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The Rohlin Property for Actions of $\mathbf{Z}^2$ on UHF Algebras

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1 Introduction

A noncommutative Rohlin type theorem was introduced by A. Connes for the classification of single automorphisms of von Neumann algebras[2, 3]. This was generalized, for example, by A.Ocneanu [19, 20], for systems of several commuting automorphisms of finite von Neumann algebras, and more generally for actions of discrete amenable groups. On the other hand this was also generalized in the framework of C^*-algebras. The Rohlin property for single automorphisms of a certain class of C^*-algebras was established [1, 5, 6, 11, 12, 13, 14]. In particular a noncommutative Rohlin type theorem for single automorphisms of UHF algebras (and some AF algebras) was shown and the automorphisms with the Rohlin property were classified up to outer conjugacy by A. Kishimoto [13, 14]. Here we present a generalization of the above results for UHF algebras.

2 Rohlin type theorem

Let N be a positive integer . We first define the Rohlin property for actions of $\mathbf{Z}^N$ on unital C^*-algebras. This is a simple generalization of that in the case of $N=1$ [13]. Let $\xi_1,\ldots,\xi_N$ be the canonical basis of $\mathbf{Z}^N$ i.e.

$$\xi_i \equiv (0,\ldots,0,1,0,\ldots,0),$$

where 1 is in the i-th component, and let $I=(1,\ldots,1)$ throughout this paper. For $m=(m_1,\ldots,m_N)$ and $n=(n_1,\ldots,n_N)\in\mathbf{Z}^N$, $m\leq n$ means $m_i\leq n_i$ for each $i=1,\ldots,N$.

Definition 1 Let N be a positive integer. Let A be a unital C^*-algebra and let α be an action of $\mathbf{Z}^N$ on A i.e. α is a group homomorphism from $\mathbf{Z}^N$ into the automorphisms $Aut(A)$ of A. Then α is said to have *the Rohlin property* if for any $m\in\mathbf{N}^N$ there exist $m^{(1)},\ldots,m^{(R)}\in\mathbf{N}^N$ with $m^{(1)},\ldots,m^{(R)}\geq m$ and which satisfy the following condition: For any $\varepsilon>0$ and finite subset F of A there exist projections

$$e_g^{(r)}\ :\ r=1,\ldots,R,\ g\in\mathbf{Z}^N/m^{(r)}\mathbf{Z}^N$$

in A satisfying

$$\sum_{r=1}^{R} \sum_{g \in \mathbf{Z}^N/m^{(r)}\mathbf{Z}^N} e_g^{(r)} = 1 ,$$

$$\|[x,\ e_g^{(r)}]\| < \varepsilon, \tag{1}$$

$$\|\alpha_g(e_h^{(r)}) - e_{g+h}^{(r)}\| < \varepsilon$$

for any $x \in F$, $r = 1, \ldots, R$, $g \in \mathbf{Z}^N$ with $0 \le g \le m^{(r)} - I$ and $h \in \mathbf{Z}^N/m^{(r)}\mathbf{Z}^N$.

Remark 2 Let A be a UHF algebra. Then from the standard technique, we can replace the condition (1) by

$$[x,\ e_g^{(r)}] = 0 .$$

An action α of $\mathbf{Z}^N$ on A is determined by an N-tuple $(\alpha_{\xi_1}, \ldots, \alpha_{\xi_N})$ of commuting automorphisms of A. In terms of $(\alpha_{\xi_1}, \ldots, \alpha_{\xi_N})$ we can restate the Rohlin property for α as follows: For any $n, m \in \mathbf{N}$ with $1 \le n \le N$ there exist positive integers $m^{(1)}, \ldots, m^{(R)} \ge m$ which satisfy the following condition: For any $\varepsilon > 0$ and finite subset F of A there exist projections

$$e_0^{(r)}, \ldots, e_{m^{(r)}-1}^{(r)} \ : \ r = 1, \ldots, R$$

in A satisfying

$$\sum_{r=1}^{R} \sum_{j=0}^{m^{(r)}-1} e_j^{(r)} = 1 ,$$

$$\|[x,\ e_j^{(r)}]\| < \varepsilon$$

for each $r = 1, \ldots, r$, $j = 0, \ldots, m^{(r)} - 1$ and $x \in F$, and

$$\|\alpha_{\xi_n}(e_j^{(r)}) - e_{j+1}^{(r)}\| < \varepsilon ,$$

$$\|\alpha_{\xi_{n'}}(e_j^{(r)}) - e_j^{(r)}\| < \varepsilon$$

for each $n' = 1, \ldots, N$ with $n' \neq n$, $r = 1, \ldots R$ and $j = 0, \ldots, m^{(r)} - 1$, where $e_{m^{(r)}}^{(r)} \equiv e_0^{(r)}$.

In [13] A. Kishimoto introduced a notion of uniform outerness for automorphisms of C^*-algebras and he showed, if the algebras are UHF, this notion is equivalent to the usual outerness for automorphisms of the von Neumann algebras obtained through GNS representations associated with traces.

Definition 3 Let A be a unital C^*-algebra and let α be an automorphism of A. Then α is said to be *uniformly outer* if for any $a \in A$, nonzero projection $p \in A$ and $\varepsilon > 0$ there exist projections $p_1, \ldots, p_n$ in A such that

$$p = \sum_{i=1}^{n} p_i ,$$

$$\|p_i a \alpha(p_i)\| < \varepsilon$$

for $i = 1, \ldots, n$.

Theorem 4 [13] *Let A be a UHF algebra and let α be an automorphism of A. Then the following conditions are equivalent:*

(1) *α is uniformly outer.*

(2) *The weak extension of α to an automorphism of $\pi_\tau(A)''$ is outer, where τ denotes a unique tracial state on A and π_τ is the GNS representation of A associated with τ.*

We recall a Rohlin type theorem for automorphisms of UHF algebras.

Theorem 5 [14] *Let α be an automorphism of a UHF algebra A. Then the following conditions are equivalent:*

(1) *α has the Rohlin property.*

(2) *α^m is uniformly outer for any $m \in \mathbf{Z} \setminus \{0\}$.*

We show the two-dimensional version of the above theorem, namely

Theorem 6 *Let α be an action of $\mathbf{Z}^2$ on a UHF algebra A. Then the following conditions are equivalent:*

(1) *α has the Rohlin property.*

(2) *α_g is uniformly outer for any $g \in \mathbf{Z}^2 \setminus \{0\}$.*

Once we establish this theorem, we have immediately

Corollary 7 *Let α be an action of $\mathbf{Z}^2$ on a UHF algebra A. Then the following conditions are equivalent:*

(1) *α has the Rohlin property as an action of $\mathbf{Z}^2$ on A.*

(2) *α_g has the Rohlin property as an automorphism of A for any $g \in \mathbf{Z}^2 \setminus \{0\}$.*

3 Conjugacy

We introduce three types of conjugacy for actions of $\mathbf{Z}^N$ on C^*-algebras and discuss their relation.

Definition 8 Let α be an action of $\mathbf{Z}^N$ on a unital C^*-algebra A and let u be a mapping from $\mathbf{Z}^N$ into the unitaries $U(A)$ of A. Then u is said to be a *1-cocycle* for α if

$$u_{g+h} = u_g \alpha_g(u_h)$$

for any $g, h \in \mathbf{Z}^N$

If α and β are actions of $\mathbf{Z}^N$ on a unital C^*-algebra A, then for each $\varepsilon \geq 0$ and $\gamma \in Aut(A)$ we write $\alpha \overset{\gamma,\varepsilon}{\approx} \beta$ if

$$\|\alpha_{\xi_i} - \gamma \circ \beta_{\xi_i} \circ \gamma^{-1}\| \leq \varepsilon$$

for $i = 1, \ldots, N$ and write $\alpha \overset{\gamma}{\cong} \beta$ (or $\alpha \cong \beta$ simply) instead of $\alpha \overset{\gamma,0}{\approx} \beta$.

Definition 9 Let α and β be as above. Then
(1) α and β are said to be *approximately conjugate* if for any $\varepsilon > 0$ there exists an automorphism γ of A such that $\alpha \overset{\gamma,\varepsilon}{\approx} \beta$.
(2) α and β are said to be *cocycle conjugate* if there exist an automorphism γ of A and a 1-cocycle u for α such that

$$Ad\, u_g \circ \alpha_g = \gamma \circ \beta_g \circ \gamma^{-1}$$

for any $g \in \mathbf{Z}^N$.
(3) α and β are said to be *outer conjugate* if there exist an automorphism γ of A and unitaries $u_1, \ldots, u_N$ in A such that

$$Ad\, u_i \circ \alpha_{\xi_i} = \gamma \circ \beta_{\xi_i} \circ \gamma^{-1}$$

for $i = 1, \ldots, N$.

Of course cocycle conjugacy implies outer conjugacy. Moreover one has

Proposition 10 *Let A be a simple, separable unital C^*-algebra with a unique tracial state τ and let α, β be actions of $\mathbf{Z}^N$ on A. If α and β are approximately conjugate then they are cocycle conjugate.*

Remark 11 If α and β are automorphisms of a UHF algebra with the Rohlin property and they are outer conjugate, then they are approximately conjugate by the stability property. Hence the three notions of conjugacy defined above are equivalent for those automorphisms. But outer conjugacy does not always imply approximate conjugacy for $\mathbf{Z}^N$ actions. See Remark 18 for a counter example.

4 Product type actions

We discuss product type actions of $\mathbf{Z}^2$ on UHF algebras. As in the case of single automorphisms, the Rohlin property for these actions is deeply related to a notion of uniform distribution of points in $\mathbf{T}^2$. We first state this notion as a proposition whose proof is found in [1].

Proposition 12 *Let $(S_k \mid k \in \mathbf{N})$ be a sequence of finite sequences in $\mathbf{T}^N$ i.e.*

$$S_k = (s_{k,p} \mid p = 1, \ldots, n_k),$$

$$s_{k,p} \in \mathbf{T}^N$$

for each $k \in \mathbf{N}$ and $p = 1, \ldots n_k$. Then the following conditions on $(S_k \mid k \in \mathbf{N})$ are equivalent.

(1)

$$\lim_{k\to\infty} \frac{1}{n_k} \sum_{p=1}^{n_k} f(s_{k,p}) = \int_{\mathbf{T}^n} f(s)ds$$

for any $f \in C(\mathbf{T}^N)$, where ds denotes the normalized Haar measure on $\mathbf{T}^N$.

(2)

$$\lim_{k\to\infty} \frac{1}{n_k} \sum_{p=1}^{n_k} s_{k,p}^l = 0$$

for any $l = (l_1, \ldots, l_N) \in \mathbf{Z}^N \setminus \{0\}$, where s^l denotes $s_1^{l_1} \cdots s_N^{l_N}$ for each $s = (s_1, \ldots, s_N) \in \mathbf{T}^N$.

(3)

$$\lim_{k\to\infty} \frac{1}{n_k} \nu_k \left(\prod_{i=1}^{N} [\,\theta_1^{(i)}, \theta_2^{(i)}) \right) = (2\pi)^{-N} \prod_{i=1}^{N} (\theta_2^{(i)} - \theta_1^{(i)})$$

for any $0 \le \theta_1^{(i)} \le \theta_2^{(i)} < 2\pi$, where ν_k is defined by

$$\nu_k(S) \equiv \sharp\{\, p \mid 1 \le p \le n_k \text{ and } \arg(s_{k,p}) \in S \,\}$$

for each subset S of $\prod_{i=1}^{N}[\,0, 2\pi)$ and $\sharp F$ denotes the cardinality of the set F.

Moreover suppose that $(n_k \mid k \in \mathbf{N})$ have the following property: For any $n \in \mathbf{N}$ there exists positive integer k_0 such that for any positive integer $k \ge k_0$ there exist positive integers $n_k^{(1)}, \ldots, n_k^{(N)} \ge n$ which satisfy $n_k = n_k^{(1)} \cdots n_k^{(N)}$. Then the above conditions are also equivalent to

(4) For any $\varepsilon > 0$ there exist positive integers k_0 and n_0 such that for any k, m, $n_k^{(1)}, \ldots, n_k^{(N)} \in \mathbf{N}$ satisfying $k \ge k_0$, $n_k^{(1)}, \ldots, n_k^{(N)} \ge n_0$ and $n_k =$

$m \cdot n_k^{(1)} \cdots n_k^{(N)}$ *there exists an* m *to 1 surjection* φ *from* $\{1, \ldots, n_k\}$ *onto* $\{1, \ldots, n_k^{(1)}\} \times \cdots \times \{1, \cdots, n_k^{(N)}\}$ *satisfying*

$$\left| s_{k,p} - (\exp\left(2\pi i \cdot \frac{(\varphi(p))_1}{n_k^{(1)}}\right), \ldots, \exp\left(2\pi i \cdot \frac{(\varphi(p))_N}{n_k^{(N)}}\right)) \right| < \varepsilon \tag{2}$$

for any k *and* p, *where* $|s| \equiv \max\{ |s_p| \, : \, 1 \leq p \leq N\}$ *for each* $s \in \mathbf{T}^N$ *and* $(\varphi(p))_i$ *denotes the* i*-th component of* $\varphi(p)$.

If S_k satisfies the estimate (2) for some φ as above, then S_k is said to be $(n_k^{(1)}, \ldots, n_k^{(N)}; \varepsilon)$*-distributed.* If one of the conditions of the above proposition holds then $(S_k \,|\, k \in \mathbf{N})$ is said to be *uniformly distributed.* Now we state

Definition 13 Let A be a UHF algebra and let α be an action of $\mathbf{Z}^N$ on A. Then α is said to be a *product type action* if there exist a sequence $(m_k \,|\, k \in \mathbf{N})$ of positive integers such that $A \cong \otimes_{k=1}^{\infty} M_{m_k}(\mathbf{C})$ and

$$\alpha_g(A_k) = A_k$$

for any $g \in \mathbf{Z}^N$ and $k \in \mathbf{N}$, where A_k denotes the C^*-subalgebra of A corresponding to $M_{m_k}(\mathbf{C}) \otimes (\otimes_{l \neq k} \mathbf{C}1_{m_k})$.

Remark 14 In the situation above, if $N = 2$ then one finds unitaries $u_k^{(1)}, u_k^{(2)}$ in A_k and $\lambda_k \in \mathbf{T}$ such that

$$\alpha_{(p,q)} \lceil A_k = Ad\, {u_k^{(1)}}^p {u_k^{(2)}}^q ,$$

$$u_k^{(1)} u_k^{(2)} = \lambda_k u_k^{(2)} u_k^{(1)}$$

for any p, q. Since $u_k^{(1)}, u_k^{(2)}$ are unique up to a constant multiple, λ_k is unique. In addition $\lambda_k^{m_k} = 1$.

Let $n \in \mathbf{N}$ and let $U, V \in U(M_n(\mathbf{C}))$ satisfying $UV = VU$. Then we set $\mathrm{Sp}(U)$ to be a sequence consisting of the eigenvalues of U repeated as often as multiplicity indicates and $\mathrm{Sp}(U, V)$ is a sequence consisting of the pairs of eigenvalues of U and V with a common eigenvector, repeated as often as multiplicity indicates. Then the Rohlin property for the product type actions of $\mathbf{Z}^2$ on A with $\lambda_k = 1$ is characterized as follows.

Proposition 15 *Let* A *be a UHF algebra and let* α *be a product type action of* $\mathbf{Z}^2$ *on* A *with* $(m_k \,|\, k \in \mathbf{N})$, $(u_k^{(1)} \,|\, k \in \mathbf{N})$, $(u_k^{(2)} \,|\, k \in \mathbf{N})$, $(\lambda_k \,|\, k \in \mathbf{N})$ *as above. If* $\lambda_k = 1$ *for each* $k \in \mathbf{N}$ *then the following conditions are equivalent:*

(1) α *has the Rohlin property.*

(2) $(\,\mathrm{Sp}(\otimes_{k=m}^{n} u_k^{(1)}, \otimes_{k=m}^{n} u_k^{(2)}) \mid n = m,\ m+1,\ \dots\,)$ *is uniformly distributed for any* $m \in \mathbf{N}$.

In [14] A.Kishimoto showed for each UHF algebra A

(1) For any product type actions α and β of $\mathbf{Z}$ on A with the Rohlin property, α and β are approximately conjugate.

(2) For any action α of $\mathbf{Z}$ on A with the Rohlin property and $\varepsilon > 0$ there exist a product type action β of $\mathbf{Z}$ on A with the Rohlin property and an automorphism γ of A such that $\alpha \overset{\gamma,\varepsilon}{\approx} \beta$.

In particular there is one and only one approximate conjugacy class of actions of $\mathbf{Z}$ on A with the Rohlin property. In the case of $N = 2$ we do not know whether (2) is valid or not. In the rest of this paper we state several results for (1).

Theorem 16 *Let A be a UHF algebra and let α and β be product type actions of $\mathbf{Z}^2$ on A with the Rohlin property. Take $(\,m_k \mid k \in \mathbf{N}\,)$, $(\,\lambda_k \mid k \in \mathbf{N}\,)$ for α as in Definition 13 and Remark 14. Also take $(\,n_l \mid l \in \mathbf{N}\,)$, $(\,\mu_l \mid l \in \mathbf{N}\,)$ for β similarly. If $\lambda_k = \mu_l = 1$ for each $k, l \in \mathbf{N}$ then α and β are approximately conjugate.*

We discuss product type actions for two classes of UHF algebras. Let $(\,p_k \mid k \in \mathbf{N}\,)$ be the increasing sequence of all the prime numbers. For a sequence $(\,i_k \mid k \in \mathbf{N}\,)$ of nonnegative integers such that $\sum_{k=1}^{\infty} i_k = \infty$, let $q_k \equiv p_k^{i_k}$ and let $A \equiv \otimes_{k=1}^{\infty} M_{q_k}(\mathbf{C})$. We regard $M_{q_k}(\mathbf{C})$ as a C^*-subalgebra of A. We consider the class of product type actions α of $\mathbf{Z}^2$ on A factorized as $\otimes_{k=1}^{\infty} M_{q_k}(\mathbf{C})$. Thus α is defined in terms of unitaries $u_k^{(1)}, u_k^{(2)}$ in $M_{q_k}(\mathbf{C})$ and $\lambda_k \in \mathbf{T}$ with $u_k^{(1)} u_k^{(2)} = \lambda_k u_k^{(2)} u_k^{(1)}$ by

$$\alpha_{(p,q)} \lceil M_{q_k}(\mathbf{C}) = Ad\, {u_k^{(1)}}^p {u_k^{(2)}}^q \ .$$

Since $\lambda_k^{q_k} = 1$, we may regard λ_k as an element of $G_k \equiv \mathbf{Z}/q_k\mathbf{Z}$. We let $[\alpha]$ be the sequence $(\,\lambda_k \mid k \in \mathbf{N}\,)$ which is also considered as an element of $\prod_{k=1}^{\infty} G_k$. We define an equivalence relation in $\prod_{k=1}^{\infty} G_k$ by: $g \sim h$ if there is an n such that $g_k = h_k$ for $k \geq n$. Let 0 be the trivial sequence $(\,0, 0, \dots)$. We note that for every $g \in \prod_{k=1}^{\infty} G_k$ there is an action α of $\mathbf{Z}^2$ in the above class such that $[\alpha] = g$.

Theorem 17 (1) *If α is an action of $\mathbf{Z}^2$ in the above class and $[\alpha] \not\sim 0$, then α has the Rohlin property.*
(2) *If α and β are actions of $\mathbf{Z}^2$ in the above class and satisfy the Rohlin property, then the following are equivalent:*

(2.1) $[\alpha] \sim [\beta]$.

(2.2) *α and β are outer conjugate with each other.*

Remark 18 Let $A \equiv M_3(\mathbf{C}) \otimes M_{2^\infty}(\mathbf{C})$ and define $n \times n$ unitary matrices u_n, v_n by

$$u_n \equiv \begin{bmatrix} 1 & & & \\ & \omega_n & & \\ & & \ddots & \\ & & & \omega_n^{n-1} \end{bmatrix}, \quad v_n \equiv \begin{bmatrix} 0 & \cdots & 0 & 1 \\ 1 & \ddots & & 0 \\ & \ddots & \ddots & \vdots \\ & & 1 & 0 \end{bmatrix},$$

where $\omega_n \equiv \exp\left(2\pi i \cdot \frac{1}{n}\right)$ for each $n \in \mathbf{N}$. Using these matrices we define actions α, β of $\mathbf{Z}^2$ on A by

$$\alpha_{\xi_1} \equiv Ad\, u_3 \otimes \left(\bigotimes_{k=1}^{\infty} Ad\, u_{2^k} \right), \quad \alpha_{\xi_2} \equiv Ad\, v_3 \otimes \left(\bigotimes_{k=1}^{\infty} Ad\, v_{2^k} \right),$$

$$\beta_{\xi_1} \equiv id_{M_3(\mathbf{C})} \otimes \left(\bigotimes_{k=1}^{\infty} Ad\, u_{2^k} \right), \quad \beta_{\xi_1} \equiv id_{M_3(\mathbf{C})} \otimes \left(\bigotimes_{k=1}^{\infty} Ad\, v_{2^k} \right).$$

Then in the same way as the proof of Theorem 17, we can show that α, β have the Rohlin property and they are not approximately conjugarte. But they are clearly outer conjugate.

Let $\{ q_k \mid k \in K \}$ be a finite or infinite set of prime numbers. We next consider product type actions of $\mathbf{Z}^2$ on the UHF algebra

$$\bigotimes_{k \in K} M_{q_k^\infty}(\mathbf{C}),$$

where $M_{q_k^\infty}(\mathbf{C})$ is understood as $\otimes_{n=1}^{\infty} M_{q_k}(\mathbf{C})$.

Theorem 19 *Let A be a UHF algebra as above and let α, β be product type actions of $\mathbf{Z}^2$ on A. If α and β have the Rohlin property then α and β are approximately conjugate with each other.*

Now we sum up the results we have stated so far.

Theorem 20 *Let $(p_k \mid k \in \mathbf{N})$ be the increasing sequence of all the prime numbers. For any UHF algebra A there exist one and only one sequence $(i_k \mid k \in \mathbf{N})$ of nonnegative integers and ∞ such that $A \cong \otimes_{k=1}^{\infty} M_{p_k^{i_k}}(\mathbf{C})$, where $M_{p_k^\infty}(\mathbf{C})$ is understood as $\otimes_{n=1}^{\infty} M_{p_k}(\mathbf{C})$ ([10]). Then it follows:*

(1) *If* $\sharp\{ k \in \mathbf{N} \mid 1 \le i_k < \infty \} = \infty$ *then there are infinitely many outer conjugacy classes of product type actions of* $\mathbf{Z}^2$ *on* A *with the Rohlin property.*
(2) *If* $\sharp\{ k \in \mathbf{N} \mid 1 \le i_k < \infty \} < \infty$ *and* A *is infinite-dimensional then there is one and only one outer conjugacy class of product type actions of* $\mathbf{Z}^2$ *on* A *with the Rohlin property.*
(3) *If* A *is finite-dimensional then there is no action of* $\mathbf{Z}^2$ *on* A *with the Rohlin property.*

References

[1] O. Bratteli, D. E. Evans and A. Kishimoto, The Rohlin property for quasi-free automorphisms of the Fermion algebra, *Proc. London. Math. Soc.* (3)71(1995), 675–694.

[2] A. Connes, Periodic automorphisms of hyperfinite factors of type II_1, *Acta Sci. Math.* 39(1977), 39–66.

[3] A. Connes, Outer conjugacy class of automorphisms of factors, *Ann. Scient. Ec. Norm. Sup.* 4^e *serie* 8(1975), 383-420.

[4] E. Christensen, Near inclusion of C^*-algebras, *Acta Math.* 144(1980), 249–265.

[5] G. A. Elliott, D. E. Evans and A. Kishimoto, Outer conjugacy classes of trace scaling automorphisms of stable UHF algebras, *preprint*

[6] D. E. Evans, and A. Kishimoto, Trace scaling automorphisms of certain stable AF algebras, *preprint.*

[7] R. Exel and T. A. Loring, Almost commuting unitary matrices, *Proc. Amer. Math. Soc.* 106(1989), 913–915.

[8] R. Exel and T. A. Loring, Invariants of almost commuting unitaries, *J. Func. Anal.* 95(1991), 364–376.

[9] R. Exel, The soft torus and applications to almost commuting matrices, *Pac. J. Math.* 160(1993), 207–217.

[10] J. Glimm, On certain classes of operator algebras, *Trans. Amer. Math. Soc.* 95(1960), 318–340.

[11] R. H. Herman and A. Ocneanu, Stability for integer actions on UHF C^*-algebras, *J. Func. Anal.* 59(1984), 132–144.

[12] R. H. Herman and A. Ocneanu, Spectral analysis for automorphisms of UHF C^*-algebras, *J. Func. Anal.* 66(1986), 1–10.

[13] A. Kishimoto, The Rohlin property for shifts on UHF algebras and automorphisms of Cuntz algebras, *J. Func. Anal.* (to appear).

[14] A. Kishimoto, The Rohlin property for automorphisms of UHF algebras, *J. reine angew. Math.* 465(1995), 183–196.

[15] A. Kishimoto, A Rohlin property for one-parameter automorphism groups, *preprint*

[16] A. Kishimoto and A. Kumjian, Crossed products Cuntz algebras by quasi-free automorphisms, *preprint*

[17] T. A. Loring, K-theory and asymptotically commuting matrices, *Can. J. Math.* 40(1988), 197–216.

[18] H. Nakamura, The Rohlin Property for Actions of $\mathbf{Z}^2$ on UHF Algebras, *preprint*

[19] A. Ocneanu, A Rohlin type theorem for groups acting on von Neumann algebras, Topics in Modern Operator Theory, Birkhäuser Verlag, (1981), 247–258.

[20] A. Ocneanu, Actions of Discrete Amenable Groups on von Neumann Algebras, Lec. Note in Math. 1138, Springer Verlag, (1985).

[21] D. Voiculescu, Almost inductive limit automorphisms and embedding into AF-algebras, *Ergod. Th. & Dynam. Sys.* 6(1986), 475–484.

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