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Spaces having σ-compact-finite k-networks

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Introduction

Every CW-complex, more generally, every space dominated by locally separable metric spaces has a star-countable k-network. Also, every Lašnev space has a σ-hereditarily closure preserving (abbr,σ-HCP) k-network, and every space dominated by Lašnev subspaces has a σ-compact-finite k-network. We recall that spaces with a star-countable k-network, and spaces with a σ-HCP k-network have σ-compact-finite k-networks.

Spaces with a star-countable k-network are investigated in [IT], [LT1],[LT3], and [S]. Spaces with a σ-HCP k-network are investigated in [L], and spaces with a compact-countable k-network in [LT3] and [LT2].

In this paper, we shall investigate spaces with a σ-compact-finite k-network and around these spaces, and their applications.

All spaces are regular and T_1, and maps are continuous and onto.

Definitions Let X be a topological space, and let $\mathcal{W}$ be a collection of subsets of X. We recall that $\mathcal{W}$ is compact-finite (star-countable) if for compact subset $K\subset X$ ($Q\in\mathcal{W}$), meets at most finitely (countably) many $P\in\mathcal{W}$. Let $\mathcal{P}$ be a cover of X. Then $\mathcal{P}$ is called a k-network for X, if whenever $K\subset U$ with K compact and U open, then $K\subset\cup\mathcal{P}'\subset U$ for some finite $\mathcal{P}'\subset\mathcal{P}$. Also, $\mathcal{P}$ is called a cs*-network (cs-network) if whenever L is a sequence converging to a point $x\in X$ such that $x\in U$ with U open in X, then there exists $P\in\mathcal{P}$ such that $x\in P$, and

P contains a subsequence of L (L is eventually in P). If X has a σ-locally finite k-network (countable k-network), then X is called $\aleph$-space ($\aleph_0$-space).

Main Results

Theorem 1. Each of the following implies that Y has a σ-compact-finite k-network.

(a) Y has a star-countable k-network.

(b) Y has a σ-hereditarily closure-preserving k-network.

(c) Y is dominated by spaces with a σ-compact-finite k-network.

(d) Y is the closed image of a space X with a σ-compact-finite k-network, and one of the following properties holds.

(1) X is a k-space.

(2) X is a space with G_δ-points.

(3) X is a normal, isocompact space.

(4) X is realcompact.

(5) Each $\partial f^{-1}(y)$ is Lindelöf.

Theorem 2. (CH) Let X be a k-space with a σ-compact-finite k-network. Then X is the topological sum of $\aleph_0$-spaces iff X is locally separable.

Theorem 3. (1) Let X be a k-space with a σ-compact-finite k-network. Then X has a star-countable k-network iff every metric closed subset of X is locally separable.

(2) Let X be a sequential space with a σ-compact-finite cs*-network. Then X is the topological sum of $\aleph_0$-spaces iff every metric closed subset of X is locally separable.

Theorem 4. (1) Suppose that X is determined by a point-countable cover of locally separable metric subsets. If X has a σ-compact-finite k-network, then X has a star-countable k-network.

(2) Suppose that X is determined by a point-countable closed cover of locally separable metric subsets. If X has a point-countable cs-network, then X is a locally $\aleph_0$-space.

Theorem 5. (1) Let X be a separable space. Then each of the following implies that X is an $\aleph_0$-space.

(a) X is a Fréchet space with a point-countable k-network [GMT].

(b) (CH) X is a k-space with a σ-compact-finite k-network. (If X is mata-Lindelöf, or $\chi(X) \leqq \omega_1$, (CH) can be omitted).

(2) Let X be a cosmic space (i.e space with a countable network). If X has a point-countable cs-network, then X is an $\aleph_0$-space.

We recall canonical spaces S_{ω_1}, S_ω, and S_2. S_ω is called the sequential fan, and S_2 is the Arens' space. S_{ω_1}; S_ω is respectively the space obtained from the topological sum of ω_1; ω many convergent sequences by identifying all limit points to a single point.

Theorem 6. Let X be a k-space with a σ-compact-finite k-network. Then X is the quotient s-image of a metric space iff X contains no closed copy of S_{ω_1}.

Theorem 7. (CH) Let X be a k-space with a σ-compact-finite k-network, Then X is weakly first countable iff X contains no closed copy of S_ω.

Theorem 8. (CH) Let X be a k-space with a σ-compact-finite k-network. Then X is a Lašnev space iff X contains no closed copy of S_2.

References

[IT] Y. Ikeda and Y. Tanaka, Spaces having star-countable k-networks, Top. Proc., 18(1993), 107-132.

[L] C. Liu, Spaces with a σ-hereditarily closure preserving k-network, Top. Proc., 18(1993), 179-188.

[LT1] C. Liu and Y. Tanaka, Spaces with a star-countable k-network, and related results, to appear in Top, Appl.

[LT2] --------------------, Spaces with certain compact-countable k-networks, and questions, Q & A in Gen. Top., 14(1996), 15-37.

[LT3] --------------------, Spaces having star-countable or compact-countable k-networks, to appear

[S] M. Sakai, On spaces with star-countable k-networks, to appear