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Spherical functions on spherical homogeneous spaces and Rankin-Selberg convolution

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In this note, we study spherical functions on certain p-adic spherical homogeneous spaces. We show the existence, uniqueness and an explicit formula of the spherical functions, and study its application to Rankin-Selberg convolution. Though we treat only the orthogonal group case in this note, similar results hold for other cases.

§1. Preliminaries

1.1 In this and the next sections, we let F be a non-archimedean local field of characteristic different from 2, and denote by o the integer ring of F. Fix a prime element π of F and put $q = \#(o/\pi o)$. Let $|\cdot|$ be the normalized valuation of F ($|\pi| = q^{-1}$). We denote by F_n^m the space of $m \times n$ matrices whose entries are in F. For a symmetric matrix S of degree m and $x \in F_n^m$, we put $S[x] = {}^t x S x$. For a real number α, we denote by $[\alpha]$ the integer with $[\alpha] \le \alpha < [\alpha] + 1$.

1.2 Let m be a positive integer and put $n = \left[\frac{m}{2}\right]$. Let S_m be a symmetric matrix of degree m given by

$$S_m = \begin{cases} \begin{bmatrix} 0 & J_n \\ J_n & 0 \end{bmatrix} & \text{if } m \text{ is even} \\ \begin{bmatrix} 0 & 0 & J_n \\ 0 & 2 & 0 \\ J_n & 0 & 0 \end{bmatrix} & \text{if } m \text{ is odd} \end{cases}$$

where $J_n = \begin{bmatrix} 0 & & 1 \\ & \cdot^{\cdot^{\cdot}} & \\ 1 & & 0 \end{bmatrix} \in GL_n(F)$. Denote by G_m (or $O(m)$) the orthogonal group of S_m: $G_m = O(m) = \{ g \in GL_m(F) \mid {}^t g S_m g = S_m \}$. Let $K_m = G_m(o)$ be a maximal open compact subgroup of G_m. We normalize the Haar measure dg on G_m so that $\mathrm{vol}(K_m) = 1$.

1.3 We define an embedding ι_m of G_m into G_{m+1} as follows:

(a) If $m = 2n$ is even,

$$\iota_m\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a & 0 & b \\ 0 & 1 & 0 \\ c & 0 & d \end{bmatrix}$$

where $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in G_m$ is the block decomposition corresponding to the partition $m = n + n$.

(b) If $m = 2n + 1$ is odd,

$$\iota_m\left(\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}\right) = \begin{bmatrix} a_1 & \frac{a_2}{2} & \frac{a_2}{2} & a_3 \\ b_1 & \frac{b_2+1}{2} & \frac{b_2-1}{2} & b_3 \\ b_1 & \frac{b_2-1}{2} & \frac{b_2+1}{2} & b_3 \\ c_1 & \frac{c_2}{2} & \frac{c_2}{2} & c_3 \end{bmatrix}$$

where $\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \in G_m$ is the block decomposition corresponding to the partition $m = n + 1 + n$.

1. 4 For an integer r ($1 \le r \le n = \left[\frac{m}{2}\right]$), let

$$N_{m,r} = \{\, v_{m,r}(x, y) := \begin{bmatrix} 1_r & -J_r{}^t x S_{m-2r} & J_r(y - \frac{1}{2} S_{m-2r}[x]) \\ 0 & 1_{m-2r} & x \\ 0 & 0 & 1_r \end{bmatrix} \mid x \in F_n^{m-2r},\ y \in \mathrm{Alt}_r(F) \,\}$$

and

$$M_{m,r} = \{\, \mu_{m,r}(a, h) := \begin{bmatrix} a & & 0 \\ & h & \\ 0 & & \tilde{a} \end{bmatrix} \mid a \in GL_r(F),\ h \in G_{m-2r} \,\},$$

where $\mathrm{Alt}_r = \{\, y \in F_r^r \mid {}^t y + y = 0 \,\}$ and $\tilde{a} = J_r{}^t a^{-1} J_r$ for $a \in GL_r$. Then $P_{m,r} = N_{m,r} M_{m,r}$ is a maximal parabolic subgroup of G_m.

1. 5 Let $T_m = \{\, \mathbf{d}_m(t_1, \cdots, t_n) \mid t_1, \cdots, t_n \in F^\times \,\}$ be a maximal F-split torus of G_m, where $\mathbf{d}_m(t_1, \cdots, t_n)$ denotes the matrix $\mathrm{diag}(t_1, \cdots, t_n, t_n^{-1}, \cdots, t_1^{-1})$ if m is even and $\mathrm{diag}(t_1, \cdots, t_n, 1, t_n^{-1}, \cdots, t_1^{-1})$ if m is odd. For $t = \mathbf{d}_m(t_1, \cdots, t_n) \in T_m$, put $\delta_m(t) = d(tvt^{-1})/dv = \prod_{i=1}^{n} |t_i|^{m-2i}$, where dv is a Haar measure on $N_{m,n}$ (= a maximal unipotent subgroup of G_m). Denote by $X_{unr}(T_m)$ the group of unramified characters of T_m. We let the Weyl group $W_m := N_{G_m}(T_m)/T_m$ act on $X_{unr}(T_m)$ by $(w\chi)(t) = \chi(w^{-1}tw)$.

1. 6 Let $\mathcal{H}_m = \mathcal{H}(G_m, K_m)$ be the Hecke algebra of (G_m, K_m). For $\chi \in X_{unr}(T_m)$, let ϕ_χ be a function on G_m defined by $\phi_\chi(vtk) = (\delta_m^{1/2}\chi)(t)$ for $v \in N_{m,n}$, $t \in T_m$, $k \in K_m$. Define a **C**-homomorphism $\chi^\wedge$ of $\mathcal{H}_m$ to **C** by

$$\chi^\wedge(\varphi) = \int_{G_m} \phi_\chi(g)\,\varphi(g)\,dg \qquad (\varphi \in \mathcal{H}_m).$$

Then $\chi \mapsto \chi^\wedge$ gives rise to a bijection between $W_m \backslash X_{unr}(T_m)$ and $\mathrm{Hom}_{\mathbf{C}}(\mathcal{H}_m, \mathbf{C})$ (cf. [Sa]).

1. 7 Let $T_r^* = \left\{ \begin{bmatrix} t_1 & & 0 \\ & \ddots & \\ 0 & & t_r \end{bmatrix} \mid t_1, \cdots, t_r \in F^\times \right\}$ be a maximal split torus of $GL_r(F)$. Let $\xi \in X_{unr}(T_r^*)$ and $\chi \in X_{unr}(T_m)$. We often identify ξ and χ with $(\xi_1, \cdots, \xi_r) \in (\mathbf{C}^\times)^r$ and $(\chi_1, \cdots, \chi_n) \in (\mathbf{C}^\times)^n$ determined by $\xi(\mathrm{diag}(\pi^{k_1}, \cdots, \pi^{k_r})) = \xi_1^{k_1} \cdots \xi_r^{k_r}$ and $\chi(\mathbf{d}_m(\pi^{\ell_1}, \cdots, \pi^{\ell_n})) = \chi_1^{\ell_1} \cdots \chi_n^{\ell_n}$ for $(k_1, \cdots, k_r) \in \mathbf{Z}^r$ and $(\ell_1, \cdots, \ell_n) \in \mathbf{Z}^n$, respectively. We define the L-factor $L(\xi \otimes \chi; s)$ by

$$L(\xi \otimes \chi; s) = \prod_{1 \le i \le r,\ 1 \le j \le n} \{(1 - \xi_i \chi_j q^{-s})(1 - \xi_i \chi_j^{-1} q^{-s})\}^{-1}.$$

We also define the L-factors $L(\xi, \mathrm{Sym}^2; s)$ and $L(\xi, \mathrm{Alt}^2; s)$ by

$$L(\xi, \mathrm{Sym}^2; s) = \prod_{1 \le i \le j \le r} (1 - \xi_i \xi_j q^{-s})^{-1}, \quad L(\xi, \mathrm{Alt}^2; s) = \prod_{1 \le i < j \le r} (1 - \xi_i \xi_j q^{-s})^{-1}.$$

§2. Local spherical functions

2. 1 Let m' and r be non-negative integers and put $m = m' + 2r + 1$. Let

$$G = G_m,\ K = K_m,\ T = T_m,\ \mathcal{H} = \mathcal{H}_m,\ n = \left[\frac{m}{2}\right]$$

$$G' = G_{m'},\ K' = K_{m'},\ T' = T_{m'},\ \mathcal{H}' = \mathcal{H}_{m'},\ n' = \left[\frac{m'}{2}\right]$$

and identify G' with a subgroup of G via $g' \mapsto \mu_{m,r}(1, \iota_{m'}(g'))$.

2. 2 Let $U = U_{m,r} = N_{m,r} \cdot \{ \mu_{m,r}(z, 1) \mid z \in Z_r \}$ where Z_r is the group of upper unipotent matrices in $GL_r(F)$. Throughout this section, we fix an additive character ψ of F with conductor o. We define a character ψ_U of U by

$$\psi_U(v_{m,r}(x, y)\, \mu_{m,r}(z, 1)) = \psi(x_{n'+1,1} - \varepsilon_m\, x_{n'+2,1} + \sum_{i=1}^{r-1} z_{i,i+1})$$

for $x \in M_{m-2r,r}(F)$, $y \in Alt_r(F)$ and $z \in Z_r$, where we put

$$\varepsilon_m = \begin{cases} 1 & \text{if } m \text{ is even} \\ 0 & \text{if } m \text{ is odd.} \end{cases}$$

It is easy to see that G' normalizes U and fixes ψ_U.

2. 3 For $(\chi', \chi) \in X_{unr}(T') \times X_{unr}(T)$, let

$$\Omega(\chi', \chi) = \{ \mathcal{W} : G \mapsto \mathbf{C} \mid$$

$$\text{(i) } \mathcal{W}(uk'gk) = \psi_U(u)\, \mathcal{W}(g) \quad (u \in U, k' \in K', g \in G, k \in K)$$

$$\text{(ii) } \varphi' * \mathcal{W} * \varphi = \chi'^\wedge(\varphi')\, \chi^\wedge(\varphi)\, \mathcal{W} \quad (\varphi' \in \mathcal{H}', \varphi \in \mathcal{H}) \}.$$

Here

$$(\varphi' * \mathcal{W} * \varphi)(g) = \int_{G'} dx' \int_G dx\ \varphi'(x')\, \mathcal{W}(x'gx)\, \varphi(x).$$

We call $\Omega(\chi', \chi)$ the space of spherical functions on G attached to (χ', χ).

2.4 Remark

(i) Let $\mathbf{G} = G' \times G$ and $\mathbf{H} = (UG')^{diag} \subset \mathbf{G}$. Then $\mathbf{H}$ is a spherical subgroup of $\mathbf{G}$ and $\mathcal{W} \in \Omega(\chi', \chi)$ may be regarded as a spherical function of $(\mathbf{G}, \mathbf{H})$ (cf. [GP]).

(ii) When $m' = 0$ or 1, these functions are the usual Whittaker functions. Bump, Friedberg and Furusawa [BFF] have studied the spherical functions in the case $m' = 2$, and Murase and Sugano [MS] considered the case $r = 0$.

2. 5 Let $L_n = \mathbf{Z}^n$ and $L_n^+ = \{ (\ell_1, \cdots, \ell_n) \in L_n \mid \ell_1 \geq \cdots \geq \ell_n \geq 0 \}$. For $\ell = (\ell_1, \cdots, \ell_n) \in L_n$, put $t_m(\ell) = \mathbf{d}_m(\pi^{\ell_1}, \cdots, \pi^{\ell_n}) \in T_m$. We define $t_{m'}(\ell') \in T_{m'}$ for $\ell' \in L_{n'}$ similarly. Let g_o be an element of G given by

$$g_o = \begin{cases} \mu_{m,r}(1, \begin{bmatrix} A & 0 \\ 0 & \tilde{A} \end{bmatrix}) & \text{if } m \text{ is even} \\ \mu_{m,r}(1, \begin{bmatrix} 1_{n'} & -2{}^tX & -{}^tXXJ_{n'} \\ 0 & 1 & X \\ 0 & 0 & 1_{n'} \end{bmatrix}) & \text{if } m \text{ is odd} \end{cases}$$

where $X = (1, \cdots, 1) \in F^{n'}$ and $A = \begin{bmatrix} 1_{n'} & {}^tX \\ 0 & 1 \end{bmatrix} \in GL_{n'+1}(F)$. For $(\ell', \ell) \in L_{n'} \times L_n$, put $g(\ell', \ell) = t_{m'}(\ell')\, g_o\, t_m(\ell) \in G$.

2. 6 Theorem (Cartan decomposition) *We have*

$$G = \coprod UK' \cdot g(\ell', \ell) \cdot K \quad (\textit{disjoint union})$$

where ℓ' *runs over* $L_{n'}^+$ *and* ℓ *over* $L_r \times L_{n-r}^+$.

2. 7 Corollary *For* $\mathcal{W} \in \Omega(\chi', \chi)$, *we have*

$$\operatorname{Supp} \mathcal{W} \subset \coprod UK' \cdot g(\ell', \ell) \cdot K$$

where ℓ' *runs over* $L_{n'}^+$ *and* ℓ *over* L_n^+.

2. 8 Using the Cartan decomposition (Corollary 2.7) and a similar method of [Shin] and [Ka], we obtain the following existence and uniqueness of spherical functions:

Theorem *For* $(\chi', \chi) \in X_{unr}(T') \times X_{unr}(T)$, *there uniquely exists* $\mathcal{W}_{\chi',\chi} \in \Omega(\chi', \chi)$ *with* $\mathcal{W}_{\chi',\chi}(1) = 1$. *In particular, we have* $\dim_{\mathbf{C}} \Omega(\chi', \chi) = 1$.

2. 9 For $\chi \in X_{unr}(T)$, we put

$$\Delta_m(\chi) = \prod_{1 \leq i < j \leq n} (1 - \chi_i^{-1}\chi_j)(1 - \chi_i^{-1}\chi_j^{-1}) \times \begin{cases} 1 & \text{if } m \text{ is even} \\ \prod_{1 \leq i \leq n} (1 - \chi_i^{-2}) & \text{if } m \text{ is odd.} \end{cases}$$

We define $\Delta_{m'}(\chi')$ for $\chi' \in X_{unr}(T')$ similarly. For $(\chi', \chi) \in X_{unr}(T) \times X_{unr}(T')$, we put

$$\mathcal{D}(\chi', \chi) = \Delta_{m'}(\chi')^{-1} \Delta_m(\chi)^{-1} \prod_{\substack{1 \le i \le n' \\ 1 \le j \le n}} (1 - q^{-1/2}(\chi'_i \chi_j^{-1})^{\eta_{ij}}) (1 - q^{-1/2}(\chi'_i \chi_j)^{-1})$$

where $\eta_{ij} = \begin{cases} 1 & \text{if } j \le r + i \\ -1 & \text{if } j > r + i. \end{cases}$ Put

$$Q_{m'} = \begin{cases} (1 - q^{-n'}) \prod_{1 \le i \le n'-1} (1 - q^{-2i}) & \text{if } m' = 2n' \\ \prod_{1 \le i \le n'} (1 - q^{-2i}) & \text{if } m' = 2n' + 1. \end{cases}$$

2. 10 The following explicit formula can be proved by a method similar to that of [CS].

Theorem *For $(\chi', \chi) \in X_{unr}(T) \times X_{unr}(T')$, let $\mathcal{W}_{\chi',\chi} \in \Omega(\chi', \chi)$ be as in Theorem 2.8. Then, for $(\ell', \ell) \in L^+_{n'} \times L^+_n$, we have*

$$\mathcal{W}_{\chi',\chi}(g(\ell', \ell)) = \frac{1}{Q_{m'}} \sum_{w' \in W_{m'}, w \in W_m} \mathcal{D}(w'\chi', w\chi)$$

$$\times (w'\chi' \cdot \delta_{m'}^{1/2})(t_{m'}(\ell')) \cdot (w\chi \cdot \delta_m^{1/2})(t_m(\ell)).$$

§3. Application to Rankin-Selberg convolution

3. 1 Let $G = G_m$ and $G^* = G_{m-1}$ be the orthogonal group of S_m and S_{m-1} defined over $\mathbf{Q}$. We regard G^* as a subgroup of G via ι_{m-1}. Let r be an integer with $1 \le r \le \left[\frac{m-1}{2}\right]$. Let $P^* = N_{m-1,r} M_{m-1,r}$ be a maximal parabolic subgroup of G^* and put $G' = G_{m'}$ with $m' = m - 2r - 1$. Then $\mu^* = \mu_{m-1,r}$ gives an isomorphism of $GL_r \times G'$ onto $M_{m-1,r}$.

3. 2 Let φ be an automorphic form on $GL_r(\mathbf{A})$ with central character ω. Assume that φ is right-invariant under $\prod_{p<\infty} GL_r(\mathbf{Z}_p)$ and square integrable over $GL_r(\mathbf{Q})\backslash GL_r(\mathbf{A})^1$, where $GL_r(\mathbf{A})^1 = \{ g \in GL_r(\mathbf{A}) \mid |\det(g)|_{\mathbf{A}} = 1 \}$. We also let f be an automorphic form on $G'(\mathbf{A})$ right-invariant under $\prod_{p<\infty} G'(\mathbf{Z}_p)$ and square integrable over $G'(\mathbf{Q})\backslash G'(\mathbf{A})$. Define a function $\phi(\ ;\varphi\otimes f)$ on $G^*(\mathbf{A})\times\mathbf{C}$ by

$$\phi(\nu^*\,\mu^*(a,g')\,k^*, s;\varphi\otimes f)=\varphi(a)\,f(g')\,|\det a|_{\mathbf{A}}^{s+(m'+r-1)/2},$$

where $\nu^*\in N_{m-1,r}(\mathbf{A})$, $a\in GL_r(\mathbf{A})$, $g'\in G'(\mathbf{A})$ and $k^*\in K^*_\infty\prod_{p<\infty} G^*(\mathbf{Z}_p)$ (K^*_∞ is a suitable maximal compact subgroup of $G^*(\mathbf{R})$). The Eisenstein series

$$E(g^*, s;\varphi\otimes f)=\sum_{\gamma\in P^*(\mathbf{Q})\backslash G^*(\mathbf{Q})}\phi(\gamma g^*, s;\varphi\otimes f)$$

is absolutely convergent for $\mathrm{Re}(s)>>0$ and continued to a meromorphic function of s on the whole $\mathbf{C}$.

3. 3 Let F be a cusp form on $G(\mathbf{A})$ right-invariant under $\prod_{p<\infty} G(\mathbf{Z}_p)$. The object of this section is to study the following Rankin-Selberg convolution

$$Z_{F,\varphi\otimes f}(s)=\int_{G^*(\mathbf{Q})\backslash G^*(\mathbf{A})} F(g^*)\,E(g^*, s-\frac{1}{2};\varphi\otimes f)\,dg^*.$$

The function $Z_{F,\varphi\otimes f}(s)$ is continued to a meromorphic function of s on the whole $\mathbf{C}$.

3. 4 Let $U=U_{m,r}\subset G$ and $\psi_U\in \mathrm{Hom}(U(\mathbf{A}),\mathbf{C}^\times)$ be as in §2.2 replacing ψ with the additive character $\psi_{\mathbf{A}}$ of $\mathbf{Q}\backslash\mathbf{A}$ such that $\psi_{\mathbf{A}}(x_\infty)=\exp(2\pi i x_\infty)$ for $x_\infty\in\mathbf{R}$. We set

$$\mathcal{W}_{f,F}(g)=\int_{U(\mathbf{Q})\backslash U(\mathbf{A})} du\int_{G'(\mathbf{Q})\backslash G'(\mathbf{A})} dg'\,f(g')\,\psi_U(u)^{-1}\,F(u\,\mu^*(1,g')\,g)$$

for $g\in G(\mathbf{A})$ and

$$W_\varphi(x)=\int_{Z_r(\mathbf{Q})\backslash Z_r(\mathbf{A})}\psi_{\mathbf{A}}(\sum_{i=1}^{r-1} z_{i,i+1})\,\varphi(zx)\,dz$$

for $x\in GL_r(\mathbf{A})$.

3. 5 Unfolding the Eisenstein series in the integral of $Z_{F,\,\varphi\otimes f}(s)$, we get

Proposition (The basic identity)

$$Z_{F,\,\varphi\otimes f}(s) = \int_{(\mathbf{A}^\times)^r} W_\varphi(\mathrm{diag}(t_1, \cdots, t_r))\, \mathcal{W}_{f,F}(\mu^*(\mathrm{diag}(t_1, \cdots, t_r), 1))$$

$$\times \prod_{i=1}^{r} |t_i|_{\mathbf{A}}^{s-(m+r+1)/2+2i}\, d^\times t_1 \cdots d^\times t_r.$$

3. 6 We now assume that φ, f and F are Hecke eigenform. Let $\xi_p \in X_{unr}(T_r^*(\mathbf{Q}_p))$, $\chi'_p \in X_{unr}(T_{m'}(\mathbf{Q}_p))$ and $\chi_p \in X_{unr}(T_m(\mathbf{Q}_p))$ be the corresponding Satake parameters at p. For each p, the restriction of $\mathcal{W}_{f,F}$ to $G(\mathbf{Q}_p)$ belongs to $\Omega(\chi'_p, \chi_p)$. Then Theorem 2.8 implies that

$$\mathcal{W}_{f,F}(g) = \mathcal{W}_{f,F}^{(\infty)}(g_\infty) \prod_{p<\infty} \mathcal{W}_{\chi'_p,\,\chi_p}(g_p)$$

for $g = g_\infty \prod_{p<\infty} g_p \in G(\mathbf{A})$, where $\mathcal{W}_{f,F}^{(\infty)}$ is the restriction of $\mathcal{W}_{f,F}$ to $G(\mathbf{R})$. It is well-known that a similar fact holds for W_φ:

$$W_\varphi(x) = W_\varphi^{(\infty)}(x_\infty) \prod_{p<\infty} W_{\xi_p}(x_p)$$

for $x = x_\infty \prod_{p<\infty} x_p \in GL_r(\mathbf{A})$, where W_{ξ_p} is the p-adic Whittaker function attached to ξ_p on $GL_r(\mathbf{Q}_p)$ with $W_{\xi_p}(1) = 1$ (cf. [Shin]) and $W_\varphi^{(\infty)}$ is the restriction of $W_\varphi^{(\infty)}$ to $GL_r(\mathbf{R})$. Therefore we obtain the Euler product decomposition for $Z_{F,\,\varphi\otimes f}(s)$:

$$Z_{F,\,\varphi\otimes f}(s) = Z_{F,\,\varphi\otimes f}^{(\infty)}(s) \prod_{p<\infty} Z_p(s),$$

$$Z_{F,\,\varphi\otimes f}^{(\infty)}(s) = \int_{(\mathbf{R}^\times)^r} W_\varphi^{(\infty)}(\mathrm{diag}(t_1, \cdots, t_r))\, \mathcal{W}_{f,F}^{(\infty)}(\mu^*(\mathrm{diag}(t_1, \cdots, t_r), 1))$$

$$\times \prod_{i=1}^{r} |t_i|_\infty^{s-(m+r+1)/2+2i}\, d^\times t_1 \cdots d^\times t_r\,,$$

$$Z_p(s) = \int_{(\mathbf{R}^\times)^r} W_{\xi_p}(\mathrm{diag}(t_1, \cdots, t_r))\, \mathcal{W}_{\chi'_p, \chi_p}(\mu^*(\mathrm{diag}(t_1, \cdots, t_r), 1))$$

$$\times \prod_{i=1}^{r} |t_i|_p^{s-(m+r+1)/2+2i}\, d^\times t_1 \cdots d^\times t_r .$$

3. 7 By using Theorem 2.10 and Shintani's explicit formula for W_{ξ_p} ([Shin]), we obtain the following:

Theorem

$$Z_p(s) = \frac{L(\xi_p \otimes \chi_p; s)}{L(\xi_p \otimes \chi'_p; s + 1/2)} \times \begin{cases} L(\xi_p, \mathrm{Sym}^2, 2s)^{-1} & \textit{if } m \textit{ is even} \\ L(\xi_p, \mathrm{Alt}^2, 2s)^{-1} & \textit{if } m \textit{ is odd.} \end{cases}$$

3. 8 Remark Similar results hold for the integral of F on O(m) against the restriction to O(m) of Eisenstein series on O(m+1).

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