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NONEXISTENCE OF STABLE INTEGRAL CURRENTS

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1. INTRODUCTION

In this paper, we discuss the nonexistence of stable integral p-currents in compact submanifolds of m-dimensional submanifolds in Euclidean spaces $\mathbf{R}^{m+1}$ and $\mathbf{R}^{m+2}$. The existence theorems due to Federer and Fleming in [3] state that for any compact Riemannian manifold M, any non-trivial integral homology class in $\mathcal{H}_p(M,Z)$ corresponds to a stable integral current, where $\mathcal{H}_p(M,Z)$ is the pth singular homology group with integer coefficients. The Federer-Fleming theorem and the calculus of variations were employed by Lawson and Simons [4] in the study of the topology and geometry of submanifolds of standard spheres. They proved the following:

Thorem LS1. *There are no stable integral p-currents in spheres of dimension m, where $0 < p < m$.*

In [4], Lawson and Simons conjectured the follwing:

Conjecture of Lawson and Simons. *There are no stable integral p-currents in any compact, simply-connected Riemannian manifold of dimension m which is 1/4-pinched, where $0 < p < m$.*

This conjecture is also open now. It is very well-known that any compact, simply-connected Riemannian manifold of dimension m which is 1/4-pinched is homeomorphic to sphere. On the other hand, any compact connected convex hypersurface in $\mathbf{R}^{m+1}$ is diffeomorphic to sphere. So we can consider the following problem:

Problem. *Do there exist stable integral p-currents in any compact connected convex hypersurface in $\mathbf{R}^{m+1}$.*

For this problem, the first author proved the following in [1]:

Theorem C. *Let M be an m-dimensional compact hypersurface with positive Ricci curvature in $\mathbf{R}^{m+1}$. Assume that M does not admit any point $x \varepsilon M$ at which M has only two distinct principal curvatures λ_1 and λ_0 such that*

$$max\{\lambda_1,\lambda_0\} \geq (3+2\sqrt{2})min\{\lambda_1,\lambda_0\}.$$

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Then for any $p\varepsilon(0,m)$, *there are no stable integral* p-*currents in* M *and*

$$\mathcal{H}_p(M,Z)=0.$$

Corollary 1. *Let* M *be an* m-*dimensional compact hypersurface in* $\mathbf{R}^{m+1}$ *with sectional curvature* $K(M)>A\lambda^2(A\geq\frac{1}{3+2\sqrt{2}}$ *is constant). Then for any* $p\varepsilon(0,m)$, *there are no stable integral* p-*currents in* M *and*

$$\mathcal{H}_p(M,Z)=0.$$

Lawson and Simons [4] also studied the nonexistence of stable integral p-currents in compact submanifolds of standard spheres $S^m(c)$. They showed that

Thorem LS2. *Let* N *be an* n-*dimensional compact submanifold of* $S^m(c)$. *Then, for given integer* $p\epsilon(0,n)$ *there is no stable integral* p-*current in* N *and* $\mathcal{H}_p(N,Z)=\mathcal{H}_{n-p}(N,Z)=0$ *if*

$$\sum_{i,\alpha}[2\|h'(e_i,e_\alpha)\|^2-\langle h'(e_i,e_i),h'(e_\alpha,e_\alpha)\rangle]<p(n-p)c$$

is satisfied for every $x\epsilon N$ *and any orthonormal basis* $\{e_i,e_\alpha\}$ *of* T_xN, $i=1,\cdots,p$; $\alpha=p+1,\cdots,n$. *Here* h' *is the second fundamental form of* N *in* $S^m(c)$.

We emphasize that their techniques can be applied in more general settings. This observation makes it possible to obtain further results on the non-existence of stable integral currents.

Throughout this paper let N be an n-dimensional compact Riemannian submanifold in an m-dimensional Riemannian manifold M which is a submanifold of $\mathbf{R}^{m+1}$ or $\mathbf{R}^{m+2}$. Let h' be the second fundamental form of N in M and $\{e_i,e_\alpha\}$ an orthonormal basis of the tangent space T_xN for a point $x\varepsilon N$ where $1\leq i\leq p$ and $p+1\leq\alpha\leq n$. The purpose of this paper is to prove the following statement (A) for various ambient submanifolds of Euclidean spaces:

(A) For given integer $p\epsilon(0,n)$ there is no stable integral p-current in N and $\mathcal{H}_p(N,Z)=\mathcal{H}_{n-p}(N,Z)=0$.

The same notation and formulas as in [1] and [2] will be used throughout this paper. We first prove the generalized version of the Lawson-Simons theorem [4] as follows.

Theorem 1 (Cheng and Shiohama [2]). *Let* N *be an* n-*dimensional compact submanifold of* M^m *which is an* m-*dimensional hypersurface of an* $m+1$- *dimensional Euclidean space* $\mathbf{R}^{m+1}$. *Then the statement (A) is true if*

$$\text{(0)}\qquad \sum_{i,\alpha}[2\|h'(e_i,e_\alpha)\|^2-\langle h'(e_i,e_i),h'(e_\alpha,e_\alpha)\rangle]<2p(p-n)\lambda(x)^2$$

is satisfied for every $x\epsilon N$ *and any orthonormal basis* $\{e_i,e_\alpha\}$ *of* T_xN, $i=1,\cdots,p$; $\alpha=p+1,\cdots,n$. *Here* h' *is the second fundamental form of* N *in* M^m, *and* $\lambda(x)^2$ *is the maximum at* x *of the square of principal curvatures of* M^m.

Remark 1. If M^m is $\mathbf{R}^m$, then $\lambda=0$. The condition (0) then becomes

$$\sum_{i,\alpha}[2\|h'(e_i,e_\alpha)\|^2-\langle h'(e_i,e_i),h'(e_\alpha,e_\alpha)\rangle]<0.$$

Hence Xin's result in [5] is a special case of our Theorem 1.

In addition to the assumptions of Theorem 1, if M has nonnegative Ricci curvature, then a simple computation shows that the sectional curvature of M is non-negative. In this case we prove

Theorem 2 (Cheng and Shiohama [2]). *Let N be an n-dimensional compact submanifold of M^m and M^n an m-dimensional hypersurface with nonnegative Ricci curvature of an $m+1$- dimensional Euclidean space $\mathbf{R}^{m+1}$. Then the statement (A) holds if*

$$\sum_{i,\alpha}[2\|h'(e_i,e_\alpha)\|^2 - \langle h'(e_i,e_i), h'(e_\alpha,e_\alpha)\rangle] < \frac{1}{4}p(p-n)\lambda(x)^2$$

is satisfied for any $x\varepsilon N$ and any orthonormal basis $\{e_i, e_\alpha\}$ of T_xN, $i = 1,\cdots,p$; $\alpha = p+1,\cdots,n$. Here h' is the second fundamental form of N in M^m, and $\lambda(x)^2$ is the maximum at x of the square of principal curvatures of M^m.

As a corollary to Theorem 2 we obtain a slight extension of the Lawson-Simons theorem for product Riemannian manifolds:

Corollary 2, (Cheng and Shiohama [2]). *Let N be an n-dimensional compact submanifold of $\mathbf{R}^k \times S^{m-k}(c)$. Then the statement (A) is true if*

$$\sum_{i,\alpha}[2\|h'(e_i,e_\alpha)\|^2 - \langle h'(e_i,e_i), h'(e_\alpha,e_\alpha)\rangle] < 0$$

is satisfied for every $x\epsilon N$ and any orthonormal basis $\{e_i, e_\alpha\}$ of T_xN, $i = 1,\cdots,p$; $\alpha = p+1,\cdots,n$. Here h' is the second fundamental form of N in $\mathbf{R}^k \times S^{m-k}(c)$.

Example (cf. Zhang [7]). We consider the ellipsoid

$$M = \{(x_1,\cdots,x_{m+1})\epsilon\mathbf{R}^{m+1}; \sum_{i=1}^{m} x_i^2 + \frac{x_{m+1}^2}{c^2} = 1\}.$$

Let N be a compact submanifold of M. By direct computation, we can prove that if for every $x\epsilon N$ and any orthonormal basis $\{e_i, e_\alpha\}$ of T_xN, $i = 1,\cdots,p$; $\alpha = p+1,\cdots,n$,

$$\sum_{i,\alpha}[2\|h'(e_i,e_\alpha)\|^2 - \langle h'(e_i,e_i), h'(e_\alpha,e_\alpha)\rangle] < n(n-p)c$$

then the statement (A) holds. In this case, h' is the second fundamental form of N in M.

We next discuss the case where M is a Riemannian product manifold: $M = M_1 \times M_2$, and each M_i $(i = 1,2)$ is an m_i-dimensional hypersurface in $\mathbf{R}^{m_i+1}$ of positive Ricci curvature.

Theorem 3 (Cheng and Shiohama [2]). *Let N be an n-dimensional compact submanifold of $M = M_1^{m_1} \times M_2^{m_2}$, where $M_i^{m_i}(i = 1,2)$ is an m_i-dimensional hypersurface with positive Ricci curvature of an $m_i + 1$-dimensional Euclidean space $\mathbf{R}^{m_i+1}$. Then statement (A) is true if*

$$\sum_{i,\alpha}[2\|h'(e_i,e_\alpha)\|^2 - \langle h'(e_i,e_i), h'(e_\alpha,e_\alpha)\rangle] < \frac{1}{4}p(p-n)(\lambda(x)^2 + \mu(x)^2)$$

is satisfied for every $x \epsilon N$ and any orthonormal basis $\{e_i, e_\alpha\}$ of T_xN, $i = 1,\cdots,p$; $\alpha = p+1,\cdots,n$. Here h' is the second fundamental form of N in M and $\lambda(x)^2$ and $\mu(x)^2$ are the maximums at x of the square of principal curvatures of $M_1^{m_1}$ and $M_2^{m_2}$ respectively.

With the same notation as in Theorem 3, we prove the following:

Theorem 4 (Cheng and Shiohama [2]). *Let $M_i^{m_i}$ $(i = 1, 2)$ have distinct principal curvatures other than two. Then the statement (A) holds if*

$$\sum_{i,\alpha}[2\|h'(e_i,e_\alpha)\|^2 - \langle h'(e_i,e_i), h'(e_\alpha,e_\alpha)\rangle] < 0$$

is satisfied for every $x \epsilon N$ and any orthonormal basis $\{e_i, e_\alpha\}$ of T_xN, $i = 1,\cdots,p$; $\alpha = p+1,\cdots,n$. Here h' is the second fundamental form of N in M.

Corollary 3 (Cheng and Shiohama [2]). *Let N be an n-dimensional compact submanifold of $M = S^{m_1}(c_1) \times S^{m_2}(c_2)$, where $S^{m_i}(c_i)(i = 1,2)$ is an m_i-dimensional sphere of constant curvature c_i. Then the statement (A) holds if*

$$\sum_{i,\alpha}[2\|h'(e_i,e_\alpha)\|^2 - \langle h'(e_i,e_i), h'(e_\alpha,e_\alpha)\rangle] < 0$$

is satisfied for every $x \epsilon N$ and any orthonormal basis $\{e_i, e_\alpha\}$ of T_xN, $i = 1,\cdots,p$; $\alpha = p+1,\cdots,n$. Where h' is the second fundamental form of N in M.

Remark 2. When $c_1 = c_2 = 1$, Corollary 3 reduces to Zhang's Theorem 2 in [6].

Proof of Theorems

Let D be the Euclidean flat connection of $\mathbf{R}^{m+1}$. Since N is a submanifold of M^m, N is also a submanifold in $\mathbf{R}^{m+1}$. Let ∇' and ∇ denote the Levi-Civita connections of N with respect to M^m and $\mathbf{R}^{m+1}$, respectively. Also let $\chi(T^\perp(\mathbf{R}^{m+1}, N))$, $\chi(T^\perp(\mathbf{R}^{m+1}, M^m))$ and $\chi(T^\perp(M^m, N))$ be the respective spaces of normal vector fields. For any simple p-vector $\xi \epsilon \wedge^p T_xN$ and for any vector field V tangent to N, let Ψ be the flow generated by V. Define

$$Q_\xi(V) = \frac{d^2\|\Psi_{t*}\xi\|}{dt^2}|_{t=0}.$$

Proof of Theorem 1. Since N is a submanifold of M^m, it is also a submanifold in $\mathbf{R}^{m+1}$. Let ∇' and ∇ denote the Levi-Civita connections of N with respect to M^m

and $\mathbf{R}^{m+1}$ respectively. The shape operator A_η determined by $\eta\epsilon\chi(T^\perp(\mathbf{R}^{m+1}, N))$ is given by

$$(1) \qquad -A_\eta Y = (D_Y \eta)^T,$$

where $Y\epsilon\chi(TN)$. If $\eta\epsilon\chi(T^\perp(M^m, N))$, then

$$(2) \qquad \begin{aligned} A_\eta Y &= -(D_Y\eta)^T = -(\bar{\nabla}_Y \eta + \bar{h}(\eta, Y))^T \\ &= -(\bar{\nabla}_Y \eta)^T = A'_\eta Y, \end{aligned}$$

where $Y\epsilon\chi(TN)$, $\bar{\nabla}$ and $\bar{h}$ are the Levi-Civita connection and the second fundamental form on M^m with respect to $\mathbf{R}^{m+1}$ and A'_η is the shape operator determined by $\eta\epsilon\chi(T^\perp(M^m, N))$. If η is the normal to M^m, then

$$(3) \qquad A_\eta Y = (\bar{A}_\eta Y)^T,$$

where $\bar{A}_\eta$ is the so-called shape operator determined by $\eta\epsilon\chi(T^\perp(\mathbf{R}^{m+1}, M^m))$.

Let $(\mathcal{S}, \xi)$ be an oriented p-rectifiable set. For $x\epsilon\mathcal{S}$, we have a tangent p-space $T_x\mathcal{S} \subset T_xN$. Choose an orthonormal basis $\{e_i, e_\alpha\}$ of T_xN such that $\{e_i\}$ is an orthonormal basis of $T_x\mathcal{S}$ and $\xi = e_1 \wedge \cdots \wedge e_p$. Suppose that $\{\eta_u\}$ is an orthonormal basis of $T_x^\perp(\mathbf{R}^{m+1}, N)$. Let $A_u = A_{\eta_u}$. Then $\{e_i, e_\alpha, \eta_u\}$ is an orthonormal basis of $\mathbf{R}^{m+1}$. Hence

$$(4) \qquad \mathrm{tr}Q_\xi = \sum Q_\xi(e_i) + \sum Q_\xi(e_\alpha) + \sum Q_\xi(\eta_u).$$

Making use of the proof given for the Theorem appearing in [1], we have

$$(5) \qquad \mathrm{tr}Q_\xi = \sum_{j,\alpha,u} [2\langle A_u(e_j), e_\alpha\rangle^2 - \langle A_u(e_\alpha), e_\alpha\rangle\langle A_u(e_j), e_j\rangle].$$

At a point $x\epsilon N$, we take an orthonormal basis $\{\eta_1, \cdots, \eta_{m-n}, \eta\}$ of $T_x^\perp(\mathbf{R}^{m+1}, N)$ so that $\{\eta_v\}$ and η are the orthonormal bases of $T_x^\perp(M^m, N)$ and $T_x^\perp(\mathbf{R}^{m+1}, M^m)$, respectively, and

$$(6) \qquad \bar{A}_\eta(\tilde{e}_a) = -\lambda_a \tilde{e}_a \quad \text{for} \quad a = 1, \cdots, m,$$

where $\{\tilde{e}_a\}$ is an orthonormal basis of T_xM^m and $\bar{A}_\eta$ is the so-called shape operator determined by $\eta\epsilon\chi(T^{\perp}(\mathbf{R}^{m+1}, M^m))$. From (2), (3) and (5), we obtain

(7)
$$\begin{aligned}
&\mathrm{tr}Q_\xi \\
&= \sum_{v=1}^{m-n}\sum_{j,\alpha}[2\langle A_v(e_j), e_\alpha\rangle^2 - \langle A_v(e_\alpha), e_\alpha\rangle\langle A_v(e_j), e_j\rangle] \\
&+ \sum_{j,\alpha}[2\langle A_\eta(e_j), e_\alpha\rangle^2 - \langle A_\eta(e_\alpha), e_\alpha\rangle\langle A_\eta(e_j), e_j\rangle] \\
&= \sum_{v=1}^{m-n}\sum_{j,\alpha}[2\langle A'_v(e_j), e_\alpha\rangle^2 - \langle A'_v(e_\alpha), e_\alpha\rangle\langle A'_v(e_j), e_j\rangle] \\
&+ \sum_{j,\alpha}[2\langle \bar{A}_\eta(e_j), e_\alpha\rangle^2 - \langle \bar{A}_\eta(e_\alpha), e_\alpha\rangle\langle \bar{A}_\eta(e_j), e_j\rangle] \\
&= \sum_{j,\alpha}[2\|h'(e_j, e_\alpha)\|^2 - \langle h'(e_\alpha, e_\alpha), h'(e_j, e_j)\rangle] \\
&+ \sum_{j,\alpha}[2\langle \bar{A}_\eta(e_j), e_\alpha\rangle^2 - \langle \bar{A}_\eta(e_\alpha), e_\alpha\rangle\langle \bar{A}_\eta(e_j), e_j\rangle],
\end{aligned}$$

(8)
$$\begin{aligned}
&\sum_{j,\alpha}[2\langle \bar{A}_\eta(e_j), e_\alpha\rangle^2 - \langle \bar{A}_\eta(e_\alpha), e_\alpha\rangle\langle \bar{A}_\eta e_j, e_j\rangle] \\
&= \sum_{j,\alpha}\{2[\sum_a e_j^a e_\alpha^a \lambda_a]^2 - \sum_a (e_j^a)^2\lambda_a \sum_a (e_\alpha^a)^2\lambda_a\},
\end{aligned}$$

where $e_j = \sum_a e_j^a \tilde{e}_a$, $e_\alpha = \sum_a e_\alpha^a \tilde{e}_a$. Since $\|e_j\| = \|e_\alpha\| = 1$ and $\langle e_j, e_\alpha\rangle = 0$, we have

$$\sum_a (e_j^a)^2 = 1, \quad \sum_a (e_\alpha^a)^2 = 1, \quad \sum_a e_j^a e_\alpha^a = 0.$$

Making use of assertion (1) in the following Lemma 1, we get

(9)
$$\sum_{j,\alpha}[2\langle \bar{A}_\eta(e_j), e_\alpha\rangle^2 - \langle \bar{A}_\eta(e_\alpha), e_\alpha\rangle\langle \bar{A}_\eta(e_j), e_j\rangle] \le 2p(n-p)\lambda^2,$$

where λ^2 is the maximum of the square of principal curvatures of M^m at the corresponding point. According to (7), (9) and the assumption in Theorem 1, we conclude

$$\begin{aligned}
&\mathrm{tr}Q_\xi \\
&= \sum_{j,\alpha}[2\|h'(e_j, e_\alpha)\|^2 - \langle h'(e_\alpha, e_\alpha), h'(e_j, e_j)\rangle] \\
&+ \sum_{j,\alpha}[2\langle \bar{A}_\eta(e_j), e_\alpha\rangle^2 - \langle \bar{A}_\eta(e_\alpha), e_\alpha\rangle\langle \bar{A}_\eta(e_j), e_j\rangle] \\
&= \sum_{j,\alpha}[2\|h'(e_j, e_\alpha)\|^2 - \langle h'(e_\alpha, e_\alpha), h'(e_j, e_j)\rangle] \\
&+ 2p(n-p)\lambda^2 < 0.
\end{aligned}$$

Hence

$$\mathrm{tr}Q_S = \sum_{n=1}^{\infty} \int_{S_n} n\mathrm{tr}Q_{\xi_n} d\mathcal{H}^p(x) < 0.$$

This implies that there are no stable integral p-currents in N. By Theorem FF in [1], we have

$$\mathcal{H}_p(N, Z) = \mathcal{H}_{n-p}(N, Z) = 0.$$

Lemma 1. *Let $a_1, \cdots, a_m, b_1, \cdots, b_m$ be real numbers satisfying $\sum_j a_j^2 = \sum_j b_j^2 = 1$ and $\sum_j a_j b_j = 0$. Then we have the following:*

(1) *For given real numbers $\lambda_1, \lambda_2, \cdots, \lambda_m$,*

$$2(\sum_j \lambda_j a_j b_j)^2 - \sum_j \lambda_j a_j^2 \sum_j \lambda_j b_j^2 \leq 2\lambda^2.$$

(2) *For given nonnegative real numbers $\lambda_1, \lambda_2, \cdots, \lambda_m$,*

$$2(\sum_j \lambda_j a_j b_j)^2 - \sum_j \lambda_j a_j^2 \sum_j \lambda_j b_j^2 \leq \frac{1}{4}\lambda^2.$$

Here λ^2 is the maximum of λ_j^2.

Lemma 1 can be proved by the same method as the lemma in [1].

Proof of Theorem 2. Since M^m is a hypersurface in $\mathbf{R}^{m+1}$ with nonnegative Ricci curvature, we know that M^m is of nonnegative sectional curvature (see [1]). Hence assertion (2) of Lemma 1 and the same arguments appearing in the proof of Theorem 1 imply that Theorem 2 holds.

Proof of Corollary 2. Since $\mathbf{R}^k \times S^{m-k}(c)$ is a hypersurface in $\mathbf{R}^{m+1}$ with nonnegative Ricci curvature and there are only two distinct constant principal curvatures 0 and $\sqrt{c}$, we have

$$\sum_{j,\alpha}[2\langle \bar{A}_\eta(e_j), e_\alpha\rangle^2 - \langle \bar{A}_\eta(e_\alpha), e_\alpha\rangle\langle \bar{A}_\eta(e_j), e_j\rangle] \leq 0.$$

Hence

$$\begin{aligned}
&\mathrm{tr}Q_\xi \\
&= \sum_{j,\alpha}[2\|h'(e_j, e_\alpha)\|^2 - \langle h'(e_\alpha, e_\alpha), h'(e_j, e_j)\rangle] \\
&+ \sum_{j,\alpha}[2\langle \bar{A}_\eta(e_j), e_\alpha\rangle^2 - \langle \bar{A}_\eta(e_\alpha), e_\alpha\rangle\langle \bar{A}_\eta(e_j), e_j\rangle] \\
&= \sum_{j,\alpha}[2\|h'(e_j, e_\alpha)\|^2 - \langle h'(e_\alpha, e_\alpha), h'(e_j, e_j)\rangle] \\
&< 0.
\end{aligned}$$

Thus

$$\mathrm{tr}Q_S = \sum_{n=1}^{\infty} \int_{S_n} n\mathrm{tr}Q_{\xi_n} d\mathcal{H}^p(x) < 0.$$

We obtain the result that there are no stable integral p-currents in N and

$$\mathcal{H}_p(N, Z) = \mathcal{H}_{n-p}(N, Z) = 0.$$

This proves Corollary 2.

Using proofs similar to that given for Theorem 1, the following Lemma 2 yields Theorems 3 and 4.

Lemma 2 (Cheng and Shiohama [2]). *Let a_i, b_i, d_j and e_j be real numbers $(i = 1, \cdots, m_1; j = 1, \cdots, m_2)$ satisfying $\sum_i a_i^2 + \sum_j e_j^2 = \sum_i b_i^2 + \sum_j d_j^2 = 1$ and $\sum_i a_i b_i + \sum_j d_j e_j = 0$. Then the following holds:*

(1) *For given positive real numbers λ_i and μ_j,*

$$2[(\sum_i \lambda_i a_i b_i)^2 + (\sum_j \mu_j d_j e_j)^2] - \sum_i \lambda_i a_i^2 \sum_i \lambda_i b_i^2 - \sum_j \mu_j d_j^2 \sum_j \mu_j e_j^2 \le \frac{1}{4}(\lambda^2 + \mu^2). \tag{11}$$

(2) *For given positive real numbers λ_i and μ_j, where neither the number of distinct λ_i nor the number of distinct μ_j is not two,*

$$2[(\sum_i \lambda_i a_i b_i)^2 + (\sum_j \mu_j d_j e_j)^2] - \sum_i \lambda_i a_i^2 \sum_i \lambda_i b_i^2 - \sum_j \mu_j d_j^2 \sum_j \mu_j e_j^2 \le 0, \tag{12}$$

where λ^2 and μ^2 are the maximum of λ_i^2 and μ_j^2, respectively.

This Lemma 2 can be proved by making use of the method of Lagrange multipliers (see [2] for in details).

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