数理解析研究所講究録
947 巻 1996 年 121-132

Analysis of Parties' Power in the House of Councilors with Nonsymmetric Shapley-Owen Index: Using the Quantification Method III

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1 Introduction

Power indices are used for evaluating the voters' influence on group decision making. The Shapley-Shubik index, defined by Shapley and Shubik [1954], is one of the most well-known. It has been applied to many voting systems and it has been proved to be useful in evaluating the voters' power. See Lucas [1983] and Straffin [1994]. The nonsymmetric generalization of the Shapley-Shubik index was developed by Owen [1971], and it was modified later by Shapley [1977]. Today, it is called the nonsymmetric Shapley-Owen index. See also Owen and Shapley [1989]. The basic premise of this index is that coalition formation depends on the voters' ideological similarities.

Regretfully, the number of applications of the nonsymmetric Shapley-Owen index is small. Although this index assumes voters are not symmetric, it assumes, at the same time, bills arise everywhere and at random. Previous studies have since modified the model in their own way to deal with this nonsymmetry of bills. For example, they have used the factor analysis in common to show the voters' differences. That is, they have followed Shapley's modification. See Frank and Shapley [1981], Rapoport and Golan [1985], Rabinowitz and Macdonald [1986], and Ono and Muto [1995].

In this paper, we will focus on another assumption of the nonsymmetric Shapley-Owen index, the unboundedness of the profile space. We will analyze the same voting situation as in Ono and Muto [1995], the House of Councilors in Japan during 1989–1992; but we will use the quantification method III to position the voters and bills in the ideological space, following Owen's nonsymmetric generalization.

2 The Shapley-Shubik Index and Its Nonsymmetric Generalization

Let $N = \{1, 2, \cdots, n\}$ be the set of voters and let $v : 2^N \to R$ with $v(\emptyset) = 0$ be a characteristic function, where 2^N is the set of all subsets (coalitions) $S \subseteq N$ and R is the set of real numbers. To deal with voting situations, the following assumptions are made:

1. $\forall S \subseteq N : v(S) \in \{0, 1\}$.
2. $v(N) = 1$.
3. **(Monotonicity)** $\forall S, T \subseteq N : (S \subset T \Rightarrow v(S) \leq v(T))$.
4. **(Properness)** $\forall S \subset N : v(S) + v(N \setminus S) \leq 1$, where $N \setminus S$ is the complement of S with respect to N.

The first assumption means v takes only the values 1 and 0. If the members in S can pass the bill regardless of the others' votes, we define $v(S) = 1$; otherwise, $v(S) = 0$. Sets S with $v(S) = 1$ are called winning coalitions, and sets with $v(S) = 0$ losing coalitions. The set of all winning coalitions and of all losing coalitions are denoted by $\mathcal{W}$ and $\mathcal{L}$, respectively. And secondly, grand coalition N is assumed to always be winning. Monotonicity Assumption means that sets including a winning coalition are winning, while sets included in a losing coalition are losing. Finally, Properness Assumption excludes the case where two or more disjoint coalitions are winning. The games with the preceding assumptions are called voting games.

We next define a special class of voting games called weighted majority games. Let $w_i > 0$ be the weight of voter $i \in N$, or the number of his votes. All the members in N vote either yea or nay. The issue passes if the number of yea-votes is more than or equal to the quota q, where $\frac{1}{2}\sum_{i\in N} w_i < q \leq \sum_{i\in N} w_i$. The characterisic function v is given as

$$v(S) = \begin{cases} 1 & \text{if } \sum_{i\in S} w_i \geq q, \\ 0 & \text{if } \sum_{i\in S} w_i < q. \end{cases}$$

For simplicity, we denote this game by $[q; w_1, \cdots, w_n]$.

Suppose voters join a coalition one after another and eventually form the grand coalition. Then there exists a unique voter,called a pivot, who joins and thereby turns a losing coalition into a winning one.

Definition 2.1 *Take an ordering of n voters and voter $i \in N$. Let S be the set of voters preceding i. Then voter i is called a pivot for the ordering if $v(S \cup \{i\}) = 1$ and $v(S) = 0$.*

Each of the $n!$ orderings of n voters has a unique pivot. The Shapley-Shubik index of a voter is the probability of his being a pivot when every ordering is equally probable[1].

Definition 2.2 *The Shapley-Shubik index of a voting game* (N, v) *is the n-vector* $\varphi(v) = (\varphi_1(v), \cdots, \varphi_n(v))$ *where*

$$\varphi_i(v) = \sum_{S \in \mathcal{L}, S \cup \{i\} \in \mathcal{W}} \frac{s!(n-s-1)!}{n!}, \quad i = 1, \cdots, n. \tag{1}$$

Here s is the number of members in S.

The Shapley-Shubik index assumes that all orderings are formed with equal probability. But, in political voting situations, some orderings would be more probable than the others. For instance, take voters 1, 2 and 3, a liberalist, a centrist and a conservative, respectively. Since the extreme voters 1 and 3 are opposed to each other, ordering 132 is less likely to be formed than 123 or 321. Here ordering ijk implies that i is the first to join, j is the second and k is the last, i.e., the coalition is formed in the order of i, j, k.

To take the nonsymmetry of voters into account, Owen [1971] introduced an ideology profile space. It is a multidimensional real space, and each dimension corresponds to a particular ideology. For example, right versus left, conservative versus progressive, and so on. Every voter is placed in this space depending on his ideological position. Each proposed bill has ideologies as well, and is placed in the space. If a bill is proposed, voters whose ideological positions are close to the bill would enthusiastically support it. Owen thus supposed that voters form a coalition according to their Euclidean distances from the proposed bill. More precisely, for each bill, the closest voter joins first, the second-closest joins next, and so on. The most distant voter joins last. Owen further supposed that bills appear at random in this ideology profile space.

If there is only one ideological axis, say a left-right axis, then the three voters' case above can be depicted in terms of a line as in figure 1: for each $i = 1, 2, 3$, x^i denotes voter i's position. The space is divided into four regions E_1, E_2, E_3 and E_4 by the midpoints of the line segments x^1x^2, x^1x^3 and x^2x^3. For any bill in region E_1, voter 1 is the closest, 2 is the next and 3 is the most distant; and thus the grand coalition is formed in the order of 123. Similarly in regions E_2, E_3, E_4, corresponding orderings are 213, 231, 321, respectively. It is to be noted that regions E_2 and E_3 (producing orderings 213 and 231, respectively) are bounded intervals; while regions E_1 and E_4 (producing orderings 123 and 321, respectively) are unbounded. This means that if bills (or issues) arise at random

[1] The Shapley-Shubik index is the wellknown Shapley value of voting games. This index is derived from axioms like the Shapley value. Axioms which uniquely give this index are given in Dubey [1975].

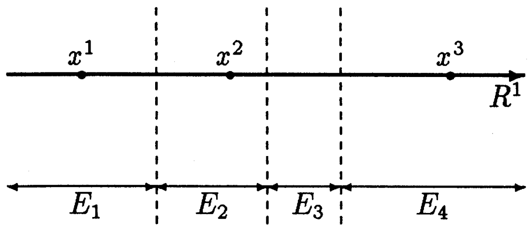
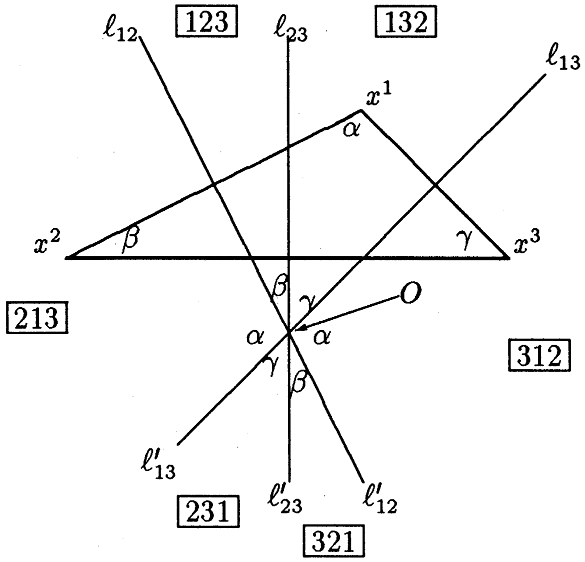


Figure 1: Three voters' locations in a unidimensional profile space

in the whole real line, orderings 231 and 213 appear only in a negligibly few occasions; and the other two orderings 123 and 321 appear with equal probability of 1/2. In the simple majority case, voter 2 is pivotal in the both orderings; thus 2 has the whole power.



* $\boxed{ijk}$ denotes the ordering in each sector.

Figure 2: Three voters' location in a two-dimensional profile space

More orderings may appear in a two-dimensional space. Figure 2 depicts the case with three voters: each x^i denotes voter i's position. Since there are three points, we have three perpendicular bisectors of each pair of points. In this figure, the line ℓ_{ij}-ℓ'_{ij} represents the perpendicular bisector of x^i and x^j, $i, j = 1, 2, 3, i \neq j$. For example, bills in the sector formed by half-lines $O\ell_{13}$ and $O\ell'_{12}$ produce ordering 312; because for any bill in the region voter 3 is the most enthusiastic supporter, voter 1 is the next, and 2 is the least enthusiastic. Assuming issues appear at random in the whole two-dimensional space, we obtain that the ordering 312 is produced with probability $\alpha/2\pi$ where α is the angle formed by two lines $O\ell_{13}$ and $O\ell'_{12}$. For each of the other regions, we can find in a similar manner which ordering is produced and how it is probable: these orderings are given in figure 2. Thus in the simple majority case where the second voter is always a pivot, the nonsymmetric Shapley-Shubik index is

$$(\frac{\alpha}{\pi}, \frac{\beta}{\pi}, \frac{\gamma}{\pi}).$$

In a space with more dimensions, the same results are obtained. For details, see Owen [1971].

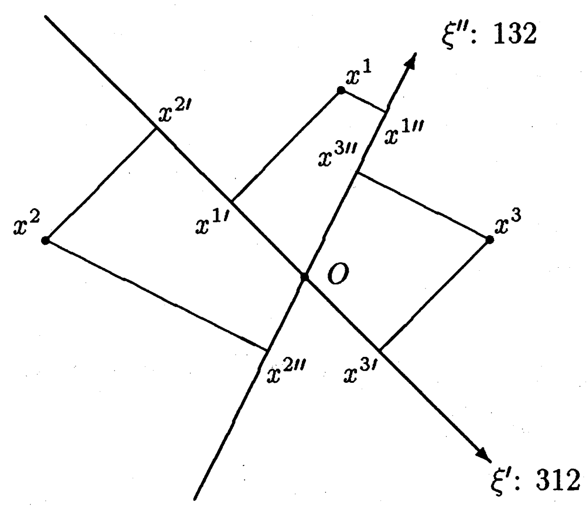


Figure 3: Shapley's construction of orderings

The Owen's method requires a highly dimensional space; $(n-1)$-dimensions are necessary for the case with n voters; otherwise, it cannot determine the origin of the space. Thus, this method is not very practical for cases with many voters. Shapley [1977] developed the following alternative method which enables us to find the nonsymmetric index using a space with lesser dimensions. See also Owen and Shapley [1989]. Take an ideology profile space with any dimension. Similarly to the Owen's method, voters are represented by points in the space. Issues are, however, represented not by points but by directed arrows (or vectors) passing through the origin of the space. Now take two voters, i and j, and an issue ξ. Positions of the two voters are represented by points x^i and x^j. Drop perpendiculars from x^i and x^j to ξ; and denote their foots by x'^i and x'^j. Shapley assumed voter i prefers ξ more than j does, or i precedes j with respect to ξ, if $||\vec{Ox'^i}|| > ||\vec{Ox'^j}||$ where O is the origin of the space and $||\vec{Ox'^i}||$ (resp. $||\vec{Ox'^j}||$) is the signed distance along the arrow ξ between O and x'^i (resp. x'^j). The inequality means that voter i's projection to ξ is greater; and thus the voter supports the issue more enthusiastically. In figure 3, for issue ξ' we have $||\vec{Ox'^3}|| > ||\vec{Ox'^1}|| > ||\vec{Ox'^2}||$; thus ordering 312 is produced. Similarly we obtain 132 for issue ξ''. If we turn the arrow around the origin assuming issues arise at random, we find for each ordering the sector in which it is produced. For each ordering, the proportion of the corresponding angle to 2π gives the probability that the ordering appears. We thus obtain the nonsymmetric index by finding out a pivotal voter in each ordering. Note, that in the Shapley's method, we may take the origin in an arbitrary position, where the same indices are obtained even if the origins are different. Thus, it can define the nonsymmetric Shapley-Owen index in less than $(n-1)$-dimmensional space. It should be also noted that Shapley's method gives the Owen's nonsymmetric index when it is applied to the case with $(n-1)$-dimensional space (n is the number of voters); hence, we may say that the Shapley's device is a generalization of the Owen's method. These two facts are shown without much difficulty. The nonsymmetric index found by the Shapley's method is often called the Shapley-Owen index. For detailed comparison of these two methods, see Ono-Muto(1995).

3 An Application: Power Distribution in the House of Councilors

There have been many applications to evaluate voters' power distribution using the symmetric Shapley-Shubik index; however, so far, only a few real cases have been studied using the nonsymmetric Shapley-Owen index.

One theoretical drawback which makes applications difficult is the assumption of bills' symmetry, *i.e.*, that bills arise at random in the ideology profile space. Let us review some studies using the nonsymmetric Shapley-Owen index, and see how they have modified the index so as to include bills' nonsymmetry.

The first study was the analysis of power distribution in the U.S. Supreme Court by Frank and Shapley [1981]. They used the factor analysis to construct a profile space. In the Court, it might be natural to assume the uniformity of the issues because issues are given exogenously. Further, the profile space was adjusted so that the issues distributed uniformly. Using the adjusted positions, they obtained the nonsymmetric Shapley-Owen index. It was a faithful application of the nonsymmetric Shapley-Owen index. They were able to analyze with only two- and three-dimensional spaces; the analysis with more than four-dimensional space is virtually impossible.

Another study was the analysis of the U.S. Presidential Election done by Rabinowitz and Macdonald [1986]. They also used the factor analysis and constructed a two-dimensional profile space. But, they found the appearance of issues were not uniform. Thus, they restricted the area to a sector in which most of the issues are included. Eventually, only about a third of the area was considered, and uniformity assumption of issues was used in this area. Here, the highly dimensional analysis still seemed to be difficult.

A case with more strongly biased issues was studied by Ono and Muto [1995]. It was the analysis of the parties' power in the House of Councilors in Japan. The profile space was constructed by the factor analysis as well. The issue distribution was too biased; thus, they used only observed issues instead. Since this distribution was discrete, they can easily analyze with highly dimensional spaces.

It is noted that the factor analysis has so far been used to construct the profile spaces. It means that each voter is considered as a point while each issue as a vector, or a direction. It is similar to Shapley's idea of constructing profile space. How would we interpret it using Owen's idea? In Owen's idea, issues are also expressed as a point. However, inner points such as those in E_2 and E_3 in figure 1 are ultimately ignored. But, these inner points might be more probable because the issues are submitted by the members in the congress. If the bills in E_2 and E_3 frequently arise, voter 1 and 3 will also have some power in simple majority games. Thus, if it is necessary to consider issues' nonsymmetry, these inner points should be more emphasized than the outer ones. However, according to Frank and Shapley or Rabinowitz and Macdonald, to exclude the outer region is not very easy because the space is continuous. But, when we use only observed data, all we have to do is calculate each issue's position as a point.

The quantification method III is a method to position voters and issues so that their similarity can be explained well in the sense of maximizing the correlation coefficient. Further it assumes the data is binary just as our data is, although the factor analysis assumes normally distributed data which enables us to consider the distance between each voter and each issue.

Now, we will evaluate the parties' power in the House of Councilors in Japan during the period 1989-1992, by using the quantification method III. In Japan, the Liberal Democratic Party (LDP) had held a majority and thus a dictatorial position both in the House of Representatives and in the House of Councilors for several decades up until 1989. Hence the LDP had total power while the other parties were completely powerless both in terms of the symmetric Shapley-Shubik index and of the nonsymmetric Shapley-Owen index. But, in the election of July 1989, some opposing parties made remarkable progress and the LDP lost their dictatorial power in the House of Councilors, though they were still the largest party. We analyzed the parties' power in the House of Councilors during that period 1989–1992 using the data concerning parties' "yea/nay" patterns in nonunanimous votes[2]. During this period, the Komeito (Komei), the Democratic Socialist Party (DSP), and the Japan Communist Party (JCP) were similar in size. However, it has been often said that the former two parties had much stronger power compared to the JCP, because they are ideologically located between the LDP and the Social Democratic Party of Japan (SDPJ). We will see if that fact has been proved true in terms of the power index.

There were more than ten parties and 15 independent members in July 1989 (just after the election). The detailed data is shown in Ono and Muto [1995]. Though each member of the House has the right to vote of his own free will, usually, members belonging to the same party vote jointly. Thus, we formulate the voting system in the House as a weighted majority game in which voters and weights are parties and the numbers of their seats, respectively. In what follows, we have chosen the six largest parties to represent voters to simplify the analysis. Some members in other mini parties and independents are included in one of the six parties if they always follow the party's decisions. Others are eliminated from the game, but the quota of the game is still given as the number of "yea" votes needed to win even if all the eliminated members vote "nay". The weighted majority game, based on the data of 1989, is given as follows:

quota	*LDP*	*SDPJ*	*Komei*	*JCP*	*DSP*	*Rengo*
127;	109	74	21	14	10	12

The symmetric Shapley-Shubik index of this game is given as

LDP	*SDPJ*	*Komei*	*JCP*	*DSP*	*Rengo*
0.567	0.117	0.117	0.067	0.067	0.067.

It gives equal power to the JCP, the DSP, and the Rengo. We also give the nonsymmetric Shapley-Owen indices with uniformly distributed bills for comparison:

dimension	*LDP*	*SDPJ*	*Komei*	*JCP*	*DSP*	*Rengo*
1	0	0.5	0	0	0.5	0
2	0.248	0.139	0.136	0.046	0.431	0.

See Frank and Shapley [1981] and Ono and Muto [1995] for the detailed way we obtain these indices.

[2]In the House of Councilors, half the members are up for election every three years.

Table 1: Pattern of yea/nay combinations (1989–1992)

type	LDP	SDPJ	Komei	JCP	DSP	Rengo	number
A	Y	Y	Y	N	Y	Y	85
B	Y	N	N	N	N	N	18
C	Y	N	N	N	Y	N	9
D	Y	N	Y	N	Y	Y	6
E	N	Y	Y	Y	Y	Y	6
F	Y	N	Y	N	Y	N	5
G	Y	N	Y	Y	Y	Y	3
H	Y	Y	Y	N	Y	N	1

Sources: Sangiin Kaigiroku [1989–1992]

Let us construct the profile space from the quantification method III. Using the data of the six parties' voting pattern given in table 1, we can obtain each voter's profile point.

In the unidimensional profile space, parties position in the order of the LDP, the DSP, the SDPJ, the Komei, the Rengo and the JCP, from the left. Each closed circle is the profile of voters; each open one is of bills. Take a bill of type A; then the closest voter for it is the DSP, the second closest one is the SDPJ, and so on. After all, the ordering

$$\text{DSP} \longrightarrow \text{SDPJ} \longrightarrow \text{Komei} \longrightarrow \text{Rengo} \longrightarrow \text{LDP} \longrightarrow \text{JCP}$$

is formed. The pivot voter is the LDP. This type of issues can be interperted, for example, that the LDP submits bills closer to four ideologically similar voters (the SDPJ, the Komei, the DSP and the Rengo) so as to make the bills pass. Similarly, ordering for each of the eight types of issues is obtained. Then the nonsymmetric Shapley-Owen index is

LDP	*SDPJ*	*Komei*	*JCP*	*DSP*	*Rengo*
0.714	0.241	0.000	0.045	0.000	0.000.

In the quantification method III, there is no convincing criterion which tells whether or not the unidimensional space describes the real situation well. This method positions voters and bills so as to maximize the correlation coefficient. The size of the correlation coefficient could inform how well they are explained. In this case, the correlation coefficient is 0.457. Thus, we should conclude unidimension is insufficient to explain.

Figure 4 is the two-dimensional profile space. The six closed circles describe parties' positions, and the eight open ones describe bills' position. In the same way as above, we make an ordering according to parties' distances from each bill. Then the nonsymmetric Shapley-Owen index is

LDP	*SDPJ*	*Komei*	*JCP*	*DSP*	*Rengo*
0.759	0.000	0.241	0.000	0.000	0.000.

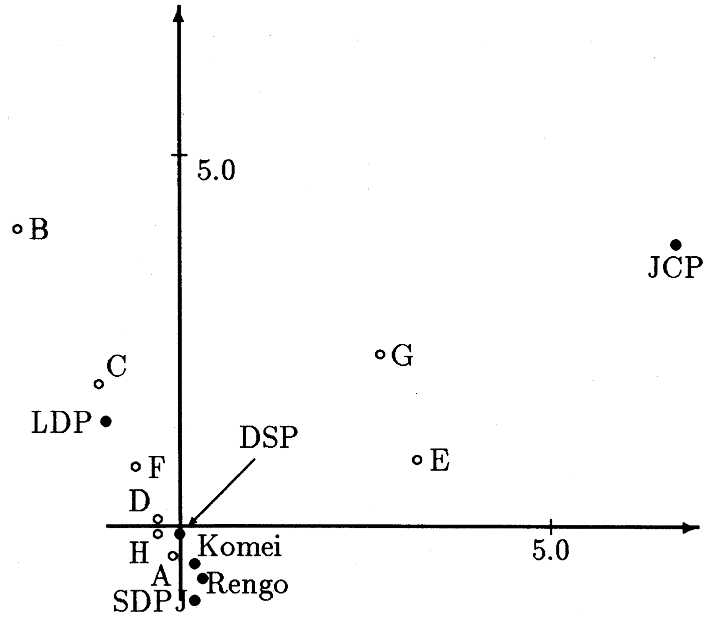


Figure 4: Two-dimensional profile space

In this figure, it is clear that these six parties are devided into 3 groups; the LDP, the JCP, and the others. The parties positions in the last group is very alike and sensitive. But, when we analyze with the unformity assumption of bills, their positions are very important. In this analysis, for example, inside parties like the DSP and the Komei do not have a chance to be the most enthusiastic supporter nor the most stubborn opponent. If we take the bill of type A, however, the Komei is the most enthusiastic to this bill in two-dimensional space. More than 60% of the bills are of this type. Nevertheless, the area producing orderings beginning with the Komei has been ignored because it is bounded.

As discussed in Ono-Muto [1995], using only observed data makes the analysis with more than three dimensions easy. As the number of dimension increases, there appear two or more parties who have almost the same distance from an issue. For example, five parties except the JCP have almost same distance from issues of type A. In this case, we assume 5! orderings of the five parties arise with same probability. Then, from five dimensional profile space, obtained index is

LDP	*SDPJ*	*Komei*	*JCP*	*DSP*	*Rengo*
0.563	0.170	0.147	0.005	0.045	0.071.

This index is similar to the one obtained from five dimensional profile space with factor analysis.

4 Concluding Remarks

In this paper, we analyzed the power distribution of House of Councilors in Japan using the nonsymmetric Shapley-Owen index. Though the nonsymmetric Shapley-Owen index has been applied to some cases, uniformity assumption of issues have required the profile space to be modified. On the other hand, unboundedness of the profile space has not been discussed. The unboundedness makes some issues unusable, even if they are very important and frequently observed. In this paper, we used the distribution of observed raw data as in Ono-Muto [1995]. But, using the quantification method III to construct the profile space, we obtained the nonsymmetric Shapley-Owen index with the idea of distance between an issue and a voter in Owen [1971]. If we use only observed data, it is easy to increase the dimension of the profile space. Further, we found that the index obtained from the quantification method III is very similar to the one obtained from the factor analysis.

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