数理解析研究所講究録
967 巻 1996 年 71-82

Flexible boundaries in deformations of hyperbolic 3-manifolds

Michihiko FUJII and Sadayoshi KOJIMA

藤井 道彦 (横浜市立大), 小島 定吉 (東工大)

§0. Introduction.

This gives a detailed description of a process of caluculations performed in the paper [3] with the same title.

Let M be a cusped hyperbolic 3-manifold with non-empty geodesic boundary. A small Dehn filling deformation of M on the cusps can be performed so that the boundary is kept to be geodesic. Then assigning to each deformation a hyperbolic structure on the boundary, we get a map B_M from the space of such deformations to the Teichmüller space of ∂M. See [3] for precise argument about this fact or §1 for its review.

In this note, we give examples of M so that we can explicitly show B_M is a local embedding at complete structure. Especially we will describe concrete calculations to see such a phenomenon. By using a polyhedral decomposition of M given in §2, we will compute the derivative of B_M at the complete structure by hand in the later sections.

Neumann-Reid [5] and Fujii [2] discovered examples of M such that B_M is a constant map. In both of these cases, we can see it by some geometric reasons. In contrast to them, we need some calculations in the case that B_M is a local embedding.

§1. The map B_M.

We will define the map B_M. Let N be a noncompact, orientable, complete hyperbolic 3-manifold of finite volume, and $\overline{\rho}_0 : \pi_1(N) \to \mathrm{PSL}_2(\mathbf{C})$ its holonomy representation. According to Thurston, $\overline{\rho}_0$ has a lift $\rho_0 : \pi_1(N) \to \mathrm{SL}_2(\mathbf{C})$. Since $\mathrm{SL}_2(\mathbf{C})$ is an algebraic set. the space of representations $\mathrm{Hom}(\pi_1(N), \mathrm{SL}_2(\mathbf{C}))$ is also an algebraic set. To each representation

ρ, associated is its character χ_ρ. Culler and Shalen [1] showed that the irreducible component of $\mathrm{Hom}(\pi_1(N), \mathrm{SL}_2(\mathbf{C}))$ containing ρ_0 is mapped by this correspondence onto a closed affine variety X. The preimage of a character χ_ρ near χ_{ρ_0} consists of conjugate representations to ρ. Thus a small neighborhood of χ_{ρ_0} in X is bijectively identified with the set of conjugacy classes of $\mathrm{SL}_2(\mathbf{C})$-representations near the conjugacy class of ρ_0. Note that this small neighborhood is also identified with the set of conjugacy classes of $\mathrm{PSL}_2(\mathbf{C})$-representations near the conjugacy class of $\overline{\rho}_0$.

It has been known by the local rigidity together with the Poincaré duality argument as in [4] that the complex dimension of X is equal to the number of cusps of N and that the character of ρ_0 is a smooth point. If we choose a set of meridional elements $\{m_j\}$ for all cusps of N, then the traces of these elements turn out to be a local coordinate of X near the conjugacy class of ρ_0.

Now, suppose that M is an orientable complete hyperbolic 3-manifold of finite volume with both cusps and compact geodesic boundaries. Let DM be the double of M along the boundary and ρ_0 be a holonomy representation of DM. DM admits an obvious involution τ switching the sides. Fix the set of meridians m_j's closed under τ, and choose a small neighborhood U of χ_{ρ_0} so that the traces of m_j's become a local coordinate of X near χ_{ρ_0}. Then the obvious involution τ on DM induces an involution on U which fixes a diagonal set $\mathcal{D}_M$ in U. It is a smooth submanifold of real dimension $= \#\{\text{cusps of } DM\}$, which will be our deformation space of M.

LEMMA 1. *The restriction of a representation ρ near ρ_0 whose conjugacy class is in $\mathcal{D}_M$ to $\pi_1(\partial_0 M)$ is fuchsian, where $\partial_0 M$ is a component of the boundary ∂M.*

See [3] for the proof.

Assigning the hyperbolic structure of the boundary to such a deformation DM_ρ where $\rho \in \mathcal{D}_M$, we get a map

$$B_M : \mathcal{D}_M \to \mathcal{T}(\partial M)$$

where $\mathcal{T}$ is the Teichmüller space of ∂M.

§2. Construction of Examples and Results.

Consider the Whitehead link $L = K_1 \cup K_2$ in S^3. Removing a thin tubular neighborhood of K_2 from the complement of L, we obtain a manifold W with one compact toral boundary and one toral end. Choose an arc Σ connecting two points on ∂W as in Figure 1.

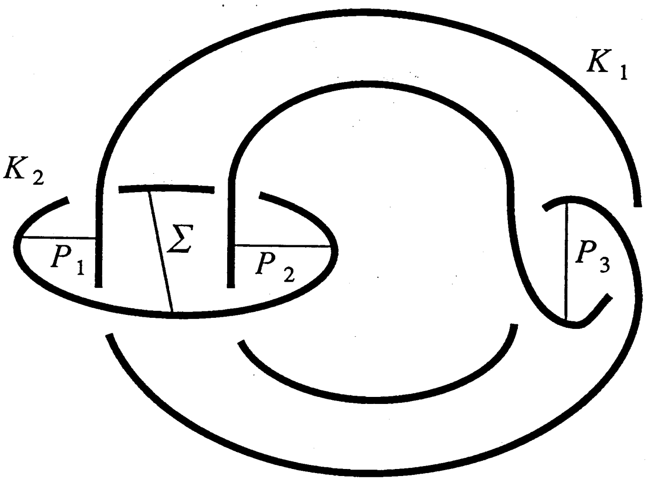

Figure 1

To give hyperbolic orbifold structures O_n's on W with singular set Σ indexed by natural numbers $n \geq 2$, we recall the fact, for instance in [7], that the regular ideal octahedron is a fundamental domain to create the hyperbolic manifold homeomorphic to the Whitehead link complement. Replace the regular ideal octahedron by the truncated octahedron as in Figure 2, where the dihedral angle along each edge connecting truncated faces is $\pi/2n$ and that of each edge through ∞ is $\pi/2$. Then the faces topologically identified to creat the Whitehead link complement are still isometric and the identification gives a hyperbolic orbifold O_n underlying on W where the singular set is Σ with rotation angle $2\pi/n$.

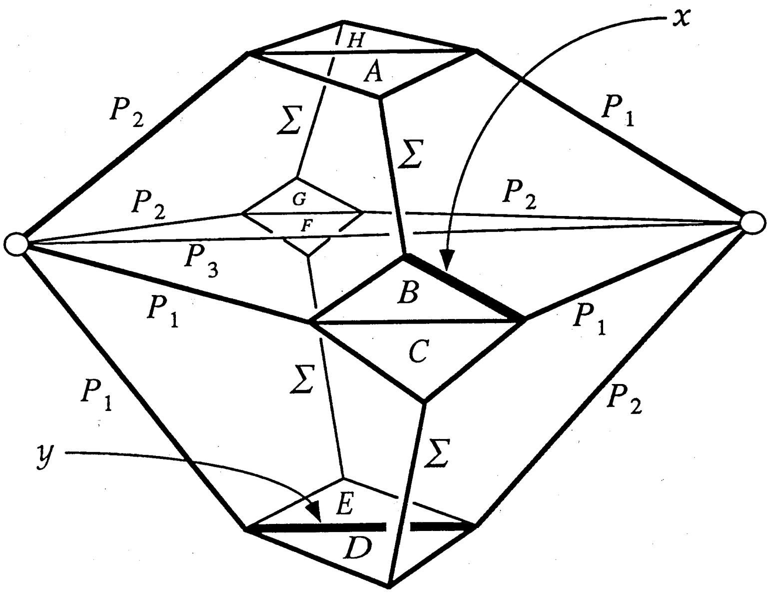

Figure 2

Since O_n has one cusp, the space $\mathcal{D}_{O_n}$ has real dimension 2. Also O_n has a toral boundary with two cone point of rotation angle $2\pi/n$. Hence the Teichmüller space of ∂O_n is homeomorphic to $\mathbf{R}^4$. Our goal is

THEOREM. *The derivative of the map $B_{O_n} : \mathcal{D}_{O_n} \to \mathcal{T}(\partial O_n)$ at the complete structure has rank* 2.

Taking n-fold cyclic branched covers along Σ, we have

COROLLARY. *There are infinitely many hyperbolic 3-manifold M with both a cusp and a boundary such that the map $B_M : \mathcal{D}_M \to \mathcal{T}(\partial M)$ is a local embedding near the complete structure.*

§3. Truncated Tetrahedra.

The truncated octahedron to creat O_n is decomposed into four congruent truncated tetrahedra as in Figure 3.

In this section, we will give a parametrization of isometry classes of truncated tetrahedra.

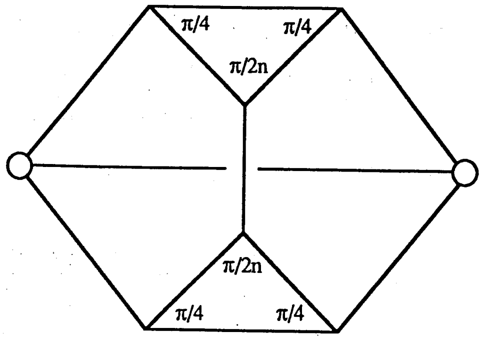

Figure 3

Label the triangular faces by A, B, and their edges by A_i, B_i $(i = 1, 2, 3)$ as in Figure 4. We call each of these edges *an external edge*, and denote the length of A_j and B_j by a_j and b_j respectively.

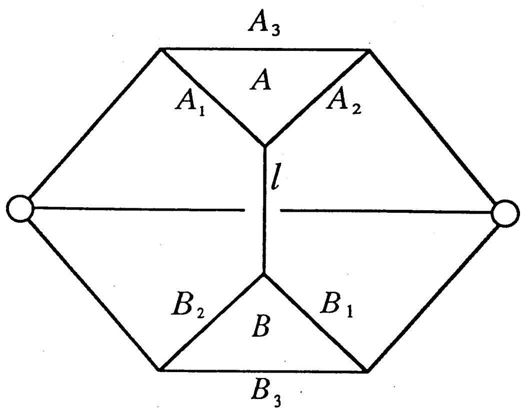

Figure 4

These lengths are subject to two identities.

One is the following. If we let l be the length of the edge shared by two pentagonal faces, then regarding it as a bottom of the left pentagon, we obtain an expression of l in terms of a_1 and b_2,

$$\cosh l = \frac{\cosh a_1 \cosh b_2 + 1}{\sinh a_1 \sinh b_2}.$$

Simultaneously, if we regard it as the bottom of the right pentagon, we obtain an expression of l in terms of a_2 and b_1,

$$\cosh l = \frac{\cosh a_2 \cosh b_1 + 1}{\sinh a_2 \sinh b_1}.$$

Then since these two are the same quantity, we obtain one identity involving edge lengths.

$$\frac{\cosh a_1 \cosh b_2 + 1}{\sinh a_1 \sinh b_2} - \frac{\cosh a_2 \cosh b_1 + 1}{\sinh a_2 \sinh b_1} = 0. \tag{1}$$

The other concerns with angles. By the hyperbolic cosine rule for the top triangle, we have

$$\cos \theta_{\mathrm{top}} = \frac{\cosh a_1 \cosh a_2 - \cosh a_3}{\sinh a_1 \sinh a_2},$$

where θ_{top} is the angle between A_1 and A_2. If we look at the bottom triangle, then the corresponding angle θ_{bottom} has an expression in terms of b_j's.

$$\cos \theta_{\mathrm{bottom}} = \frac{\cosh b_1 \cosh b_2 - \cosh b_3}{\sinh b_1 \sinh b_2}.$$

They represent the same dihedral angle, and we obtain another relation,

$$\cos \theta_{\mathrm{top}} - \cos \theta_{\mathrm{bottom}} = 0. \tag{2}$$

It is not hard to verify that the set of six length variables subject to the relations (1) and (2) parametrizes isometry classes of labelled truncated tetrahedra.

§4. Gluing Consistency.

We will parametrize the deformation of O_n in terms of the parametrization of truncated tetrahedra given in §3.

To create nonsingular but not necessarily complete hyperbolic orbifold structure on W, it is sufficient to verify gluing consistency which consists of the isometricity conditions for faces to be identified, and the cone angle conditions along edges. We will see when these are satisfied.

If the external edges to be identified have the same length, then the isometricity condition for face identification is satisfied. Since there are twelve such pairs, there are twelve simple identities in the variables we must obviously require. For simplicity, we just assign the same variable to each pair to be identified from the beginning and reduce the number of the variables to the half. Here let us choose the followings as such twelve variables.

$$b_1,\ e_3,\ a_2,\ c_2,\ d_1,\ e_1,\ f_2,\ g_1,\ h_2,\ a_3,\ c_3,\ g_3.$$

Then the relations of type (1) and (2) for the four truncated tetrahedra become dependent after gluing. In fact, reading off the lengths of the bottom edges of the pentagonal faces in order, we can see that one of the four equations of type (1), say the equation corresponding to the truncated tetrahedron parametrized by $\{g_j, h_j\}$, becomes a consequence of the other three.

To compute the cone angle conditions along edges, we label them by P_1, P_2, P_3 and Σ as in Figure 1. The dihedral angle of each edge is described in terms of the lengths of external edges as the above expression of θ_{top}. To obtain nonsingular orbifold structure, the total sum of dihedral angles around the first three edges must be 2π and the last $2\pi/n$. These constrains give four identities. The last one is independent from the others, however one of the first three identities is a consequence of the other two. To see this, recall that a toral section of the end always admits a similarity structure. Then the total sum of angles of triangles appeared in the horospherical triangulation is $4 \times 2\pi$. It is equal to the sum of the total sum of dihedral angles along P_1 and P_2 and twice of that of P_3. Then we need the following three equations:

$$\begin{aligned}
&\textstyle\sum_{e \in \mathcal{E}_1} \ (\text{the dihedral angle along } e) - 2\pi = 0, \\
&\textstyle\sum_{e \in \mathcal{E}_2} \ (\text{the dihedral angle along } e) - 2\pi = 0, \\
&\textstyle\sum_{e \in \mathcal{E}_\Sigma} \ (\text{the dihedral angle along } e) - 2\pi/n = 0,
\end{aligned}$$

where $\mathcal{E}_j$ (resp. $\mathcal{E}_\Sigma$) is a set of edges of the truncated tetrahedra which are glued to be P_j (resp. Σ).

We thus have obtained ten relations with twelve variables from gluing consistency. These relations define a map

$$f : \mathbf{R}^{12} \longrightarrow \mathbf{R}^{10},$$

such that its zero set $\mathcal{W} = f^{-1}(0)$ consists of the points in $\mathbf{R}^{12}$ satisfying the gluing consistency.

Denote by x and y the two variables indicated in Figure 2, and by $z_1, \cdots z_{10}$ the other 10 variables as follows:

$$\begin{aligned}
x \ &= b_1, \quad y = e_3, \\
z_1 \ &= a_2, \ z_2 = c_2, \ z_3 = d_1, \ z_4 = e_1, \ z_5 = f_2, \ z_6 = g_1, \ z_7 = h_2, \ z_8 = a_3, \ z_9 = c_3, \ z_{10} = g_3.
\end{aligned}$$

Also let $\boldsymbol{w}_0 \in \mathbf{R}^{12}$ be the point corresponding to the complete hyperbolic structure.

Then we obtain

$$\left(\frac{\partial f_i}{\partial z_j}(\boldsymbol{w}_0)\right) = \begin{pmatrix} -\alpha & 0 & 0 & \alpha & \alpha & 0 & 0 & 0 & 0 & 0 \\ \alpha & -\alpha & -\alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha & \alpha & -\alpha & -\alpha & 0 & 0 & 0 \\ \beta & 0 & 0 & -\beta & \beta & 0 & 0 & \sqrt{2}\beta & -\sqrt{2}\beta & 0 \\ \beta & \beta & -\beta & 0 & 0 & 0 & 0 & 0 & \sqrt{2}\beta & 0 \\ 0 & 0 & 0 & \beta & -\beta & \beta & -\beta & 0 & 0 & -\sqrt{2}\beta \\ 0 & -\beta & \beta & 0 & 0 & \beta & -\beta & -\sqrt{2}\beta & 0 & \sqrt{2}\beta \\ \gamma & \gamma & -\sqrt{2}\delta & \gamma & -\sqrt{2}\delta & -\sqrt{2}\delta & -\sqrt{2}\delta & \delta & 2\delta & 0 \\ -\sqrt{2}\delta & -\sqrt{2}\delta & \gamma & -\sqrt{2}\delta & \gamma & \gamma & \gamma & \delta & 0 & 2\delta \\ 2\beta & 2\beta & 0 & 0 & 2\beta & 0 & 2\beta & 2\sqrt{2}\beta & \sqrt{2}\beta & \sqrt{2}\beta \end{pmatrix},$$

$$\left(\frac{\partial f_i}{\partial x}(\boldsymbol{w}_0)\right)^T = (-\alpha, \alpha, 0, -\beta, -\beta, 0, 0, \gamma, -\sqrt{2}\delta, 0),$$

$$\left(\frac{\partial f_i}{\partial y}(\boldsymbol{w}_0)\right)^T = (0, 0, 0, 0, -\sqrt{2}\beta, \sqrt{2}\beta, 0, \delta, \delta, 0),$$

where $\alpha = -\dfrac{s^2}{c\sqrt{2c(1+c)}}$, $\beta = -\dfrac{s^2}{\sqrt{2c(1+c)}}$, $\gamma = \dfrac{2c-1}{\sqrt{2c(1+c)}}$, $\delta = \dfrac{1}{2\sqrt{c(1+c)}}$, $c = \cos\dfrac{\pi}{2n}$, $s = \sin\dfrac{\pi}{2n}$.

Then we can see that the rank of the matrix $\left(\dfrac{\partial f_i}{\partial z_j}(\boldsymbol{w}_0)\right)$ is 10. Also it is not hard to find the unique solutions of the following two linear equations in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$ respectively:

$$\begin{aligned} \left(\frac{\partial f_i}{\partial z_j}(\boldsymbol{w}_0)\right)\boldsymbol{u} &= -\left(\frac{\partial f_i}{\partial x}(\boldsymbol{w}_0)\right), \\ \left(\frac{\partial f_i}{\partial z_j}(\boldsymbol{w}_0)\right)\boldsymbol{v} &= -\left(\frac{\partial f_i}{\partial y}(\boldsymbol{w}_0)\right). \end{aligned}$$

In fact, if we denote the unique solutions by $\boldsymbol{u}_0$ and $\boldsymbol{v}_0$ respectively, we have

$$\begin{aligned} \boldsymbol{u}_0^T &= (-1, 1, -1, -1, 1, 1, -1, 0, 0, 0), \\ \boldsymbol{v}_0^T &= (0, -\sqrt{2}/2, \sqrt{2}/2, \sqrt{2}/2, -\sqrt{2}/2, 0, 0, -1, 0, 0). \end{aligned}$$

Then we obtained

LEMMA 2. *$\mathcal{W} = f^{-1}(0)$ is a 2-dimensional smooth manifold near $\boldsymbol{w}_0$ and we have two paths on $\mathcal{W} \subset \mathbf{R}^{12}$,*

$$\xi(t) = \boldsymbol{w}_0 + \boldsymbol{x}t + (\textit{higher order}), \qquad \eta(t) = \boldsymbol{w}_0 + \boldsymbol{y}t + (\textit{higher order}),$$

such that

$$\boldsymbol{x} = \begin{pmatrix} 1 \\ 0 \\ \boldsymbol{u}_0 \end{pmatrix}, \qquad \boldsymbol{y} = \begin{pmatrix} 0 \\ 1 \\ \boldsymbol{v}_0 \end{pmatrix},$$

where the y-component of $\xi(t)$ and the x-component of $\eta(t)$ are constant, and the x-component of $\xi(t)$ and the y-component of $\eta(t)$ have no terms of degrees n ($n \geq 2$).

§5. Dehn Filling Space and Computation.

The space $\mathcal{DF}_{O_n}$, the Dehn filling parameter space of O_n, is the set of complex lengths of the preferred meridian m for the cusp. The squares of the elements of $\mathcal{DF}_{O_n}$ turn out to be a local coordinate of $\mathcal{D}_{O_n}$ near the complete structure.

Consider a map

$$G : \mathcal{W} \longrightarrow \mathcal{DF}_{O_n},$$

which assigns to each element of $\mathcal{W}$ the corresponding Dehn filling parameter.

All inner angles of flat triangles, produced by cutting off the neighborhoods of the ideal vertices of the truncated tetrahedra along horospheres, are described explicitly in terms of the twelve parameters $x, y, z_1, \ldots, z_{10}$, by using hyperbolic trigonometry. Following [6] [7], the complex length $G(\boldsymbol{w})$ of m is described by the angles of the triangles. Then by direct computations, we can verify that the rank of the Jacobian of G at $\boldsymbol{w}_0$ is 2. Thus we have

LEMMA 3. *G is a local diffeomorphism at $\boldsymbol{w}_0 \in \mathcal{W}$.*

Letting $\mathcal{L}$ be the complex length of m, the trace of m is expressed by $2\cosh\frac{\mathcal{L}}{2}$. Since the complex length corresponding to the complete structure is 0, the natural map $\mathcal{DF}_{O_n} \to \mathcal{D}_{O_n}$ is a 2-fold branched covering around the complete structure. Hence the composition π of G with the natural map is also a 2-fold covering branched at $\boldsymbol{w}_0$. π is in fact the map locally looks like a square function : $z \longrightarrow z^2$.

The lengths L_i of geodesic segments S_i $(i = 1, \ldots, 4)$ which are illustrated by thick lines in Figure 5 cause a quadruple (L_1, L_2, L_3, L_4) which defines a global coordinate of $\mathcal{T}(\partial O_n)$.

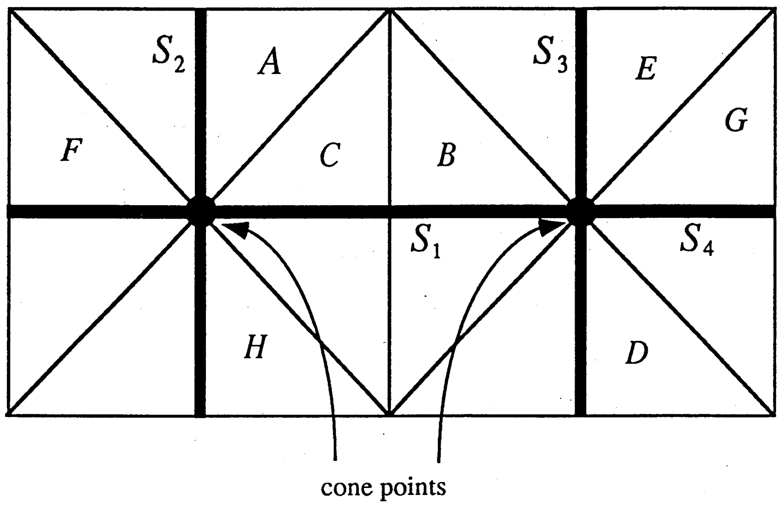

Figure 5

Let $\widetilde{B}$ be a map assigning to the element of $\mathcal{W}$ the corresponding hyperbolic structure of the boundary. Then its induced map from $\mathcal{D}_{O_n}$ is B_{O_n}. Let $\widetilde{B}_i$ (resp. B_i) be the composition of $\widetilde{B}$ (resp. B_{O_n}) with L_i.

$$\begin{tikzcd}
\mathcal{W} \arrow[d, "\pi"'] \arrow[dr, "\widetilde{B}"] & & \\
\mathcal{D}_{O_n} \arrow[r, "B_{O_n}"'] & \mathcal{T}(\partial O_n) \arrow[r, "{(L_1, L_2, L_3, L_4)}"'] & \mathbf{R}^4
\end{tikzcd}$$

Now consider a quadrilateral in general. If the lengths of four sides and one of diagonals are known, then the length of the other diagonal can be expressed in terms of them by hyperbolic trigonometry. Applying this to the quadrilateral in Figure 5 which is made of two triangular faces, we have an expression of $\widetilde{B}_i$ as a function of our length parameters.

Because of the local picture of π, letting $\bar{\xi}(t) = \pi \circ \xi(\sqrt{t})$ and $\bar{\eta}(t) = \pi \circ \eta(\sqrt{t})$, we obtain smooth paths on $\mathcal{D}_{O_n}$ such that its tangent vectors

$$v = \frac{d}{dt}\bar{\xi}(t)|_{t=0}\ , \qquad w = \frac{d}{dt}\bar{\eta}(t)|_{t=0}$$

are nontrivial. The images of these vectors by the derivative dB_i are now expressed by

$$\begin{aligned} dB_i(v) &= \left.\frac{dB_i(\overline{\xi}(t))}{dt}\right|_{t=0} = \left.\frac{d\widetilde{B}_i(\xi(\sqrt{t}))}{dt}\right|_{t=0}, \\ dB_i(w) &= \left.\frac{dB_i(\overline{\eta}(t))}{dt}\right|_{t=0} = \left.\frac{d\widetilde{B}_i(\eta(\sqrt{t}))}{dt}\right|_{t=0}. \end{aligned}$$

To carry out the actual computation of the right hand sides, we use the Taylor expansions of $\xi(t)$ and $\eta(t)$ up to the second degree. They can be deribed from the formula,

$$\frac{d^2 f_i(\xi)}{dt^2}(0) = \sum_{j,k} \frac{\partial^2 f_i}{\partial z_j \partial z_k}(\boldsymbol{w}_0)\frac{d\xi_k}{dt}(0)\frac{d\xi_j}{dt}(0) + \sum_j \frac{\partial f_i}{\partial z_j}(\boldsymbol{w}_0)\frac{d^2\xi_j}{dt^2}(0).$$

By using it, we obtained the following:

$$\left(\frac{d^2\xi_i}{dt^2}(0)\right) = \begin{pmatrix} 0 \\ 0 \\ \sqrt{2}\sqrt{1+c}/\sqrt{c} \\ \sqrt{2}\sqrt{1+c}/\sqrt{c} \\ 0 \\ 0 \\ \sqrt{2}\sqrt{1+c}/\sqrt{c} \\ 0 \\ \sqrt{2}\sqrt{1+c}/\sqrt{c} \\ 4\sqrt{1+c}/\sqrt{c} \\ 2\sqrt{1+c}/\sqrt{c} \\ 2\sqrt{1+c}/\sqrt{c} \end{pmatrix}, \quad \left(\frac{d^2\eta_i}{dt^2}(0)\right) = \begin{pmatrix} 0 \\ 0 \\ \sqrt{2}(1+s)/4s\sqrt{c(1+c)} \\ \sqrt{2}(1-c+5s+3cs)/8s\sqrt{c(1+c)} \\ \sqrt{2}(1+s)\sqrt{1+c}/8s\sqrt{c} \\ \sqrt{2}(1+s)\sqrt{1+c}/8s\sqrt{c} \\ \sqrt{2}(1-c+5s+3cs)/8s\sqrt{c(1+c)} \\ 0 \\ \sqrt{2}(1+s)/4s\sqrt{c(1+c)} \\ \sqrt{2}(1-c+3s+cs)/2s\sqrt{c(1+c)} \\ \sqrt{2}(1-c+s+cs)/2s\sqrt{c(1+c)} \\ \sqrt{2}(1-c+s+cs)/2s\sqrt{c(1+c)} \end{pmatrix},$$

where $c = \cos\frac{\pi}{2n}$, $s = \sin\frac{\pi}{2n}$.

By performing rather lengthy but direct computations by hand, we verified the following:

LEMMA 4.

$$dB_1(v) = -\frac{1}{\sqrt{c}} \quad (<0), \qquad dB_2(v) = \frac{1}{\sqrt{c}} \quad (>0),$$

$$dB_1(w) = \frac{s^2(1-c)}{4\sqrt{c}} \quad (>0), \quad dB_2(w) = \frac{s+1}{8s\sqrt{c}} \quad (>0),$$

where $c = \cos\dfrac{\pi}{2n}$ *and* $s = \sin\dfrac{\pi}{2n}$.

Lemma 4 shows that these tangent vectors on $\mathcal{D}_{O_n}$ go to a linearly independent pair in the tangent space of $\mathcal{T}(\partial O_n)$ at the original structure and we complete the proof of Theorem.

References

[1] M. Culler and P.B. Shalen, *Varieties of group representations and splittings of 3-manifolds*, Ann. Math. **117** (1983), 109-146.

[2] M. Fujii, *Deformations of a hyperbolic 3-manifold not affecting its totally geodesic boundary*, Kodai Math. J. **16** (1993), 441-454.

[3] M. Fujii and S. Kojima, *Flexible boundaries in deformations of hyperbolic 3-manifolds.* (Preprint)

[4] C.D. Hodgson and S.P. Kerckhoff, *Rigidity of hyperbolic cone-manifolds and hyperbolic Dehn surgery.* (Preprint)

[5] W.D. Neumann and A.W. Reid, *Rigidity of cusps in deformations of hyperbolic 3-orbifolds*, Math. Ann. **295** (1993), 223-237.

[6] W.D. Neumann and D. Zagier, *Volumes of hyperbolic 3-manifolds*, Topology **24** (1985), 307-332.

[7] W.P. Thurston, The geometry and topology of 3-manifolds, Lect. Notes, Princeton Univ., 1978.

Michihiko Fujii
Department of Mathematical Sciences
Yokohama City University
22-2 Seto, Kanazawa-ku, Yokohama, Kanagawa-ken 236, Japan
e-mail: mfujii@yokohama-cu.ac.jp

Sadayoshi Kojima
Department of Mathematical and Computing Sciences
Tokyo Institute of Technology
Ohokayama, Meguro, Tokyo 152, Japan
e-mail: sadayosi@is.titech.ac.jp