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Value Distribution Theory over Function Fields and a Diophantine Equation

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§1. Introduction.

This is a report of the author's late work [No9].

S. Lang [L] conjectured in 1974 that a hyperbolic algebraic variety defined over a number field has only finitely many rational points, and its analogue over function fields. For subvarieties of Abelian varieties the function field analogue was dealt with by M. Raynaud [R], and lately G. Faltings [F] proved this conjecture for subvarieties of Abelian varieties over number fields. On the other hand, the author [No6] proved the function field analogue in general case (cf. also [No1], [No2]). That is, let $\bar{\pi}: \bar{X} \to \bar{R}$ be a compact complex fiber space with irreducible general fibers such that there is a non-empty smooth Zariski open subset $R \subset \bar{R}$ and the restriction $\pi = \bar{\pi}|X : X \to R$ with $X = \bar{\pi}^{-1}(R)$ over R is hyperbolic. Then there are only finitely many compact complex spaces $Y_\mu, \mu = 1, \ldots, \mu_0$, and holomorphic mappings $\phi_\mu : R \times Y_\mu \to X$ such that any holomorphic section σ of (X, π, R) is given by $\sigma(t) = \phi(t, y)$ with some $y \in Y_\mu$, provided that some geometric condition on the boundary fibers is satisfied. See Y. Imayoshi-H. Shiga [IS], M. Zaidenberg [Z], and M. Suzuki [Su1], [Su2] for non-compact versions of this result.

In the case of curves (Fermat, Catalan, Thue equations, etc.) defined over function fields, R.C. Mason [Ma1], J. Silberman [Si] and J. Mueller [Mu] obtained similar or more precise finiteness properties by making use of a different method which relies on the function field analogue of "abc-conjecture" of Masser-Oesterlé. The function field analogue of "abc-conjecture" was proved in more general form by R.C. Mason [Ma1], [Ma2], J. Voloch [Vo] and W. Brownawell-D. Masser [BrM]. They actually proved a version of "abc-conjecture" in several variables, which is nothing but a special case of Nevanlinna-Cartan's second main theorem with truncated counting functions applied to algebraic case (see [C, (3)] and §1).

In §2 of this note we discuss the Nevanlinna-Cartan theory over function fields and explain the main results. In §3 we apply the same idea to obtain a finiteness theorem for S-units points of a Diophantine equation over number fields. §4 is devoted to examples.

For general references of Diophantine problems in the present direction, see S. Lang [L1], [L2], [L4], P. Vojta [V] and [No3], [No5], [No8], and for hyperbolic manifolds and the Nevanlinna theory, see S. Kobayashi [K1], [K2], [NO], S. Lang [L3] and [No8].

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§2. Value distribution theory over function fields and main results.

We first demonstrate the Nevanlinna-Cartan theory over function fields. We will systematically use current equations combined with the method of Cartan [C], which make the arguments more geometric and simpler than those employed in [V] or [BrM]. Let k be an algebraically closed field of characteristic 0, and let K be a function field of one variable with genus g over the constant field k. The First Main Theorem (F.M.T.) is nothing but Poincaré duality or residue theorem. Then we prove the "Second Main Theorem with truncated counting function" over function fields:

Theorem (S.M.T.). *If $x_j \in K, 0 \leq j \leq m$, are linearly independent over k, we have*

$$(q-m-1)\mathrm{ht}((x_j)) \leq \sum_{i=1}^{q} N_m(H_i(x_j)) + m(m+1)(g-1),$$

where H_i are linear forms in general position, and $N_m(H_i(x_j))$ is the truncated counting function of zeros of $H_i(x_j)$.

This theorem plays an important role in the present work. We derive a version of "*abc*-conjecture" in several variables over function fields (Corollary (2.16); cf. [Ma2], [Vo], and [BrM], Corollary I). We also show "Ramification Theorem" over function fields, and "Generalized Borel's Lemma" over functions fields. To state the latter, we take an equation

$$a_1 x_1^d + \cdots + a_s x_s^d = 0 \qquad (s \geq 2), \tag{2.1}$$

where $a_j \in K^*$, and $d \in \mathbf{Z}, d \geq 1$. The original Borel's Lemma deal with case where $a_j = 1$ and x_j are entire functions without zero, and hence d can be increased as much as you like.

(2.2) Generalized Borel's Lemma. *Let $x_j \in K^*, 1 \leq j \leq s$, satisfy* (2.1). *Assume*

$$d > s(s-2) + (s-1)^2 \mathrm{ht}(a_1, \ldots, a_s) + (s-1)(s-2)\max\{0, g-1\}.$$

Then there is a disjoint decomposition $\{1, \ldots, s\} = \cup_{\nu=1}^{l} I_\nu$ of indices such that

(i) $|I_\nu| \geq 2$ *for all* ν;

(ii) *for arbitrary two indices $j, k \in I_\nu$, the ratio x_j/x_k is a constant.*

(iii) $\sum_{j \in I_\nu} a_j x_j^d = 0$ *for all* ν.

This is important and is used to establish the Main Theorem in below.

We consider a Diophantine equation of the following type. Let $\{M_j(z_1, \ldots, z_n)\}_{j=1}^s$ be a set of monomials of the same degree d_0 in variables $z_1, \ldots, z_n$. Let $X \subset \mathbf{P}_K^{n-1}$ be a hypersurface defined over K by equation

$$a_1 M_1^d(z_1, \ldots, z_n) + \cdots + a_s M_s^d(z_1, \ldots, z_n) = 0, \tag{2.3}$$

where $a_j \in K^* = K \setminus \{0\}$ and $d \in \mathbf{Z}$ is a positive integer.

We use the notion of n-admissibility for $\{M_j\}$ due to [MN, §2], which is used to construct hyperbolic hypersurfaces of $\mathbf{P}^n(\mathbf{C})$ for any $n \geq 2$, partially answering to a conjecture of S. Kobayashi. (Cf. §4 for a number of examples.) Essentially, n-admissibility requires the exponent vectors of many monomials are located generically, not lying in a finite union of affine hypersurfaces (cf. Remark (2.4), (v)). We will then prove that hyperbolic projective hypersurfaces constructed by [MN] enjoy the same finiteness property as mentioned in §0, when they are defined over function fields.

To state the result, we introduce some notation. Let $\mathrm{P} = (z_1, \ldots, z_n) \in X(K)$ be a K-rational point of X. We denote by $\mathrm{ht}(\mathrm{P}) = \mathrm{ht}((z_i))$ the (projective) height of the point P. We set

$$Y(\mathrm{P}) = \Big\{(u_1, \ldots, u_n) \in \mathbf{P}^{n-1}(k); \sum_j a_j M_j^d(z_1, \ldots, z_n) M_j^d(u_1, \ldots, u_n) = 0, \\ \text{and } u_j = 0 \text{ if } z_j = 0\Big\}.$$

Then $Y(\mathrm{P})$ is a projective variety defined over k. Moreover, we set

$$\mathcal{R}(\mathrm{P}) = \{(z_1 u_1, \ldots, z_n u_n) \in \mathbf{P}^{n-1}(K); (u_1, \ldots, u_n) \in Y(\mathrm{P})\} \subset X(K).$$

Main Theorem. *Let the notation be as above. Assume that $\{M_j(z_1, \ldots, z_n)\}_{j=1}^s$ is n-admissible.*

(i) *Assume that*

$$d > s(s-2).$$

Then the heights $\mathrm{ht}((z_i))$ of points of $X(K)$ are bounded, so that there is a projective variety Y over k, not necessarily irreducible, and a morphism $\Phi : Y_K \to X$ over K such that $X(K) = \Phi(Y_K)$.

(ii) *Assume that*

$$d > s(s-2) + (s-1)^2 \mathrm{ht}(a_1, \ldots, a_s) + (s-1)(s-2)\max\{0, g-1\}.$$

Then all points of $X(K)$ are defined over k; that is, $X(K) = X(k)$.

(iii) *Assume that*

$$d > s!(s!-2) + (s!-1)(s!-2)\max\{0, g-1\}.$$

Then there are only finitely many rational points $\mathrm{P}_\mu \in X(K)$, $\mu = 1, \dots, \mu_0$ $(< \infty)$ *such that*

$$X(K) = \bigcup_{\mu=1}^{\mu_0} \mathcal{R}(\mathrm{P}_\mu).$$

(2.4) *Remark.* (i) If X is defined over $\mathbf{C}$, and $\{M_j(z_1, \dots, z_n)\}_{j=1}^{s}$ is n-admissible, then the condition in (i) of the Main Theorem implies the hyperbolicity of X ([MN]).

(ii) The general theorem obtained by [No6] cannot be applied, since the present X does not satisfy in general the geometric condition on the boundary fibers needed for it (see [No2], [No7]).

(iii) There is a similar result to the Main Theorem for Abelian varieties over function fields with level structure (see A. Nadel [N] and [No4]).

(iv) The equations defining these hyperbolic hypersurfaces may be the first examples of single equations in several variables which satisfy such a finiteness property. It strongly suggests that those equations has only finitely many rational points if they are defined over number fields (cf. §3).

(v) The implication of the condition, $\{M_j\}$ being admissible, used in the above arguments is that solutions of equations

$$z_1^{\alpha_{j_\nu 1} - \alpha_{k_\nu 1}} \cdots z_m^{\alpha_{j_\nu m} - \alpha_{k_\nu m}} = 1, \qquad 1 \leq \nu \leq l$$

in $\mathbf{P}^{m-1}(k)$ with any choice of such indices $j_\nu < k_\nu$ are isolated. Therefore, even if $\{M_j(z_i)\}_{j=1}^{s}$ is not n-admissible, there is a case where the Main Theorem remains valid by generic choice of the coefficients $a_j \in K^*$ in (2.3); that is, $(a_1, \dots, a_s) \in \mathbf{P}^{s-1}(K)$ with $a_j \in K^*$ lies out of a proper algebraic subset. In this case we say that X or (2.3) is of *generic case*. If $\{M_j(z_i)\}_{j=1}^{s}$ is n-admissible, we say that X or (2.3) is of *admissible case*. Cf. [MN, Theorem (3.10), §3].

§3. S-units points.

In this section we deal with X defined by (2.3) over a number field. Let F be a number field and let S be a finite set of places of F containing all infinite places, and $\mathcal{O}_S^*$ be the set of all S-units of F. We prove Borel's Lemma for S-units by making use of Schmidt's linear subspace theorem.

Borel's Lemma for S-units. *Let $\mathcal{Z}$ be the set of all S-unit solutions of equation*

$$a_1x_1 + \cdots + a_sx_s = 0 \qquad (s \geq 2) \tag{3.1}$$

with $a_j \in F^*$. Then there is a finite decomposition $\mathcal{Z} = \cup_{\mu=1}^{\mu_0}\mathcal{Z}_\mu$ ($\mu_0 < \infty$) such that for every fixed $\mathcal{Z}_\mu$, $1 \leq \mu \leq \mu_0$, there is a decomposition of indices

$$\{1, \ldots, s\} = \bigcup_{l=1}^{m} I_l$$

satisfying the following conditions:

(i) $|I_l| \geq 2$ *for all* l.

(ii) *If we write $\mathcal{Z}_\mu = \{(x_i(\zeta)); \zeta \in \mathcal{Z}_\mu\}$ and take an arbitrarily fixed I_l, then*

$$\frac{x_j(\zeta)}{x_k(\zeta)} = c_{jk} \in \mathcal{O}_S^*$$

are independent of $\zeta \in \mathcal{Z}_\mu$ for all $j, k \in I_l$.

(iii) $\sum_{j \in I_l} a_j x_j(\zeta) = 0$ *for $\zeta \in \mathcal{Z}_\mu$ and $l = 1, 2, \ldots, m$.*

Let $X \subset \mathbf{P}_F^{n-1}$ be a projective variety defined by (2.3) with $a_j \in F^*$, where $d \geq 1$ is arbitrary. A rational point $(x_1, \ldots, x_n) \in X(F)$ is called an *S-unit point* if either $x_j = 0$ or $x_j \in \mathcal{O}_S^*$, and the set of all S-unit points of X is denoted by $X(\mathcal{O}_S^*)$.

(3.2) Theorem. *Let the notation be as above. If $\{M_j(z_1, \ldots, z_n)\}_{j=1}^{s}$ is n-admissible, then there are only finitely many S-unit points of X for any $d \geq 1$.*

Remark. (i) Since the set $\{z_1, z_2, z_3\}$ of monomials in the variables, z_1, z_2, and z_3, is 3-admissible, Theorem (5.7) generalizes Mahler's finiteness theorem ([M, p. 724, Folgerung 2]).

(ii) Here we like to emphasize the following. The Main Theorem is proved by making use of Generalized Borel's Lemma over function fields derived from the Second Main Theorem with truncated counting functions over function fields (S.M.T.). The way of the proof of the Main Theorem is completely parallel to that of the Main Theorem of [MN] proving that if X is defined over complex numbers, $a_j \in \mathbf{C}^*$, then X with (2.4) is hyperbolic. The proof of Theorem (3.2) is also based on the same idea. Therefore, if any of these key theorems was established over number fields, then X defined over number fields would have only finitely many rational points. Especially, S.M.T. is naturally translated over number fields to a statement similar to W. Schmidt's linear subspace theorem with coefficient $(q-m-1-\epsilon)$ of $\mathrm{ht}((x_j))$ instead of $(q-m-1)$ and with the sum of valuations truncated at order n over the complement of a finite set of places containing all infinite places, where $m+1$ is the number of variables (cf. [Sc], [S], and [V] for W. Schmidt's linear subspace theorem). We refer this as Arithmetic Second Main Theorem (A.S.M.T.) conjecture. This is the natural generalization of abc-conjecture in several variables. A.S.M.T. would imply that

$$X(F) \text{ is finite !!}$$

§4. Examples

We recall some examples from [MN, §5]. In what follows, coefficients t and t_i are not 0. The bounds for d given below are those due to (2.4), by which the Main Theorem, (i) holds.

a) In $\mathbf{P}^3(\mathbf{C})$ we have the following examples:

$$z_1^d + \cdots + z_4^d + t(z_1 z_2 z_3)^{d/3} = 0, \qquad 3|d \geq 24.$$
$$z_1^d + \cdots + z_4^d + t(z_1 z_2 z_3 z_4)^{d/4} = 0, \qquad 4|d \geq 28.$$

The above hypersurfaces are defined by 4-admissible sets of monomials, so that they are of admissible case. Here if we simply apply (2.4) for d, we have $3|d \geq 48$ (resp. $4|d \geq 64$) in the case of the first (resp. second) example. By arguments similar to those in [MN, §5] we see that the above bounds suffice.

$$z_1^d + \cdots + z_4^d + t_1(z_1 z_2)^{d/2} + t_2(z_2 z_3)^{d/2} = 0, \qquad 2|d \geq 50.$$

This is not of admissible case, but of generic case. In the present example, one may put $t_1 = t_2 = 1$.

b) In $\mathbf{P}^4(\mathbf{C})$ we have the following examples:

$$z_1^d + \cdots + z_5^d + t_1(z_1^2 z_2)^{d/3} + t_2(z_2^2 z_3)^{d/3} + t_3(z_3^2 z_4)^{d/3} + t_4(z_4^2 z_1)^{d/3} = 0, \qquad 3|d \geq 192.$$

This is of generic case. One may put $(t_j) = (-1,-1,1,1)$.

$$z_1^d + \cdots + z_5^d + t_1(z_3 z_4^2 z_5)^{d/4} + t_2(z_1 z_2^2 z_5)^{d/4} + t_3(z_1 z_2 z_3^2)^{d/4} = 0, \qquad 4|d \geq 196.$$

This is of generic case, and one may put $t_1 = t_2 = t_3 = 1$.

$$z_1^d + \cdots + z_5^d + t_1(z_1^2 z_2)^{d/3} + t_2(z_2^2 z_3)^{d/3} + t_3(z_3^2 z_4)^{d/3} + t_4(z_4^2 z_5)^{d/3} + t_5(z_5^2 z_1)^{d/3} = 0,$$
$$3|d \geq 243.$$

This is of generic case, too, and one may put all $t_j = 1$.

$$z_1^d + \cdots + z_5^d + t_1(z_1^3 z_2)^{d/4} + t_2(z_2^3 z_3)^{d/4} + t_3(z_3^3 z_4)^{d/4} + t_4(z_4 z_1)^{d/2} = 0, \qquad 4|d \geq 256$$

This is of admissible case.

c) In $\mathbf{P}^5(\mathbf{C})$ we have the following examples.

$$\begin{aligned} &z_1^d + \cdots + z_6^d + t_1(z_1 z_2^3)^{d/4} + t_2(z_2 z_3^3)^{d/4} + t_3(z_3 z_4^3)^{d/4} \\ &+ t_4(z_4 z_5^3)^{d/4} + t_5(z_5 z_1^3)^{d/4} + t_6(z_1 z_3)^{d/2} \\ &+ t_7(z_2 z_4)^{d/2} + t_8(z_3 z_5)^{d/2} + t_9(z_4 z_1)^{d/2} = 0, \end{aligned}$$
$$4|d \geq 784.$$

This is of generic case, and one may put $t_1 = -1$ and other $t_j = 1$.

$$\begin{aligned} &z_1^d + \cdots + z_6^d \\ &+ t_1(z_1 z_2^4)^{d/5} + t_2(z_2^2 z_3^3)^{d/5} + t_3(z_3^2 z_4^3)^{d/5} + t_4(z_4 z_5^4)^{d/5} + t_5(z_5^3 z_1^2)^{d/5} \\ &+ t_6(z_1^4 z_3)^{d/5} + t_7(z_2 z_4^4)^{d/5} + t_8(z_3^3 z_5^2)^{d/5} = 0, \end{aligned}$$
$$5|d \geq 845.$$

This is of generic case, and one may put all $t_j = 1$.

$$\begin{aligned} &z_1^d + \cdots + z_6^d + t_1(z_1 z_2^3)^{d/4} + t_2(z_2 z_3^3)^{d/4} + t_3(z_3 z_4^3)^{d/4} \\ &+ t_4(z_4 z_5^3)^{d/4} + t_5(z_5 z_1^3)^{d/4} + t_6(z_1 z_3)^{d/2} + t_7(z_2 z_4)^{d/2} \\ &+ t_8(z_3 z_5)^{d/2} + t_9(z_1 z_4)^{d/2} + t_{10}(z_2 z_5)^{d/2} = 0, \end{aligned}$$
$$4|d \geq 900.$$

This is of generic case, and one may put $t_1 = -1$ and other $t_j = 1$.

$$\begin{aligned} &z_1^d + \cdots + z_6^d \\ &+ t_1(z_1 z_2^5)^{d/6} + t_2(z_2 z_3^5)^{d/6} + t_3(z_3 z_4^5)^{d/6} + t_4(z_4 z_5^2)^{d/3} + t_5(z_5 z_1^2)^{d/3} \\ &+ t_6(z_1 z_3)^{d/2} + t_7(z_2 z_4)^{d/2} + t_8(z_3 z_5)^{d/2} = 0, \end{aligned}$$
$$6|d \geq 1014.$$

This is of generic case, and one may put all $t_j = 1$.

$$\begin{aligned} &z_1^d + \cdots + z_6^d + t_1(z_1 z_2^4)^{d/5} + t_2(z_2 z_3^4)^{d/5} + t_3(z_3 z_4^4)^{d/5} + t_4(z_4 z_5^4)^{d/5} + t_5(z_5 z_1^4)^{d/5} \\ &+ t_6(z_1^2 z_3^3)^{d/5} + t_7(z_2^2 z_4^3)^{d/5} + t_8(z_3^2 z_5^3)^{d/5} + t_9(z_4^2 z_1^3)^{d/5} + t_{10}(z_5^2 z_2^3)^{d/5} = 0, \end{aligned}$$
$$5|d \geq 1125.$$

This is of admissible case.

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