数理解析研究所講究録
950 巻 1996 年 167-173

On the Church-Rosser Property of Non-E-overlapping and Weight-Preserving TRS's

三重大学工学部/沖テクノシステムズラボラトリ　五味 弘　(Hiroshi Gomi)
三重大学工学部　大山口 通夫　(Michio Oyamaguchi)
三重大学工学部　太田 義勝　(Yoshikatsu Ohta)

Abstract

A term rewriting system (TRS) is said to be weight-preserving if for any rewrite rule and any variable appearing in the both sides, the maximal weight of the variable occurrences in the left-hand-side is greater than or equal to that of the variable occurrences in the right-hand-side, and to be strongly weight-preserving if it is weight-preserving and for any rewrite rule and any variable appearing in the left-hand-side, all the weights of the variable occurrences in the left-hand-side are the same.

1 Introduction

A term rewriting system (TRS) is a set of directed equations (called rewrite rules). A TRS is Church-Rosser (CR) if any two interconvertible terms reduce to some common term by applications of the rewrite rules. This CR property is important in various applications of TRS's and has received much attention so far [1-3,5-8]. Although the CR property is undecidable for general TRS's, many sufficient conditions for ensuring this property have been obtained [1,2,5-8].

However, for nonlinear and nonterminating TRS's, only a few results on the CR property have been obtained. Our previous papers [5,6] may be pioneer ones which have first given nontrivial conditions for the CR property, though these conditions can be applied only to subclasses of right-linear TRS's. On the other hand, if we omit the right-linearity condition, then it has been shown that only the non-E-overlapping condition is insufficient for ensuring the CR property of TRS's [2]. For example, $R_0 = \{f(x,x) \to a,\ g(x) \to f(x, g(x)),\ c \to g(c)\}$, where x is a variable and f, g, a, c are function symbols, is non-E-overlapping, but not CR.

In this paper, we consider the CR property of nonlinear, nonterminating and weight-preserving TRS's. Here, a TRS is weight-preserving if there exists a weight function from the set of function symbols to the set of positive integers (called weights) satisfying the weight-preserving condition that for each rule $\alpha \to \beta$ and any variable x appearing in both α and β, the maximal weight of the x occurrences in α is greater than or equal to that of the x occurrences in β, where the weight of an x occurrence is the sum of the weights assigned to function symbols on the path from the root to the x occurrence. For example, TRS $R_1 = \{h(x,x) \to f(x, g(x))\}$, where x is a variable, weight of h is 2 and weights of f, g are 1, is weight-preserving, since the maximal weight of the x occurrences of the left-hand-side is 2 and that of the right-hand-side 2. Note that R_0 is not weight-preserving.

We first show that only the non-E-overlapping and weight-preserving properties are insufficient for ensuring the CR property. That is, the following non-E-overlapping and weight-preserving TRS R_2 is not CR:

$$R_2 = \{f(x,x) \to a,\ c \to h(c, g(c)),\ h(x, g(x)) \to f(x, h(x, g(c)))\}$$

where x is a variable and f, g, h, a, c are function symbols of weight 1.

Next, we introduce the notion of strongly weight-preserving property (stronger than the weight-preserving one). A TRS R is strongly weight-preserving if there exists a weight function such that R satisfies the weight-preserving condition and for each $\alpha \to \beta$ and for any variable x appearing in α, all the weights of the x occurrences in α are the same. For example, TRS $R_3 = \{h(g(x), g_1(g_2(x))) \to f(x, h(x, g(c)))\}$ is strongly weight-preserving, since by assigning 2 as weight of g and 1 as weights of the other symbols, R_3 satisfies the weight-preserving condition and all the weights of x occurrences of the left-hand-side are 3.

In this paper, we show that all the non-E-overlapping and strongly weight-preserving TRS's are CR (Theorem 2). We first consider the class of depth-preserving TRS's which is a subclass of weight-preserving TRS's. We show that all the non-E-overlapping and strongly depth-preserving TRS's are CR (Theorem 1). Using Theorem 1, we prove our main theorem (Theorem 2).

2 Definitions

The following definitions and notations are similar to those in [2, 5]. Let X be a set of variables, F be a finite set of function symbols and T be the set of terms constructed from X and F.

For a term M, we use $O(M)$ to denote the set of occurrences (positions) of M, and M/u to denote the subterm of M at occurrence u, and $M[u \leftarrow N]$ to denote the term obtained from M by replacing the subterm M/u by term N. The set of occurrences $O(M)$, where $M \in T$, is partially ordered by the prefix ordering: $u \leq v$ iff $\exists w, uw = v$. In this case, we denote w by v/u. If $u \leq v$ and $u \neq v$, then $u < v$. If $u \not\leq v$ and $v \not\leq u$, then u and v are said to be *disjoint* and denoted $u|v$. Let $V(M)$ be the set of variables in M, $O_x(M)$ be the set of occurrences of variable $x \in V(M)$, and $O_X(M) = \cup_{x \in V(M)} O_x(M)$ i.e., the set of variable occurrences in M. $\bar{O}(M)$ is the set of non-variable occurrences, i.e., $\bar{O}(M) = O(M) - O_X(M)$. We use $N[u \leftarrow M/u \mid u \in U]$ to denote $N[u_1 \leftarrow M/u_1, u_2 \leftarrow M/u_2, \cdots, u_n \leftarrow M/u_n]$ where $U = \{u_1, u_2, \cdots, u_n\}$, and $u_1, \cdots, u_n$ are pairwise disjoint. Here, $N[u_1 \leftarrow M/u_1, u_2 \leftarrow M/u_2, \cdots, u_n \leftarrow M/u_n] = (N[u_1 \leftarrow M/u_1, u_2 \leftarrow M/u_2, \cdots, u_{n-1} \leftarrow M_{n-1}])[u_n \leftarrow M/u_n]$ if $n > 1$.

For a term M, $H(M) = Max\{|u| \mid u \in O(M)\}$. $H(M)$ is called "height of M". The depth of occurrence $u \in O(M)$ is defined by $|u|$.

Example. $H(f(g(x))) = 2$, $H(a) = 0$, $H(g(x)) = 1$.

Let $fM_1 \cdots M_n$ where $f \in F$ and $M_i \in T$ such that $1 \leq i \leq n$. For the function f, *arity* of f is the number of arguments, i.e., $arity(f) = n$.

We use a function $w : F \to \{1, 2, 3, \cdots\}$ to assign a positive integer to each function symbol. We call w a weight function, and $w(f)$ the weight of function symbol f. For a term M, the weight $W_w(u, M)$ of occurrence u is defined as follows:

$$\begin{aligned} W_w(\varepsilon, x) &= 0 \\ W_w(\varepsilon, fM_1 \cdots M_n) &= w(f) \\ W_w(i \cdot u, fM_1 \cdots M_n) &= w(f) + W_w(u, M_i) \end{aligned}$$

where $x \in X$, $f \in F$, $arity(f) = n$, $1 \leq i \leq n$, $M_1, \cdots, M_n \in T$ and $i \cdot u \in O(M)$.

A rewrite rule is a directed equation $\alpha \to \beta$ such that $\alpha \in T - X, \beta \in T$ and $V(\alpha) \supseteq V(\beta)$. A term-rewriting system (TRS) is a set of rewrite rules.

A term M reduces to a term N if $M/u = \sigma(\alpha)$ and $N = M[u \leftarrow \sigma(\beta)]$ for some $\alpha \to \beta \in R$ and $\sigma : X \to T$. We denote this reduction by $M \xrightarrow{u} N$. In this notation u may be omitted (i.e., $M \to N$) and $\to^*$ is the reflexive-transitive closure of $\to$.

A parallel reduction $M \leftarrow\!\!\!+\!\!\!\rightarrow N$ is defined as follows:

$M \leftarrow\!\!\!+\!\!\!\rightarrow N$ iff $\exists U \subseteq O(M)$ such that
$\forall u, v \in U \quad u \neq v \Rightarrow u|v$,
$\forall u \in U \quad M/u \xrightarrow{\varepsilon} N/u$ or $N/u \xrightarrow{\varepsilon} M/u$, and
$N = M[u \leftarrow N/u \mid u \in U]$

In this case, let $R(M \leftarrow\!\!\!+\!\!\!\rightarrow N) = U$. (Note. $U = \phi$ is allowed.) Let $\leftarrow\!\!\!+\!\!\!\rightarrow^*$ be the reflexive-transitive closure of $\leftarrow\!\!\!+\!\!\!\rightarrow$.

We assume that $\gamma : M_0 \leftarrow\!\!\!+\!\!\!\rightarrow M_1 \leftarrow\!\!\!+\!\!\!\rightarrow \cdots \leftarrow\!\!\!+\!\!\!\rightarrow M_n$ in the following definitions, which will be used in Section 4.

Let $R(\gamma) = \bigcup_{0 \leq i < n} R(M_i \leftarrow\!\!\!+\!\!\!\rightarrow M_{i+1})$ and $MR(\gamma)$ be the set of *minimal* occurrences in $R(\gamma)$ under the prefix ordering.

For $u \in O(M_0)$, if there exists no $v \in R(\gamma)$ such that $v \leq u$, then γ is said to be *u-invariant.*

We denote by $\gamma[i, j]$ the subsequence $M_i \leftarrow\!\!\!+\!\!\!\rightarrow M_{i+1} \leftarrow\!\!\!+\!\!\!\rightarrow \cdots \leftarrow\!\!\!+\!\!\!\rightarrow M_j$ of γ where $i \geq 0$ and $j \leq n$.

If $M_n = N_0$, then the composition of γ and $\delta : N_0 \leftarrow\!\!\!+\!\!\!\rightarrow N_1 \leftarrow\!\!\!+\!\!\!\rightarrow \cdots \leftarrow\!\!\!+\!\!\!\rightarrow N_k$, i.e., $M_0 \leftarrow\!\!\!+\!\!\!\rightarrow M_1 \leftarrow\!\!\!+\!\!\!\rightarrow \cdots \leftarrow\!\!\!+\!\!\!\rightarrow M_n (= N_0) \leftarrow\!\!\!+\!\!\!\rightarrow N_1 \leftarrow\!\!\!+\!\!\!\rightarrow \cdots \leftarrow\!\!\!+\!\!\!\rightarrow N_k$ is denoted by $(\gamma; \delta)$.

Let $u \in MR(\gamma)$. Then, the *cut* sequence of γ at u is $\gamma/u = (M_0/u \leftarrow\!\!\!+\!\!\!\rightarrow M_1/u \leftarrow\!\!\!+\!\!\!\rightarrow \cdots \leftarrow\!\!\!+\!\!\!\rightarrow M_n/u)$.

We denote by $\gamma[\xi'/\xi]$ the sequence obtained from reduction sequence γ by replacing subsequence or cut sequence (or cut subsequence) ξ of γ by sequence ξ'.

The number of parallel reduction steps of γ is $|\gamma|_p = n$.

Note. If $\delta : M \leftarrow\!\!\!+\!\!\!\rightarrow M$, then $|\delta|_p = 1$.

Example. Let $\delta : f(c, c) \leftarrow\!\!\!+\!\!\!\rightarrow f(g(c), g(c)) \leftarrow\!\!\!+\!\!\!\rightarrow a$, then $|\delta|_p = 2$.

Let $net(\gamma)$ is the sequence obtained from γ by removing all $M_i \leftarrow\!\!\!+\!\!\!\rightarrow M_{i+1}$ satisfying that $M_i = M_{i+1}$, $0 \leq i < n$.

We use $|\delta|_{np}$ to denote $|net(\delta)|_p$.

We define the height of reduction sequence $H(\gamma)$ as $H(\gamma) = Max\{H(M_i) \mid 0 \leq i \leq n\}$.

Definitions of $< left(\gamma, h), right(\gamma, h), ldis(\gamma, h), width(\gamma, h) >$

$$left(\gamma, h) = \begin{cases} Min\{i \mid H(M_i) = h\} & \text{if } \exists i \ (0 \leq i \leq n) \text{ such that} \\ & H(M_i) = h \text{ and } \forall j (0 \leq j < i) \ H(M_j) < h \\ \perp & \text{otherwise} \end{cases}$$

$$right(\gamma,h) = \begin{cases} Max\{i \mid H(M_i)=h\} & \text{if } \exists i\ (0 \le i \le n) \text{ such that} \\ & H(M_i)=h \text{ and } \forall j (i<j\le n)\ H(M_j)<h \\ \perp & \text{otherwise} \end{cases}$$

$$ldis(\gamma,h) = \begin{cases} n - left(\gamma,h) & \text{if } left(\gamma,h) \ne \perp \\ \perp & \text{otherwise} \end{cases}$$

$$width(\gamma,h) = \begin{cases} right(\gamma,h) - left(\gamma,h) & \text{if } left(\gamma,h) \ne \perp \bigwedge right(\gamma,h) \ne \perp \\ right(\gamma,h) - left(\gamma,h') & \text{if } left(\gamma,h) = \perp \bigwedge right(\gamma,h) \ne \perp \bigwedge \\ & h' = Min\{h' \mid h' > h \bigwedge left(\gamma,h') \ne \perp\} \\ right(\gamma,h') - left(\gamma,h) & \text{if } left(\gamma,h) \ne \perp \bigwedge right(\gamma,h) = \perp \bigwedge \\ & h' = Min\{h' \mid h' > h \bigwedge right(\gamma,h') \ne \perp\} \\ \perp & \text{otherwise} \end{cases}$$

We write $P(\gamma,h)\downarrow$ if $P(\gamma,h) \ne \perp$ and otherwise $P(\gamma,h)\uparrow$ for $P \in \{left, right, ldis, width\}$.
Example. Let $\delta : f(c) \leftarrow\!\!\!+\!\!\!\rightarrow f(g(g(c))) \leftarrow\!\!\!+\!\!\!\rightarrow f(g(c)) \leftarrow\!\!\!+\!\!\!\rightarrow f(f(g(g(c)))) \leftarrow\!\!\!+\!\!\!\rightarrow f(f(c)) \leftarrow\!\!\!+\!\!\!\rightarrow g(c)$. Then, we have $left(\delta,1) = 0$, $left(\delta,2)\uparrow$, $left(\delta,3) = 1$, $ldis(\delta,1) = 5, ldis(\delta,3) = 4$, $right(\delta,1) = 5$, $right(\delta,3)\uparrow$, $width(\delta,1) = right(\delta,1) - left(\delta,1) = 5$, $width(\delta,2) = 3$, $width(\delta,3) = 2$.

Definitions of $< K_{ldis}(\gamma), K_{width}(\gamma), K_{right}(\gamma) >$

$$\begin{aligned} K_{ldis}(\gamma) &= \{(h, ldis(\gamma,h)) \mid ldis(\gamma,h)\downarrow\} \\ K_{width}(\gamma) &= \{(h, width(\gamma,h)) \mid width(\gamma,h)\downarrow\} \\ K_{right}(\gamma) &= \{(h, right(\gamma,h)) \mid right(\gamma,h)\downarrow\} \end{aligned}$$

Example. For $\delta : f(c) \leftarrow\!\!\!+\!\!\!\rightarrow f(g(g(c))) \leftarrow\!\!\!+\!\!\!\rightarrow f(g(c)) \leftarrow\!\!\!+\!\!\!\rightarrow f(f(g(g(c)))) \leftarrow\!\!\!+\!\!\!\rightarrow f(f(c)) \leftarrow\!\!\!+\!\!\!\rightarrow g(c)$ in the previous example, we have $K_{ldis}(\delta) = \{(1,5),(3,4),(4,2)\}$, $K_{width}(\delta) = \{(1,5),(2,3),(3,2),(4,0)\}$ and $K_{right}(\delta) = \{(1,5),(2,4),(4,3)\}$.

We define an ordering $<_s \subseteq N \times N$ (where $N = \{0,1,2,\cdots\}$) as follows: $(a,b) <_s (a',b') \Leftrightarrow (a < a' \bigwedge b \le b') \bigvee (a = a' \bigwedge b < b')$. Let $\le_s$ be $<_s \bigvee =$. We use $\ll_s$ to denote the multiset ordering of this ordering $<_s$. Let $\lesseqqgtr_s$ be $\ll_s \bigvee =$. We use $\{\cdots\}_m$ to denote a multiset, e.g., $\{1,1,2\}_m$.

3 Weight-Preserving TRS's

Definition of $<$ **E-overlapping TRS** $R >$

A TRS R is said to be E-overlapping iff there exists an ε-invariant reduction sequence $\sigma(\alpha_1/u) \leftarrow\!\!\!+\!\!\!\rightarrow^* \sigma'(\alpha_2)$ for some $\alpha_1 \to \beta_1, \alpha_2 \to \beta_2 \in R$, $u \in \bar{O}(\alpha_1)$ and mappings $\sigma, \sigma' : X \to T$ where $u = \varepsilon$ implies that $(\alpha_1 \to \beta_1) \ne (\alpha_2 \to \beta_2)$. In this case, the pair $(\sigma(\alpha_1)[u \leftarrow \sigma'(\beta_2)], \sigma(\beta_1))$ is called an *E-critical pair.* A TRS R is non-E-overlapping if there exist no E-critical pairs.

Definition of $<$ **depth-preserving TRS** R **([3])** $>$

A TRS R is *depth-preserving* if $\forall \alpha \to \beta \in R\, \forall x \in V(\alpha) \bigcap V(\beta)$ $\quad Max\{|v| \mid v \in O_x(\beta)\} \le Max\{|u| \mid u \in O_x(\alpha)\}$.

Example. $R_2 = \{f(x,x) \to a,\ c \to h(c,g(c)),\ h(x,g(x)) \to f(x,h(x,g(c)))\}$ (where x is a variable) given in Section 1 is depth-preserving, since for the first and the second rules, the right-hand-sides contain no variables, and for the third rule, the maximal depth of the x occurrences of the left-hand-side $h(x,g(x))$ is 2 and that of the right-hand-side $f(x,h(x,g(c)))$ is 2.

Definition of $<$ **strongly depth-preserving TRS** $R >$

A TRS R is *strongly depth-preserving* if R is depth-preserving and $\forall \alpha \to \beta \in R \; \forall x \in V(\alpha) \; \forall u, v \in O_x(\alpha) \quad |u| = |v|$.

Example. Let $R_4 = \{f(x,x) \to a, \; c \to g(c), \; g(x) \to f(x,x)\}$ and $R_5 = \{f(x,x,x) \to h(x,x,x,x,g(c)), c \to g(c)\}$ where x is a variable. Both R_4 and R_5 are strongly depth-preserving. (Note that both R_4 and R_5 are duplicating[6].)

Definition of $<$ weight-preserving TRS R $>$

For a weight function w, a TRS R is *w-weight-preserving* if $\forall \alpha \to \beta \in R \; \forall x \in V(\alpha) \bigcap V(\beta)$ $Max\{W_w(v, \beta) \mid v \in O_x(\beta)\} \leq Max\{W_w(u, \alpha) \mid u \in O_x(\alpha)\}$.

A TRS R is *weight-preserving* if R is w-weight-preserving for some weight function w.
Example. $R_6 = \{f(x,x) \to a, \; c \to h(c, g_1(g_2(c))), \; g(x) \to f(x, h(x, g(c)))\}$. R_6 is w-weight-preserving for a weight function w such that $w(g) = 2$ and the weight of the other symbols are 1. But R_6 is not depth-preserving.

Definition of $<$ strongly weight-preserving TRS R $>$

For a weight function w, a TRS R is *strongly w-weight-preserving* if R is w-weight-preserving and $\forall \alpha \to \beta \in R \; \forall x \in V(\alpha) \; \forall u, v \in O_x(\alpha) \quad W_w(u, \alpha) = W_w(v, \alpha)$.

A TRS R is *strongly weight-preserving* if R is strongly w-weight-preserving for some weight function w.
Example. $R_7 = \{f(x,x) \to a, \; c \to h(c, g_1(g_2(c))), \; h(g_3(x), g_1(g_2(x))) \to f(x, h(x, g(c)))\}$. R_7 is strongly w-weight-preserving for a weight function w such that $w(g_3) = 2$ and the weight of the other symbols are 1. But R_7 is not strongly depth-preserving.

If TRS R is depth-preserving, then R is weight-preserving, since R is w_1-weight-preserving for the weight function w_1 such that $w_1(f) = 1$ for all $f \in F$. And if R is strongly depth-preserving, then R is strongly weight-preserving.

In this section, we show that the TRS

$$R_2 = \{f(x,x) \to a, \; c \to h(c, g(c)), \; h(x, g(x)) \to f(x, h(x, g(c)))\}$$

given in Section 1 is non-E-overlapping and depth-preserving (and weight-preserving), but not CR.

Obviously, R_2 is non-E-overlapping, since there is no pair $(\alpha_1/u, \alpha_2)$ satisfying that the root (topmost) symbols of α_1/u and α_2 are the same for $\alpha_1 \to \beta_1$, $\alpha_2 \to \beta_2 \in R_2$ and $u \in \bar{O}(\alpha_1)$, except that $\alpha_1 = \alpha_2, \beta_1 = \beta_2$ and $u = \varepsilon$. It has already been explained in the above that R_2 is depth-preserving (and weight-preserving).

We can show that TRS R_2 is not CR. Note that
$c \to h(c, g(c)) \to f(c, h(c, g(c))) \to f(h(c, g(c)), h(c, g(c))) \to a$ and $c \to^* h(a, g(a))$.
Thus, $a \leftrightarrow^* h(a, g(a))$ holds, but we can show that a and $h(a, g(a))$ are not joinable.

4 Assertions

We use the following six assertions $S(n), S'(n), P(k), P'(k), Q(k)$ and $Q'(k)$ (where $n \geq 2, k \geq 0$) to prove that non-E-overlapping and strongly depth-preserving TRS R is CR.

Assertions $S(n)$ and $S'(n)$ are similar to the Elimination lemma in [4]. Assertion $Q(k)$ ensures that TRS R is CR.

Assertion $S(n)$
Let $\gamma : \sigma(\beta) \leftarrow \sigma(\alpha) \leftrightarrow\!\!\!\rightarrow^* \sigma'(\alpha) \rightarrow \sigma'(\beta)$ for some rule $\alpha \rightarrow \beta \in R$ and mappings σ, σ' where $|\gamma|_p = n$ and the subsequence $\bar{\gamma} : \sigma(\alpha) \leftrightarrow\!\!\!\rightarrow^* \sigma'(\alpha)$ is ε-invariant.
Then $\exists \delta : \sigma(\beta) \leftrightarrow\!\!\!\rightarrow^* \sigma'(\beta)$ such that the following conditions (i)-(iii) hold:
(i) $|\delta|_p \leq n - 2$
(ii) If β is a variable, then $H(\delta) < H(\gamma)$.
Otherwise, δ is ε-invariant and $H(\delta) \leq H(\gamma)$.
(iii) $K_{ldis}(\delta) \ll_s K_{ldis}(\gamma)$

Assertion $S'(n)$
Let $\gamma : \sigma(\beta) \leftarrow \sigma(\alpha) \leftrightarrow\!\!\!\rightarrow^* \sigma'(\alpha) \rightarrow \sigma'(\beta)$ for some rule $\alpha \rightarrow \beta \in R$ and mappings σ, σ' where $|\gamma|_p = n$ and the subsequence $\bar{\gamma} : \sigma(\alpha) \leftrightarrow\!\!\!\rightarrow^* \sigma'(\alpha)$ is ε-invariant.
Then $\exists \delta : \sigma(\beta) \leftrightarrow\!\!\!\rightarrow^* \sigma'(\beta)$ such that the following conditions (i)-(iii) hold:
(i) $|\delta|_p = |\gamma|_p, |\delta|_{np} \leq |\gamma|_{np} - 2$
(ii) If β is a variable, then $H(\delta) < H(\gamma)$.
Otherwise, δ is ε-invariant and $H(\delta) \leq H(\gamma)$.
(iii) $K_{ldis}(\delta) \lesseqqgtr_s K_{ldis}(\gamma)$ and $K_{right}(\delta) \lesseqqgtr_s K_{right}(\gamma)$.

Note that γ satisfies the same condition in $S(n)$ and $S'(n)$.

Assertion $P(k)$
Let $\gamma : M \leftrightarrow\!\!\!\rightarrow^* \sigma(\alpha) \rightarrow \sigma(\beta)$ for some rule $\alpha \rightarrow \beta \in R$ and mapping σ where $H(\gamma) \leq k$ and the subsequence $\bar{\gamma} : M \leftrightarrow\!\!\!\rightarrow^* \sigma(\alpha)$ is ε-invariant.
Then, there exists $\delta : M \leftrightarrow\!\!\!\rightarrow^* N \leftrightarrow\!\!\!\rightarrow^* \sigma(\beta)$ for some N such that the following conditions (i)-(iii) hold:
(i) $H(\delta) \leq H(\gamma)$
(ii) $M \rightarrow^* N$
(iii) for the subsequence $\delta' : N \leftrightarrow\!\!\!\rightarrow^* \sigma(\beta)$ of δ, either $H(\delta') < H(\gamma)$ or δ' is ε-invariant.

Assertion $P'(k)$
Let $\gamma : M_0 \leftrightarrow\!\!\!\rightarrow M_1 \leftrightarrow\!\!\!\rightarrow M_2 \cdots \leftrightarrow\!\!\!\rightarrow M_n$ where $H(\gamma) \leq k$, the number of ε-reductions in γ is $l (> 0)$ and each ε-reduction is $M_i \xrightarrow{\varepsilon} M_{i+1}$ for some i $(0 \leq i < n)$. Let $M_{i_1} \xrightarrow{\varepsilon} M_{i_1+1}, \cdots, M_{i_l} \xrightarrow{\varepsilon} M_{i_l+1}$ be the ε-reductions of γ, $0 \leq i_1 < i_2 \cdots < i_l < n$. Then, there exist i_j $(1 \leq j \leq l)$ and $\delta : M_0 \leftrightarrow\!\!\!\rightarrow^* N \leftrightarrow\!\!\!\rightarrow^* M_{i_j+1}$ for some N such that the following conditions (i)-(iii) hold:
(i) $H(\delta) \leq H(\gamma[0, i_j + 1])$
(ii) $M_0 \rightarrow^* N$
(iii) for the subsequence $\delta' : N \leftrightarrow\!\!\!\rightarrow^* M_{i_j+1}$ of δ, either $H(\delta') < H(\gamma[0, i_j + 1])$ holds or $i_j = i_l$ and δ' is ε-invariant.

Assertion $Q(k)$
Let $\gamma : M \leftrightarrow\!\!\!\rightarrow^* N$ where $H(\gamma) \leq k$.
Then, $\exists \delta : M \leftrightarrow\!\!\!\rightarrow^* L \leftrightarrow\!\!\!\rightarrow^* N$ for some L such that $H(\delta) \leq k$, $M \rightarrow^* L$ and $N \rightarrow^* L$.

Assertion $Q'(k)$
Let $\gamma_i : M \leftrightarrow\!\!\!\rightarrow^* M_i$, where $H(\gamma_i) \leq k$, $1 \leq i \leq n$ and $n \geq 2$.
Then, $\exists \delta : M \leftrightarrow\!\!\!\rightarrow^* N$ for some N such that $H(\delta) \leq k$ and $\forall i$ $(1 \leq i \leq n)$ $M_i \rightarrow^* N$.

We can prove $S(n) \bigwedge S'(n)$ by induction on $n \geq 2$, and that $P(k) \Rightarrow P'(k)$ and $Q(k) \Rightarrow Q'(k)$ for any $k \geq 0$. Using these results, we can prove $P(k) \bigwedge Q(k)$ by induction on $k \geq 0$. These proofs are omitted.

5 Conclusion

By $Q(k)$ where $k \geq 0$, we have the following result.

Theorem 1
All the non-E-overlapping and strongly depth-preserving TRS's are CR.

For any strongly w-weight-preserving TRS R, we construct a strongly depth-preserving TRS $\bar{R}$ which can simulate reductions of R. For this purpose, we define a set of new function symbols $\bar{F}$ and a translation $\psi : F \to \bar{F}^*$ as follows:

$$\bar{F} = \{f_1, f_2, \cdots, f_k \mid f \in F, w(f) = k\}$$

where $arity(f_i) = 1,\ 1 \leq i < k$ and $arity(f_k) = arity(f)$

$$\psi(f) = f_1 \cdot f_2 \cdot \cdots \cdot f_k \text{ for } f \in F \text{ of } w(f) = k$$

Here, $(f_1 \cdot f_2 \cdot \cdots \cdot f_k)M_1 \cdots M_n = f_1(f_2 \cdots (f_k M_1 \cdots M_n) \cdots)$ for $M_1, \cdots, M_n \in T$.

Translation ψ is extended to the translation: $T \to \bar{T}^*$ as follows:

$$\begin{aligned} \psi(x) &= x \quad \text{for } x \in X \\ \psi(fM_1 \cdots M_n) &= \psi(f)\psi(M_1) \cdots \psi(M_n) \quad \text{for } f \in F, M_1, \cdots, M_n \in T \end{aligned}$$

Here, $\bar{T}$ is the set of terms constructed from X and $\bar{F}$.

Using this translation ψ, we define a new TRS $\bar{R}$ by

$$\bar{R} = \{\psi(\alpha) \to \psi(\beta) \mid \alpha \to \beta \in R\}$$

It is straightforward that if R is non-E-overlapping and strongly w-weight-preserving, then $\bar{R}$ is non-E-overlapping and strongly depth-preserving.

And $\bar{R}$ is CR iff R is CR. Hence, we have the following result.

Theorem 2

All the non-E-overlapping and strongly weight-preserving TRS's are CR.

Example. If the third rule of R_2 in Section 1 is replaced by $h(g_1(g_2(x)), g(x)) \to f(x, h(x, g(c)))$, then we obtain a new TRS

$$R_2' = \{f(x,x) \to a,\ c \to h(c, g(c)),\ h(g_1(g_2(x)), g(x)) \to f(x, h(x, g(c)))\}$$

which is non-E-overlapping and strongly w-weight-preserving for a weight function w such that $w(g) = 2$ and the weight of the other symbols are 1, so that Theorem 2 ensures that R_2' is CR, though R_2 is not CR.

Finally, we remark that Theorem 1 can be extended a little bit, i.e., TRS R is CR if R is non-E-overlapping and satisfies that $\forall \alpha \to \beta \in R\, \forall x \in V(\alpha) \bigcap V(\beta) \quad Min\{|u| \mid u \in O_x(\alpha)\} \geq Max\{|v| \mid v \in O_x(\beta)\}$.

References

[1] N.Dershowitz and J.-P.Jouannaud, Rewrite Systems, Handbook of Theoretical Computer Science, Vol.B, ed. J.van Leeuwen, pp.243-320, North-Holland, Amsterdam(1990).

[2] G.Huet, Confluent reductions:abstract properties and applications to term rewrite systems, J.ACM, 27, pp.797-821(1980).

[3] K.Matsuura, M.Oyamaguchi and Y.Ohta, On the E-overlapping property of nonlinear term rewriting systems, LA symposium(Winter)(1994).

[4] M.Ogawa and S.Ono, On the uniquely converging property of nonlinear term rewriting systems, Technical Report of IEICE, COMP 89-7, pp.61-70(1989).

[5] M.Oyamaguchi and Y.Ohta, On the Confluent Property of Right-Ground Term Rewriting Systems, Trans,IEICE Japan, J76-D-I, pp.39-45(1993).

[6] Y.Toyama and M.Oyamaguchi, Church-Rosser Property and Unique Normal Form Property of Non-Duplicating Term Rewriting Systems, LA Symposium(Winter) (1993) and 4th International Workshop on CTRS(1994).

[7] Y.Ohta, M.Oyamaguchi and Y.Toyama, On the Church-Rosser Property of Simple-Right-Linear TRS's, Trans. IEICE Japan, J78-D-I, pp.263-268(1995).

[8] M.Oyamaguchi and Y.Toyama, On the Church-Rosser property of E-overlapping and simple-right-linear TRS's, Technical Report of IEICE, COMP 94-29, pp.47-56(1994).