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A metrical result on transcendence measures in certain fields

(Abstract)

by

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Let $\mathbb{F}_q$ be a finite field of characteristic p with q elements. Then we denote by $A = \mathbb{F}_q[T]$ the ring of polynomials in a variable T over $\mathbb{F}_q$, $K = \mathbb{F}_q(T)$ the quotient field of A, and $K_\infty = \mathbb{F}_q((1/T))$ the field of formal power series in $1/T$ over $\mathbb{F}_q$.

The purpose of the present talk is to give transcendence measures for almost all elements of K_∞ with respect to a Haar measure on K_∞.

As in Sprindžuk [4], we have a non-archimedean absolute value $|\cdot|$ on K_∞. That is, for any element $\omega \in K_\infty$ of the form

$$\omega = \sum_{\nu=\ell}^{\infty} a_\nu T^{-\nu} \quad (a_\nu \in \mathbb{F}_q,\ \nu \in \ell, \ell + 1, \ldots),$$

we define $|\omega| = q^{-\ell}$ if $a_\ell \neq 0$, $|\omega| = 0$ if $a_\nu = 0$ for all $\nu \geq \ell$. We know that K_∞ is complete and locally compact with respect to the metric defined by this absolute value. Let μ be the Haar measure normalized by

$$\mu(\mathbb{D}) = 1, \text{ where } \mathbb{D} = \{\omega \in K_\infty;\ |\omega| \leq q^{-1}\}.$$

For any nonzero polynomial $P(x) = \sum_{i=1}^{d} a_i x^i \in A[x]$, we define *logarithmic height* $\hat{h}(P)$ of P by

$$\hat{h}(P) = \log_q \max_{0 \le i \le d} |a_i|,$$

where $\log_q x$ means the logarithmic function with base q. Then our main result is stated as follows.

Theorem. *Let* ε *be an arbitrary positive number. Then, for almost all* $\omega \in K_\infty$ *(w.r.t.* μ*), we have*

$$|P(\omega)| \ge q^{-(3+\varepsilon)dh} \min(1, |\tilde{\omega}|)^d$$

for all nonzero polynomials $P \in A[x]$ *with* $\deg P \le d$, $\hat{h}(P) \le h$, *and* $\max(d,h) \ge c(\omega,\varepsilon)$, *where* $\tilde{\omega} = \omega -$ *(the constant term of* ω*) and* $c(\omega,\varepsilon)$ *is a positive constant depending only on* ω *and* ε.

We see that this lower bound is fairy good. In fact, by Hilfssatz 3 of Bundschuh [2], for any $\omega \in K_\infty$ and any positive integers d,h, there exists a nonzero polynomial $P \in A[x]$ with $\deg P \le d$, $\hat{h}(P) \le h$ satisfying

$$|P(\omega)| \le q^{-d(h-1)-1} \max(1, |\omega|)^d.$$

For the proof of the theorem, We refer to the paper [1].

References

[1] M. Amou, A metrical result on transcendence measures in certain fields, preprint.

[2] P. Bundschuh, Transzendenzmasse in Körpern formaler Laurentreihen, J. reine angew. Math. 300 (1978), 411-432.

[3] G.V. Chudnovsky, Contributions to the theory of transcendental numbers, Amer. Math. Soc., Mathematical surveys and monographs, no. 19, Providence, R. I., 1984.

[4] V.G. Sprindžuk, Mahler's problem in metric number theory, Amer. Math. Soc., Translations of mathematical monographs, vol. 25, Providence, R. I., 1969.

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