tag{3.31}$$

in the present example. Equation (3.31) represents two curves intersecting at a stationary state given by $a_s = 0.947$ and $\theta_s = 0$ which is seen to be an unstable node from the preceding stability analysis. Thus, inside the above orbit, the repulsion of solitons occurs due to topographic effect.

d. $\lambda = +1, \ \Gamma < 4\gamma$

In the parameter range $\Gamma < 4\gamma$, there exist no stationary amplitudes. The figure corresponding to Fig. 5 is drawn in Fig. 6 for the values of $\Gamma = -1.0$ and $\gamma = 0.05$.

The properties of the orbits of solitons are similar to those of the case b. When solitons enter from the right, they always pass through topography. The net effect is a phase shift given by (3.29).



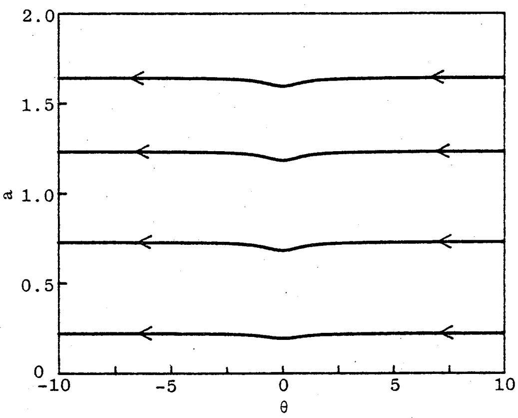


FIG. 5. As in FIG. 3 except $\lambda = +1.0$. **FIG. 6.** As in FIG. 3 except $\lambda = +1.0$ and $\Gamma = -1.0$.

E. Effect of small dissipation

In the analysis developed so far, the effect of dissipation has been neglected. In real physical phenomena, however, there exist many situations where the dissipation would play an important role. For instance, in the description of long internal waves in the stratified lower atmosphere, turbulent dissipation would become to be significant [24]. In order to elucidate the effect of dissipation on the dynamics of algebraic solitons in addition to topographic effect, we consider the following model equation

$$u_t + \Gamma u_x - 4uu_x - Hu_{xx} = \gamma B_x + \mu u_{xx}, \tag{3.32}$$

where μ is a small positive parameter which measures the magnitude of dissipation. When $\gamma = 0$, Eq. (3.32) reduces to the so-called BO-Burgers equation introduced in [24,25].

While Eq. (3.32) can be used to investigate various problems associated with it, we shall here focus our attention on the study of the dynamics of an algebraic soliton under the action of small dissipation. For the purpose, it is appropriate to employ the soliton perturbation theory developed previously.

If we introduce again the profile of the bottom topography given by (3.3), the perturbation equations for the amplitude and position of the soliton are now written as follows:

$$\frac{da}{dt} = -\frac{8\lambda\gamma(\frac{1}{a}+\frac{1}{b})\xi}{[\xi^2 + (\frac{1}{a}+\frac{1}{b})^2]^2} - \mu a^3, \tag{3.33a}$$

$$\frac{d\xi}{dt} = \Gamma - a + \frac{4\lambda\gamma}{a^2}\frac{\xi^2 - (\frac{1}{a}+\frac{1}{b})^2}{[\xi^2 + (\frac{1}{a}+\frac{1}{b})^2]^2}. \tag{3.33b}$$

When compared to the corresponding equations (3.15) in the absence of dissipation, the dissipative effect appears only in the amplitude equation (3.33a). Owing to the dissipative term μa^3, however, the complete integrability of the system is lost perfectly. Instead, the dynamics of the soliton are found to exhibit a variety of phenomena which have never been observed in the absence of dissipation. Here we shall first seek stationary solutions of Eqs. (3.33) and subsequently examine their stability against small perturbations.

1. Stationary solution

It now follows from the equations $da_s/dt = d\xi_s/dt = 0$ that the amplitude and position of the soliton corresponding to the stationary state are obtained by solving the nonlinear algebraic equations

$$\frac{(\Gamma - a_s)^2 + (\mu a_s)^2}{\Gamma - a_s + \mathrm{sgn}(\Gamma - a_s)\sqrt{(\Gamma - a_s)^2 + (\mu a_s)^2}} = \frac{2\lambda\gamma}{(a_s+1)^2}, \tag{3.34a}$$

$$\xi_s = \frac{1}{\mu a_s^2}(a_s+1)\left[\Gamma - a_s - \mathrm{sgn}(\Gamma - a_s)\sqrt{(\Gamma - a_s)^2 + (\mu a_s)^2}\right], \tag{3.34b}$$

where we have put $b = 1$. When μ tends to zero, these expressions reduce to Eqs. (3.20).

It is evident that the characteristics of stationary states given by (3.34) depend on the parameters Γ, γ and μ. We shall particularly pay our attention to elucidating the effect of μ while fixing Γ and γ to typical values. A plot of the stationary amplitude a_s as a function of μ is shown in Fig. 7(a) for the values of the parameters $\Gamma = 1.0$ and $\gamma = 0.05$ where Newton's method has been used to solve Eq. (3.34a).

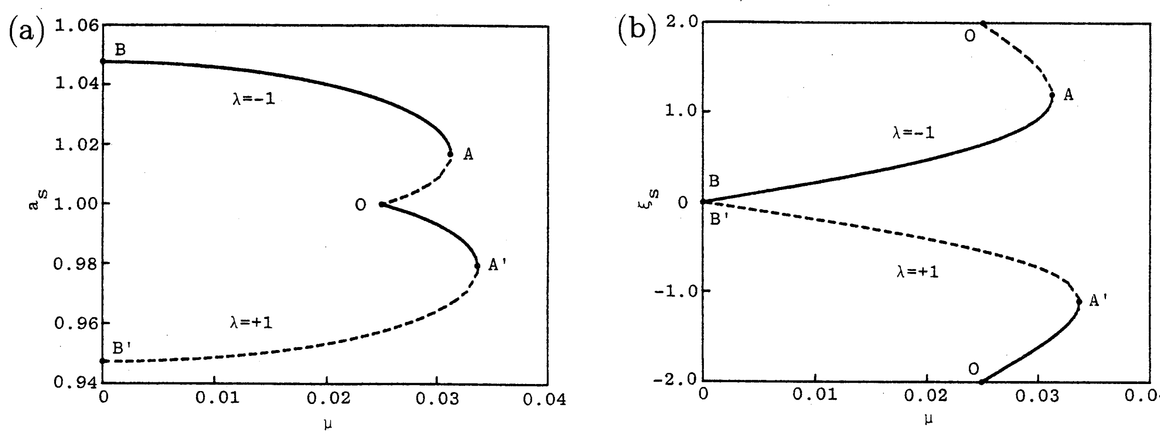


FIG.7. Plot of the stationary amplitude a_s and the stationary position ξ_s as a function of μ for the values of the parameters $\Gamma = 1.0$ and $\gamma = 0.05$.

Also, the corresponding plot of the stationary position ξ_s is given in Fig. 7(b) for the same parameter values. The branch OAB in the figures represents the stationary states for $\lambda = -1$ whereas the branch OA$'$B$'$ corresponds to those for $\lambda = +1$. In the case of $\lambda = -1$, one can see from Fig. 7(a) that for the range of the parameter $0 < \mu < 0.025$, there is only one stationary amplitude for a given μ. If μ exceeds the value 0.025, a new amplitude appears and hence we find a total of two stationary amplitudes. This happens until μ reaches a value $\mu = 0.0314$ which corresponds to the point A in the figure. Furthermore, if μ exceeds the point A, no stationary amplitudes are found to exist. This last phenomenon is easily understood from the physical point of view. Indeed, when the magnitude of μ exceeds certain critical value ($\mu = 0.0314$ in the present example), the dissipative effect becomes dominant and hence it suppresses both nonlinear and topographic effects. Consequently, the amplitude of the soliton decreases indefinitely without approaching any stationary value. The similar explanation is given to the case $\lambda = +1$ where the value of μ corresponding to the point A$'$ is 0.0338.

2. Linear stability

In order to study the stability of the stationary solutions obtained above, we have performed the linear stability analysis . Applying the same procedure as that used for the system of equations (3.15), we have arrived at the following conclusion.

For $\lambda = -1$, the solutions on the branch AB are stable, representing stable focuses because in this case the corresponding two eigenvalues are found to be complex conjugate with a negative real part. On the other hand, the solutions on the branch OA become unstable with one of the two eigenvalues being real and positive and hence these may be termed unstable nodes. For $\lambda = +1$, the solutions on the branch A$'$B$'$ become unstable nodes whereas those on the branch OA$'$ are seen to be stable focuses.

IV. SUMMARY AND OUTLOOK

In this paper, we have studied the interaction of an algebraic soliton with localized bottom topography on the basis of the fBO equation which is derived from a simple two-layer fluid system by means of an asymptotic approach. Under appropriate scaling hypothesis, the topographic effect is shown to become a small perturbation to the BO equation. This enables us to apply a direct soliton perturbation theory to the system under consideration.

In spite of the introduction of a simple profile of the bottom topography, the present analysis exhibits a variety of phenomena. In particular, the prediction of the capture and repulsion of a soliton by topography is interesting because of the possibility of their appearance in lower atmospheric systems, for instance where a mountain may be modeled by a localized topography. We have also found that the above phenomena are closely related to the stability characteristic of the system while employing the linear stability analysis.

Although the dynamical system describing the change of the amplitude and position of the soliton is shown to be completely integrable, an addition of small dissipation alters drastically the properties of solutions. In fact, for certain range of the dissipation parameter, we have found the two types of stationary states for each bottom topography in which one is stable and another is unstable against small perturbation.

In conclusion, we shall point out that in comparison with a large number of works concerning the fKdV equation, there is a dearth of study devoted to the fBO equation. Indeed, many subjects are left for future studies. Among them, of practical importance is to examine whether disturbances generated in the vicinity of topography can propagate far upstream under the condition that the basic flow is near resonance, i.e. when the Froude number is nearly unity. This problem may be resolved by integrating Eq. (3.2) numerically with zero initial condition $u(x,0) = 0$. In the analysis developed in this paper, we have addressed ourselves to soliton solutions. As is well-known, however, the BO equation exhibits an exact periodic solution expressed in terms of trigonometric functions [26,27]. The modulation of periodic wave train due to external forcing is an interesting problem which may be analyzed by the similar procedure to that used for the corresponding problem within the framework of the fKdV equation [6,28].

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