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Approximation algorithms for scheduling problems with generalized due dates

(Abstract)

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1 Introduction

We are concerned with scheduling n independent jobs $J_1, J_2, \ldots, J_n$ on a single machine so as to minimize a given objective function involving generalized due dates. We make the following assumptions about feasibility of schedules.

1. The scheduling period is the interval $[0, \infty)$.
2. The machine is continuously available from the beginning, and it cannot process more than one job at a time.
3. The processing times $p_1, p_2, \ldots, p_n$ of jobs $J_1, J_2, \ldots, J_n$ are positive numbers known in advance, and they are independent of schedules.
4. Preemption is not permitted, that is, each job, once started, must be completed without interruption before another job is started.
5. All jobs are available for processing from the beginning.

The objective functions we are interested in involve generalized due dates proposed by Hall [3]. To illustrate the difference between the traditional view of due dates and Hall's view, consider the concept of lateness of a job in a schedule. In the traditional view, each job J_i has associated with it not only a processing time p_i but also a due date d_i. All due dates $d_1, d_2, \ldots, d_n$ are known in advance and they are independent of schedules. In Hall's view, no job has its own due date in advance. Instead, only a non-decreasing sequence

$$\delta_1 \leq \delta_2 \leq \cdots \leq \delta_n$$

of numbers, called generalized due dates, is given. In both cases, for each $1 \leq i \leq n$, every schedule S determines uniquely

1. the job $J_{S(i)}$ in the ith position of schedule S, that is, an order (sequence)

$$(S(1), S(2), \ldots, S(n))$$

in which the jobs are processed on the machine, and

2. the completion time $C_i(S)$ of job J_i in schedule S.

The lateness of the ith job in S, that is, the lateness $L_{S(i)}(S)$ of job $J_{S(i)}$ in S under the traditional view is given by

$$L_{S(i)}(S) = C_{S(i)}(S) - d_{S(i)},$$

whereas the lateness $L^H_{S(i)}(S)$ of job $J_{S(i)}$ in S according to Hall's view is given by

$$L^H_{S(i)}(S) = C_{S(i)}(S) - \delta_i.$$

For example, if

$$\begin{aligned} p_1 &= 3, \quad p_2 = 2, \quad p_3 = 5, \\ d_1 &= 4, \quad d_2 = 7, \quad d_3 = 10, \\ \delta_1 &= 4, \quad \delta_2 = 7, \quad \delta_3 = 10, \end{aligned}$$

then, for the permutation schedule given by the sequence (J_1, J_3, J_2), we have

$$\begin{aligned} L_1 &= -1, \quad L_2 = 3, \quad L_3 = -5, \\ L^H_1 &= -1, \quad L^H_2 = 0, \quad L^H_3 = -2, \end{aligned}$$

Several authors [1, 3, 6] describe situations in which generalized due dates arise quite naturally. These include public utility planning, survey design and some types of flexible manufacturing. Obviously, the concept of generalized due dates was proposed with the aim of allowing for job independent due dates. It may, however, be also useful to consider generalized due dates as numbers through which the sequence dependent due dates are determined. Then we obtain the traditional concept and Hall's concept as two (very) special cases of sequence dependent due dates $D_1(S), D_2(S), \ldots, D_n(S)$. Taking constant (sequence independent) functions

$$D_i(S) = d_i \quad \text{for } 1 \le i \le n,$$

we obtain the traditional concept, taking

$$D_i(S) = \delta_{S^{-1}(i)} \quad \text{for } 1 \le i \le n,$$

we obtain Hall's concept.

Now it is clear that we may expect a variety of changes in results concerning the generalized due date counterparts of the traditional scheduling problems. The following table, in which the notation[1] proposed by Graham et al. [2] is used, shows that problems involving generalized due dates may be easier, harder, or equally difficult as their traditional counterparts. The table suggests that the max-problems tends to be harder and the sum-problems tend to be easier for the problems involving generalized due dates (see [4] for further details).

[1] This notation will be used throughout this paper.

Problem (notation for traditional view)	Traditional view	Hall's view
$1\|\|L_{\max}$	Polynomially solvable	Polynomially solvable
$1\|prec\|L_{\max}$	Polynomially solvable	NP-hard
$1\|r_j\|L_{\max}$	NP-hard	NP-hard
$1\|\|\sum U_j$	Polynomially solvable	Polynomially solvable
$1\|prec, p_j = 1\|\sum U_j$	NP-hard	Polynomially solvable
$1\|r_j\|\sum U_j$	NP-hard	NP-hard
$1\|\|\sum T_j$	NP-hard	Polynomially solvable
$1\|prec, p_j = 1\|\sum T_j$	NP-hard	Polynomially solvable
$1\|r_j\|\sum T_j$	NP-hard	NP-hard

Most research in scheduling involving generalized due dates has been concerned with establishing the complexity status of the problems whose traditional counterparts have regular objective functions. Little is known about the problems whose traditional counterparts have non-regular objective functions, and about approximation algorithms for the problems involving generalized due dates.

In what follows, we are concerned with several single machine problems involving non-regular objective functions and generalized due dates. The objective functions we are interested in are defined as follows.

Traditional model	Hall's model
$L_{\max}(S) = \max_{1\le i\le n} L_i(S)$	$L^H_{\max}(S) = \max_{1\le i\le n} L^H_i(S)$
$L_{\min}(S) = \min_{1\le i\le n} L_i(S)$	$L^H_{\min}(S) = \min_{1\le i\le n} L^H_i(S)$
$\Delta L(S) = L_{\max}(S) - L_{\min}(S)$	$\Delta L^H(S) = L^H_{\max}(S) - L^H_{\min}(S)$
$L_{\text{abs}}(S) = \max \|L_i(S)\|$	$L^H_{\text{abs}}(S) = \max \|L^H_i(S)\|$

Main results can be summarized as follows. First, we show that the problems of minimizing the maximum absolute lateness and the range of lateness are NP-hard in the strong sense, both with and without allowing for machine idle time. Second, for all of these problems, we give simple efficient approximation algorithms based on the first-fit strategy. We show that they achieve the performance ratios of n for the problems of minimizing the maximum absolute lateness and of $(n+1)/2$ for the problems of minimizing the range of lateness.

2 Approximation algorithms

In this section, we present two simple approximation algorithms for the problems $1||L^H_{\text{abs}}$, $1|nmit|L^H_{\text{abs}}$, $1||\Delta L^H$, and $1|nmit|\Delta L^H$, where, following the notation of Hoogeveen [5], $nmit$ indicates that no machine idle time is allowed. The algorithms are based on the first-fit strategy.

First, we introduce Algorithm A which works for the problems of minimizing L^H_{abs} and minimizing ΔL^H without allowing for the machine idle time.

The algorithm returns the resulting schedule A as a permutation, i.e., A returns the index $A(i)$ of the job in the ith position for each i, $1 \le i \le n$.

```
Algorithm A(p_1, p_2, ..., p_n, δ_1, δ_2, ..., δ_n)
  δ_0 ← 0
  for i = 1 to n do
    a_i = δ_i − δ_{i−1}
  I ← {1, 2, ..., n}
  J ← {1, 2, ..., n}
  while I ≠ ∅ do
    Choose i such that a_i = min_{k∈I} a_k
    Choose j such that p_j = min_{k∈J} p_k
    A(i) ← j
    I ← I \ {i}
    J ← J \ {j}
  od
  Output(A)
end
```

The time complexity of Algorithm A is $O(n \log n)$, if we use a fast sorting scheme. First, we show the following lemma which plays an important role in the proofs of establishing the performance guarantees.

Lemma 1 *For each schedule S, we have*

$$\max_i \{p_{A(i)} - a_i\} \leq \max_i \{p_{S(i)} - a_i\}$$

and

$$\min_i \{p_{A(i)} - a_i\} \geq \min_i \{p_{S(i)} - a_i\}.$$

Proof. We only verify the validity of the first inequality. The proof of the second one is analogous. Without loss of generality, we assume that $p_1 \leq p_2 \leq \cdots \leq p_n$. The proof is by contradiction. Suppose that there exists a schedule S such that $p_{A(j)} - a_j > p_{S(k)} - a_k$, where j and k are such that $p_{A(j)} - a_j = \max_i\{p_{A(i)} - a_i\}$ and $p_{S(k)} - a_k = \max_i\{p_{S(i)} - a_i\}$.

Since $p_{A(j)} - a_j > p_{S(k)} - a_k \geq p_{S(j)} - a_j$, we have $p_{A(j)} > p_{S(j)}$, consequently, $A(j) > S(j)$. There are at most $(A(j) - 2)$ i's such that $i \neq j$ and $p_{S(i)} < p_{A(j)}$. But, there are at least $(A(j) - 1)$ i's such that $i \neq j$ and $a_i \leq a_j$. Therefore, there exists i, $i \neq j$ such that $a_i \leq a_j$ and $p_{S(i)} \geq p_{A(j)}$. Hence,

$$\begin{aligned} p_{S(k)} - a_k &\geq p_{S(i)} - a_i \\ &= p_{S(i)} - a_j + a_j - a_i \\ &\geq p_{A(j)} - a_j + a_j - a_i \\ &\geq p_{A(j)} - a_j. \end{aligned}$$

This contradicts the assumption. ∎

Then, we analyze the performance of the algorithm concerning the problem $1|nmit|L^H_{\text{abs}}$. Let OPT be an optimal schedule for this problem. We obtain the following bound on the performance of Algorithm A.

Theorem 1

$$L^H_{\text{abs}}(A) \leq n \times L^H_{\text{abs}}(OPT).$$

Proof. By Lemma 1, we have

$$\begin{aligned}
L^H_{\text{abs}}(OPT) &= \max\{L^H_{\max}(OPT), -L^H_{\min}(OPT)\} \\
&\geq \frac{1}{2} \times (L^H_{\max}(OPT) - L^H_{\min}(OPT)) \\
&\geq \frac{1}{2} \times \max_i |L^H_{OPT(i)} - L^H_{OPT(i-1)}| \\
&= \frac{1}{2} \times \max_i |C_{OPT(i)} - \delta_i - (C_{OPT(i-1)} - \delta_{i-1})| \\
&= \frac{1}{2} \times \max_i |p_{OPT(i)} - a_i| \\
&\geq \frac{1}{2} \times \max_i |p_{A(i)} - a_i|.
\end{aligned}$$

Let $I = \{1, 2, \ldots, n\}$, and let J and K be the set of the indices such that $p_{A(i)} \geq a_i$ for all $i \in J$ and $p_{A(i)} < a_i$ for all $i \in K$, respectively. From the definition of L^H_{abs} and a_i, it follows that

$$\begin{aligned}
L^H_{\text{abs}}(A) &= \max\{L^H_{\max}(A), -L^H_{\min}(A)\} \\
&\leq \max\{\sum_{i \in J}(p_{A(i)} - a_i), -\sum_{i \in K}(p_{A(i)} - a_i)\}.
\end{aligned}$$

First, we assume that $|J| = n/2$. Then we have

$$\begin{aligned}
L^H_{\text{abs}}(A) &\leq \max\{|J| \times \max_{i \in J}\{p_{A(i)} - a_i\}, -|K| \times \min_{i \in K}\{p_{A(i)} - a_i\}\} \\
&\leq \max\{\frac{n}{2} \times \max_i\{p_{A(i)} - a_i\}, -\frac{n}{2} \times \min_i\{p_{A(i)} - a_i\}\} \\
&\leq \frac{n}{2} \times \max_i |p_{A(i)} - a_i|.
\end{aligned}$$

Next, we assume that $|J| \leq (n-1)/2$. Then we have

$$\begin{aligned}
L^H_{\text{abs}}(A) &\leq \max\{\sum_{i \in J}(p_{A(i)} - a_i), -\sum_{i \in I \setminus J}(p_{A(i)} - a_i)\} \\
&= \max\{\sum_{i \in J}(p_{A(i)} - a_i), \sum_{i \in J}(p_{A(i)} - a_i) - \sum_{i \in I}(p_{A(i)} - a_i)\} \\
&\leq \sum_{i \in J}(p_{A(i)} - a_i) + |\sum_{i \in I}(p_{A(i)} - a_i)| \\
&\leq |J| \times \max_{i \in J}\{p_{A(i)} - a_i\} + |C_{A(n)} - \delta_n| \\
&\leq \frac{n-1}{2} \times \max_i |p_{A(i)} - a_i| + |L^H_{A(n)}|.
\end{aligned}$$

Finally, we assume that $|J| \geq (n+1)/2$. Then we have

$$\begin{aligned}
L^H_{\text{abs}}(A) &= \max\{\sum_{i \in I \setminus K}(p_{A(i)} - a_i), -\sum_{i \in K}(p_{A(i)} - a_i)\} \\
&= \max\{\sum_{i \in I}(p_{A(i)} - a_i) - \sum_{i \in K}(p_{A(i)} - a_i), -\sum_{i \in K}(p_{A(i)} - a_i)\} \\
&\leq |\sum_{i \in I}(p_{A(i)} - a_i)| - \sum_{i \in K}(p_{A(i)} - a_i) \\
&\leq |C_{A(n)} - \delta_n| - |K| \times \min_{i \in K}\{p_{A(i)} - a_i\} \\
&\leq |L^H_{A(n)}| + \frac{n-1}{2} \times \max_i |p_{A(i)} - a_i|.
\end{aligned}$$

To conclude the proof, it is sufficient to observe that

$$|L^H_{A(n)}| = |L^H_{OPT(n)}| \leq L^H_{\text{abs}}(OPT).$$

∎

Theorem 1 provides the performance ratio n between the optimal value of L^H_{abs} and the value induced by a schedule found by Algorithm A. The following theorem says that this ratio cannot be improved.

Theorem 2 *There exists an instance satisfying*

$$L^H_{\text{abs}}(A) = n \times L^H_{\text{abs}}(OPT).$$

Next, we analyze the performance of the algorithm concerning the problem $1|nmit|\Delta L^H$. Now, let OPT be an optimal schedule for this problem. We obtain the following bound on the performance of Algorithm A.

Theorem 3

$$\Delta L^H(A) \leq \frac{n+1}{2} \times \Delta L^H(OPT).$$

Theorem 3 provides the performance ratio $(n+1)/2$ between the optimal value of ΔL^H and the value induced by a schedule found by Algorithm A. The following theorem says that this ratio cannot be improved.

Theorem 4 *There exists an instance satisfying*

$$\Delta L^H(A) = \frac{n+1}{2} \times \Delta L^H(OPT).$$

By using Algorithm A, we can make an approximation algorithm for the problems $1||L^H_{\text{abs}}$ and $1||\Delta L^H$, which gives the same approximation ratios as in Theorem 1 and 3.

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References

[1] Browne, J., Dubois, D., Rathmill, K., Sethi, S. P., and Stecke, K. Classification of flexible manufacturing systems. *The FMS Magazine* (Apr. 1984), 114–117.

[2] Graham, R. L., Lawler, E. L., Lenstra, J. K., and Rinnooy Kan, A. H. G. Optimization and approximation in deterministic sequencing and scheduling: A survey. *Annals of Discrete Mathematics 5* (1979), 287–326.

[3] Hall, N. G. Scheduling problems with generalized due dates. *IEE Transactions 18* (1986), 220–222.

[4] Hall, N. G., Sethi, S. P., and Sriskandarajah, C. On the complexity of generalized due date scheduling problems. *European Journal of Operational Research 51* (1991), 100–109.

[5] Hoogeveen, J. A. Minimizing maximum earliness and maximum lateness on a single machine. Tech. Rep. BS-R9001, Centre for Mathematics and Computer Science, Amsterdam, 1990.

[6] Stecke, K. E., and Solbert, J. J. Loading and control policies for a flexible manufacturing system. *International Journal of Production Research 19* (1981), 481–490.