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Second Betti numbers of semigroup rings

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Introduction

In this article we consider the second Betti numbers which appear in minimal free resolutions of certain affine semigroup rings. In particular, we treat two kinds of affine semigroup rings. One is semigroup rings which are associated with distributive lattices. And the other is a monomial curve whose second Betti number is dependent on the base field.

Let L be a finite distributive lattice (see §1 for definitons) and A the polynomial ring $k[X_\alpha|\alpha \in L]$ over a field k with the standard grading, i.e., Deg $X_\alpha = 1$ for each $\alpha \in L$. We define $\mathcal{R}_k[L]$ to be the quotient algebra

$$\mathcal{R}_k[L] \ = A/(X_\alpha X_\beta - X_{\alpha\wedge\beta}X_{\alpha\vee\beta} \mid \alpha \not\sim \beta)$$

of A, where we write $\alpha \not\sim \beta$ if α and β are incomparable in L. Then $\mathcal{R}_k[L]$ is a typical example of an ASL as well as an affine semigroup ring. See Hibi[Hi$_1$] for detailed information. It has been conjectured that the second Betti number of $\mathcal{R}_k[L]$ is independent of a base field k. In this paper we give a combinatorial formula for the second Betti numbers of $\mathcal{R}_k[L]$, when L is a planar distributive lattice. In particular, we show that it is independent of the base field k. The algebra $\mathcal{R}_k[L]$ associated with a planar distributive lattice L can be viewed as a ladder determinantal ring of 2-minors. Hence our result may be considered as a generalization of the 2-minor case of Kurano's result([Ku]).

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§1. Preliminaries

We recall basic definitions and properties in commutative ring theory and in combinatorics concerning with the algebras $\mathcal{R}_k[L]$. Refere to [Hi$_1$] for further information. See also [Bu-He], [Hi$_2$] and [St$_1$] as gereral references.

(1.1) Let $A = k[X_1, X_2, \ldots, X_v]$ denote the polynomial ring in v-variables over a field k, which will be considered to be the graded algebra $A =$

$\bigoplus_{n \geq 0} A_n$ over k with the standard grading, i.e., Deg $X_i = 1$ for each i. Let $\mathbf{Z}$ (resp. $\mathbf{Q}$, $\mathbf{N}$) denote the set of integers (resp. rational numbers, non-negative integers). We write $A(j)$, $j \in \mathbf{Z}$, for the graded module $A(j) = \bigoplus_{n \in \mathbf{Z}} [A(j)]_n$ over A with $[A(j)]_n := A_{n+j}$. Let I be an ideal of A generated by homogeneous polynomials and R the quotient algebra A/I. When R is regarded as a graded module over A with the quotient grading, it has a graded *finite free resolution*

$$0 \longrightarrow \bigoplus_{j \in \mathbf{Z}} A(-j)^{\beta_{h,j}} \xrightarrow{\varphi_h} \cdots \xrightarrow{\varphi_2} \bigoplus_{j \in \mathbf{Z}} A(-j)^{\beta_{1,j}} \xrightarrow{\varphi_1} A \xrightarrow{\varphi_0} R \longrightarrow 0; \quad (1)$$

where each $\bigoplus_{j \in \mathbf{Z}} A(-j)^{\beta_{i,j}}$, $1 \leq i \leq h$, is a graded free module of rank $0 \neq \sum_{j \in \mathbf{Z}} \beta_{i,j} < \infty$, and where every φ_i is degree-preserving. Moreover, there exists a unique such resolution which minimizes each $\beta_{i,j}$; such a resolution is called *minimal.* If a finite free resolution (1) is minimal, then the *homological dimension* $\mathrm{hd}_A(R)$ of R over A is the non-negative integer h and $\beta_i = \beta_i^A(R) := \sum_{j \in \mathbf{Z}} \beta_{i,j}$ is called the i-th *Betti number* of R over A.

(1.2) All partially ordered sets ("poset" for short) to be considered are finite. A *poset ideal* in a poset P is a subset I such that $\alpha \in I, \beta \in P$ and $\beta \leq \alpha$ together imply $\beta \in I$. A *clutter* is a poset in which no two elements are comparable. A *lattice* is a poset L such that any two elements α and β of L have a greatest lower bound $\alpha \wedge \beta$, and a least upper bound $\alpha \vee \beta$. A subposet P of a lattice L is called a *sublattice* of L if both $\alpha \wedge \beta$ and $\alpha \vee \beta$ belong to P for all α, $\beta \in P$. We say that a lattice L is *distributive* if the equalities $\alpha \wedge (\beta \vee \gamma) = (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$ and $\alpha \vee (\beta \wedge \gamma) = (\alpha \vee \beta) \wedge (\alpha \vee \gamma)$ hold for all α, β, $\gamma \in L$. The following theorem is very importamt in lattice theory.

Fundamental theorem for finite distributive lattices. *For every finite distributive lattice L there exists a unique poset P such that $D = J(P)$, where $J(P)$ is a poset which consists of all poset ideals of P, ordered by inclusion.*

A distributive lattice $D = J(P)$ is called *planar* if P contains no three-element clutter.

(1.3) Let L be a finite distributive lattice and A the polynomial ring $k[X_\alpha | \alpha \in L]$ over a field k with the standard grading, i.e., Deg $X_\alpha = 1$ for each $\alpha \in L$. We define $\mathcal{R}_k[L]$ to be the quotient algebra

$$\mathcal{R}_k[L] \ = A/(X_\alpha X_\beta - X_{\alpha \wedge \beta} X_{\alpha \vee \beta} \mid \alpha \not\sim \beta)$$

of A, where we write $\alpha \not\sim \beta$ if α and β are incomparable in L. The algebra $\mathcal{R}_k[L]$ is a graded algebra with the quotient grading. A monomial

$X_{\alpha_1}X_{\alpha_2}\cdots X_{\alpha_q}$ of $\mathcal{R}_k[L]$ is called *standard* if $\alpha_1 \le \alpha_2 \le \cdots \le \alpha_q$ in L. The set of standard monomials is a basis of $\mathcal{R}_k[L]$ as a vector space over k.

A minimal free resolution of $\mathcal{R}_k[L]$ over A is of the form

$$\cdots \to A(-3)^{\beta_{2,3}} \oplus A(-4)^{\beta_{2,4}} \to A(-2)^{\beta_1} \to A \to \mathcal{R}_k[L] \to 0.$$

It is known that $\beta_{2,3}$ can be computed by combinatorics on L; in particular $\beta_{2,3}$ is independent of the base field k. We then focus our attention on $\beta_{2,4}$.

(1.4) We prepare some notation and terminology to state our result on β_{2_4}. Let L be a lattice. Let α and β with $\alpha \le \beta$ be elements in L. We say that α and β are *changeable* in L if there exist γ and δ with $\gamma \not\sim \delta$ in L such that $\gamma \wedge \delta = \alpha$ and $\gamma \vee \delta = \beta$. A sublattice C of L is said to be *closed* in L if the following condition is satisfied: if $\alpha, \beta \in C$ and $\gamma, \delta \in L$, and if $\gamma \wedge \delta = \alpha$, $\gamma \vee \delta = \beta$, then $\gamma, \delta \in C$. Let B be a subposet of L. We define the *closure* $C_L(B)$ of B in L to be the minimal closed sublattice including B in L.

We study two partial orders " $\le_{\text{comp}}$ " and " $\le_{\text{rlex}}$ " on $\mathbf{N}^2$ defined by

$$\begin{aligned} (a,\ b) \le_{\text{comp}} (c,\ d) &\iff a \le c \text{ and } b \le d; \\ (a,\ b) \le_{\text{rlex}} (c,\ d) &\iff b < d, \text{ or } b = d \text{ and } a \le c. \end{aligned}$$

Let L be a planar distributive lattice and $\alpha_1 \le \alpha_2 \le \cdots \le \alpha_q$, with each $\alpha_i \in L$, a "multichain" of L. Set $C = C_L(\{\alpha_1, \alpha_2, \cdots, \alpha_q\})$. We now define type C (or type $\alpha_1\alpha_2\cdots\alpha_q$) by

$$\text{type } C = \min_{f} {}_{\le_{\text{rlex}}} \{f(\alpha_q) \in \mathbf{N}^2\},$$

where $f : C \hookrightarrow \mathbf{N}^2$ runs through all order-preserving inclusion maps with respect to " $\le_{\text{comp}}$ " from C to $\mathbf{N}^2$ with $\alpha_1 \mapsto (1,1)$. We then fix an order-preserving inclusion map $f : C \hookrightarrow \mathbf{N}^2$ with $\alpha_1 \mapsto (1,1)$ and $\alpha_q \mapsto$ type $C = (i,j)$. We identify C with the image $f(C) \subset \mathbf{N}^2$ and may regard C as a sublattice of $\mathbf{N}^2$. We then say that C is a *one-sided ladder* if $(i,1) \in C$ or $(1,j) \in C$, which is independent of the choice of inclusion maps f satisfying the above conditions.

§2. Main result

Now we state our main theorem.

(2.1) THEOREM. *Let L be a planar distributive lattice. Then $\beta_{2,4} = \beta_{2,4}^A(\mathcal{R}_k[L])$ is equal to the number of sequences $(\alpha_1, \alpha_2, \alpha_3, \alpha_4) \in L^4$ with*

$\alpha_1 \leq \alpha_2 \leq \alpha_3 \leq \alpha_4$ *such that* (i) α_1 *and* α_2 *are changeable*; (ii) α_3 *and* α_4 *are changeable*; (iii) $C_L(\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\})$ *is not a one-sided ladder. In particular, the second Betti number* $\beta_2^A(\mathcal{R}_k[L])$ *is independent of the base field* k.

A combinatorial technique developed by Eagon and Roberts [Ea-Ro] is indispensable to prove Theorem (2.1). Such technique is also essential in Andersen [An], where she studies a counterexample to a certain problem on a resolution of a determinantal ideal of a generic symmetric matrix.

(2.2) (See [Ha, pp. 48–49].) Let $R = k[m_1, \cdots, m_v] \subset k[T_0, \cdots, T_r]$ be a semigroup ring over a field k, where $m_i = T_0^{a(i)_0} \cdots T_r^{a(i)_r}$ is a monomial for each $1 \leq i \leq v$. Let $A = k[X_1, \cdots, X_v]$ be a polynomial ring. We define $\deg X_i = (a(i)_0, \cdots, a(i)_r) \in \mathbf{N}^{r+1}$. Then A is an $\mathbf{N}^{r+1}$-graded algebra over k, and the surjective map $A \to R$ given by $X_i \mapsto m_i$ is a homomorphism of $\mathbf{N}^{r+1}$-graded algebras over k. Hence, there is a unique $\mathbf{N}^{r+1}$-graded minimal free resolution of R as an A-module, and $\mathrm{Tor}_i^A(k, R)$ is also $\mathbf{N}^{r+1}$-graded for every $i \geq 0$.

We express $\mathrm{Tor}_i^A(k, R)$ in terms of reduced homology of a certain simplicial complex. Let $\lambda = (\lambda_0, \cdots, \lambda_r) \in \mathbf{N}^{r+1}$. We define a simplicial complex Σ_λ as follows. The vertex set is (a subset of) $\{X_1, \cdots, X_v\}$. A subset $\{X_{i_1}, \cdots, X_{i_s}\}$ $(1 \leq i_1 < \cdots < i_s \leq v)$ is an $(s-1)$-face of Σ_λ if and only if the monomial $m_{i_1} \cdots m_{i_s}$ divides $T^\lambda = T_0{}^{\lambda_0} \cdots T_r{}^{\lambda_r}$ and $T^\lambda / m_{i_1} \cdots m_{i_s} \in R$.

(2.3) LEMMA (cf. [St$_1$,Theorem 7.9]). *We have an isomorphism as vector spaces*

$$[\mathrm{Tor}_i^A(k, R)]_\lambda \cong \tilde{H}_{i-1}(\Sigma_\lambda, k),$$

for all $i \geq 0$ *and all* $\lambda \in \mathbf{N}^{r+1}$.

(2.4) We now apply Lemma (2.3) to our problem on $\mathcal{R}_k[L]$. Let $P = \{p_1, p_2, \cdots, p_r\}$ be an arbitrary poset and $L = J(P)$ the associated distributive lattice. It is known, e.g., [H$_1$] that

$$\mathcal{R}_k[L] \cong k[T_0^{r+1-\sharp(\alpha)} \textstyle\prod_{p_i \in \alpha} T_i \mid \alpha \text{ is a poset ideal of } P].$$

Here the right-hand side is an affine semigroup ring contained in the polynomial ring $k[T_0, T_1, \cdots, T_r]$. We define $\deg T_i = (0, \cdots, \overset{i}{\check{1}}, \cdots, 0)$. Then $\mathcal{R}_k[L]$ has a structure of $\mathbf{N}^{r+1}$-graded module over the polynomial ring $A = k[X_\alpha \mid \alpha \in L = J(P)]$ with each $\deg X_\alpha = \deg(T_0^{r+1-\sharp(\alpha)} \prod_{p_i \in \alpha} T_i)$, and there exists an $\mathbf{N}^{r+1}$-graded minimal free resolution of $\mathcal{R}_k[L]$ over A. We define a simplicial complex Σ_λ as above and $\mathrm{Deg}\, T^\lambda = \frac{\lambda_0 + \lambda_1 + \cdots + \lambda_r}{r+1}$. Hence, we have $\text{Deg}\ X_\alpha = 1$ for every $\alpha \in L$ and the above $\mathbf{N}^{r+1}$-graded minimal free resolution of $\mathcal{R}_k[L]$ over A can be regarded as $\mathbf{N}$-graded minimal free resolution of $\mathcal{R}_k[L]$ over A.

(2.5) LEMMA.

$$\beta_{2_4} = \sum_{\text{Deg}\, T^\lambda = 4} \dim_k \tilde{H}_1(\Sigma_\lambda;\ k).$$

We say that a simplicial complex Σ is spanned by $\{\sigma_1, \cdots, \sigma_s\}$ if $\Sigma = \mathbf{2}^{\sigma_1} \cup \cdots \cup \mathbf{2}^{\sigma_s}$, where $\mathbf{2}^\sigma$ is the family of all subsets of σ.

Since there exists a natural bijection between $\{\lambda \in \mathbf{N}^{r+1} \mid (\mathcal{R}_k[L])_\lambda \neq 0\}$ and the set of standard monomials $\{M = X_{\alpha_1} X_{\alpha_2} \cdots X_{\alpha_s} \mid \alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_s\}$ in $\mathcal{R}_k[L]$, we often write Σ_M or $\Sigma_{\alpha_1 \cdots \alpha_s}$ instead of Σ_λ, if $\deg M = \lambda$.

(2.6) LEMMA. *The simplicial complex Σ_M is spanned by all subsets $\{\beta_1, \beta_2, \ldots, \beta_t\} \subset L$ such that $\prod_{j=1}^t (X_{\beta_j})^{e_j} = M$ in $\mathcal{R}_k[L]$ for some integers $e_j > 0$, $1 \leq j \leq t$.*

(2.7) LEMMA. *The vertex set of the simplicial complex $\Sigma_{\alpha_1 \cdots \alpha_s}$ is $C_L(\{\alpha_1, \cdots, \alpha_s\})$, where $\alpha_1 \leq \cdots \leq \alpha_s$ in L.*

In what follows, suppose that L is a planar distributive lattice.

(2.8) LEMMA. *Let C be the closure $C_L(\{\alpha_1, \cdots, \alpha_s\})$ of $\{\alpha_1, \cdots, \alpha_s\}$ with $\alpha_1 \leq \cdots \leq \alpha_s$ in L. If* type *C is (a, b), then $a, b \leq s$.*

We now give a sketch of proof of Theorem (2.1). First, by Lemma (2.5), what we must compute is the reduced homology group $\tilde{H}_1(\Sigma_{\alpha_1\alpha_2\alpha_3\alpha_4};\ k)$ for every $\{\alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4\}$ with $\alpha_1 \leq \alpha_2 \leq \alpha_3 \leq \alpha_4$ in L. Let C denote $C_L(\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\})$. Then, by Lemma (2.8), type C is one of the following: (1,1), (2,1), (2,2), (3,1), (3,2), (3,3), (4,1), (4, 2), (4, 3) and (4, 4). For each case, with one-by-one checking we easily see that $\tilde{H}_1(\Sigma_{\alpha_1\alpha_2\alpha_3\alpha_4}; k) \cong k$ if α_1 and α_2 are changeable; α_3 and α_4 are changeable; and C is not a one-sided ladder, and that $\tilde{H}_1(\Sigma_{\alpha_1\alpha_2\alpha_3\alpha_4}; k) = 0$ otherwise. Q. E. D.

§3. Examples

In this section we give some examples of affine semigroup rings whose second Betti numbers depend on the characteristic of a base field k.

(3.1) Example. We define an affine semigroup ring R in $B := k[t_1, \cdots, t_{10}]$ as follows. Let $a(i)$ $(1 \leq i \leq 16)$ be

$$\begin{aligned}
a(1) &= (1,1,0,0,0,0,0,1,1,1),\\
a(2) &= (0,1,1,1,1,0,0,0,0,1),\\
a(3) &= (0,0,0,0,1,1,1,1,0,1),\\
a(4) &= (1,1,1,0,0,1,1,0,0,0),\\
a(5) &= (0,0,1,1,0,0,1,1,1,0),\\
a(6) &= (1,0,0,1,1,1,0,0,1,0),\\
a(7) &= (1,0,0,0,1,2,1,0,0,0),\\
a(8) &= (0,0,1,0,0,1,2,1,0,0),\\
a(9) &= (0,0,1,2,1,0,0,0,1,0),\\
a(10) &= (0,1,2,1,0,0,1,0,0,0),\\
a(11) &= (0,0,0,1,2,1,0,0,0,1),\\
a(12) &= (1,0,0,1,0,0,0,1,2,0),\\
a(13) &= (2,1,0,0,0,1,0,0,1,0),\\
a(14) &= (0,0,0,0,0,0,1,2,1,1),\\
a(15) &= (1,2,1,0,0,0,0,0,0,1),\\
a(16) &= (0,1,0,0,1,0,0,1,0,2).
\end{aligned}$$

Let $R := k[M_1, \cdots, M_{16}]$, where $M_i = t^{a(i)} (1 \leq i \leq 16)$, and $A := k[x_1, \cdots x_{16}]$.

Then $\beta_2(R)$ depends on the base field k, where $\beta_2(R)$ is the second Betti number of a minimal free resolution of R as an A-module.

Proof. Let Δ_1 (resp. Δ_2) be the simplicial complex which has the following 10 (resp. 5) maximal faces.

$$\begin{aligned}
&\{x_1,x_2,x_5,x_7\}, \quad \{x_1,x_2,x_6,x_8\}, \quad \{x_1,x_3,x_4,x_9\}, \{x_1,x_3,x_6,x_{10}\},\\
&\{x_1,x_4,x_5,x_{11}\}, \quad \{x_2,x_3,x_4,x_{12}\}, \quad \{x_2,x_3,x_5,x_{13}\}, \{x_2,x_4,x_6,x_{14}\},\\
&\{x_3,x_5,x_6,x_{15}\}, \quad \{x_4,x_5,x_6,x_{16}\}.
\end{aligned}$$

$$\left(\begin{array}{l}
\text{resp.}\\
\{x_7,x_9,x_{14},x_{15}\}, \quad \{x_7,x_{10},x_{12},x_{16}\}, \quad \{x_8,x_9,x_{13},x_{16}\},\\
\{x_8,x_{11},x_{12},x_{15}\}, \quad \{x_{10},x_{11},x_{13},x_{14}\}.
\end{array}\right)$$

Then the geometric realization of Δ_1 is homotopy-equivalent with the real projective plane $\mathbf{P}^2(\mathbf{R})$. Note that the simplicial complex with the following maximal faces

$$\{x_1,x_2,x_5\},\quad \{x_1,x_2,x_6\},\quad \{x_1,x_3,x_4\},\{x_1,x_3,x_6\},$$
$$\{x_1,x_4,x_5\},\quad \{x_2,x_3,x_4\},\quad \{x_2,x_3,x_5\},\{x_2,x_4,x_6\},$$
$$\{x_3,x_5,x_6\},\quad \{x_4,x_5,x_6\}$$

is a triangulation of $\mathbf{P}^2(\mathbf{R})$ (see, e.g. [Tr-Ho]).

And the geometric realization of Δ_2 is homotopy-equivalent with that of the complete graph with 5 vertices.

Let $\lambda = (2,2,\cdots,2) \in \mathbf{N}^{10}$. With the same situation as in (2.2), we can easily check that

$$\Sigma_\lambda = \Delta_1 \cup \Delta_2,$$

and $\Delta_1 \cap \Delta_2$ is a set of finite points. Then we have the reduced Mayer-Vietoris sequence

$$\begin{aligned}0 = \tilde{H}_1(\Delta_1 \cap \Delta_2; k) &\longrightarrow \tilde{H}_1(\Delta_1; k) \oplus \tilde{H}_1(\Delta_2; k)\\ \longrightarrow \tilde{H}_1(\Sigma_\lambda; k) &\longrightarrow \tilde{H}_0(\Delta_1 \cap \Delta_2; k) \longrightarrow \tilde{H}_0(\Delta_1; k) \oplus \tilde{H}_0(\Delta_2; k) = 0.\end{aligned}$$

Thus

$$\begin{aligned}&\dim_k \tilde{H}_1(\Sigma_\lambda; k)\\ &= \dim_k \tilde{H}_1(\Delta_1; k) \;+\; \dim_k \tilde{H}_1(\Delta_2; k) + \dim_k \tilde{H}_0(\Delta_1 \cap \Delta_2; k).\end{aligned}$$

Since $\dim_k \tilde{H}_1(\Delta_2; k)$ and $\dim_k \tilde{H}_0(\Delta_1 \cap \Delta_2; k)$ are indepedent of k, and since

$$\begin{aligned}\dim_k \tilde{H}_1(\Delta_1; k) &= \dim_k \tilde{H}_1(P^2(R); k)\\ &= \begin{cases} 0 & \text{if char} k \neq 2 \\ 1 & \text{if char} k = 2, \end{cases}\end{aligned}$$

$\dim_k \tilde{H}_1(\Sigma_\lambda; k)$ depends on the characteristic of k. By Lemma 2.3 we have the desired result. Q.E.D.

Remark. In fact we can compute the second Betti number $\beta_2(R)$ of the above example by the computer software "Macaulay." According to "Macaulay" we have $\beta_1(R) = 75$ and

$$\beta_2(R) = \begin{cases} 588 & \text{if char} k = 0 \\ 589 & \text{if char} k = 2. \end{cases}$$

(3.2) Example. We define a monomial curve R to be $k[t^{a_1}, \cdots, t^{a_{16}}]$ in $k[t]$, where

$$
\begin{aligned}
a_1 &= 11100000111, \\
a_2 &= 10111100001, \\
a_3 &= 10000111101, \\
a_4 &= 11110011000, \\
a_5 &= 10011001110, \\
a_6 &= 11001110010, \\
a_7 &= 11000121000, \\
a_8 &= 10010012100, \\
a_9 &= 10012100010, \\
a_{10} &= 10121001000, \\
a_{11} &= 10001210001, \\
a_{12} &= 11001000120, \\
a_{13} &= 12100010010, \\
a_{14} &= 10000001211, \\
a_{15} &= 11210000001, \\
a_{16} &= 10100100102.
\end{aligned}
$$

Then $\beta_2(R)$ depends on the base field k.

Proof. Put $\mu = 42222222222$. It is easy to see that Σ_μ is equal to Σ_λ in Example 3.1. Then as in Example 3.1, we have the desired result by Lemma 2.3. Q.E.D.

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