数理解析研究所講究録
998巻 1997年 141-152

ON CRYSTALLINE FUNDAMENTAL GROUPS

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1. Introduction

In arithmetic geometry, many cohomology theories, such as Betti, etale, de Rham, and crystalline ones, have been studied and various comparison theorems have been proved. In view of these, A. Grothendieck proposed the philosophy that cohomology theory is motivic and most mathematicians believe his philosophy now.

His philosophy was extended by P. Deligne to the one that the theory of pro-unipotent quotient of rational fundamental groups are also motivic. Historically, Quillen studied on usual rational fundamental groups (and homotopy groups). It was Grothendieck who defined etale fundamental groups. De Rham fundamental groups were defined by Sullivan and Chen independently. The mixed Hodge structure were constructed by Morgan and Hain independently. Deligne constructed these rational fundamental groups by using the theory of Tannakian categories and fiber functors.

If we follow the philosophy of Deligne, there should be a crystalline fundamental group for a smooth variety X over a field k of characteristic $p>0$ and there should be the comparison theorem with de Rham fundamental groups if X is liftable to characteristic zero. But Deligne defined crystalline realization of fundamental groups only for a variety which can be liftable to characteristic zero by defining crystalline Frobenius on de Rham fundamental groups. Since crystalline fundamental groups should be defined for varieties which are not necessarily liftable to characteristic zero and they should be independent of the choice of a lifting, this is not the best possible way.

In this report, we will define crystalline fundamental groups by using Tannakian categories and fiber functors, and state some property of them.

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Finally, the author would like to express his gratitude to Professor Masanobu Kaneko for giving me an opportunity to write this report, and to Professor Takeshi Saito for many advices and encouragements. The author is suppoted by JSPS research fellowship for young scientists.

2. Deligne's Definition of Fundamental Groups

In this Section, we will review on Tannakian categories and Deligne's definition of rational fundamental groups.

First we will review on usual (topological) fundamental groups. For a topological space X (with some conditions) and a point x in X, the fundamental group $\pi_1(X, x)$ of X with a base point x is defined as follows:

$$\pi_1(X, x) := (\text{loops with base point } x)/(\text{homotopies}).$$

It is not so easy to algebrize this definition. But we have the following well-known proposition:

Proposition 2.1. Let $LS(X)$ be the category of local systems of sets on X and ω_x be the functor of taking the fiber at x. Then,

1. There is an isomorphism
$$\pi_1(X, x) \cong Aut(LS(X) \xrightarrow{\omega_x} (\text{Sets})).$$
2. Via ω_x, we have an equivalence of categories
$$LS(X) \xrightarrow{\sim} (\pi_1(X, x)\text{-sets}),$$
where the left hand side is the category of sets with an action of $\pi_1(X, x)$.

We can regard the theory of Tannakian categories and fiber functors as an abstract (and rational) version of this formalism.

Now we will review on the definition of Tannakian category. In this report, we will follow the terminology of [De-Mi]. First recall the definition of tensor category briefly.

Definition 2.2. Let $\mathcal{C}$ be a category and $\otimes : \mathcal{C} \times \mathcal{C} \longrightarrow \mathcal{C}$ be a functor. Then a pair $(\mathcal{C}, \otimes)$ is called a tensor category if there exist 'associativity constraint', 'commutativity constraint', and the 'unit object' $\underline{1}$ which are compatible by 'hexagon axiom', and so on.

We will not review on the quoted words. For details, see [De-Mi, (1.1)]. As a feeling, a tensor category is a category with a structure of tensor product. If we are given a tensor category $(\mathcal{C}, \otimes)$, we can define the functor

$$\otimes_{i \in I} : \mathcal{C}^I \longrightarrow \mathcal{C}$$

naturally for a finite index set I. (See [De-Mi, (1.5)].)

Next we will review on the definition of rigidity of tensor categories. To do this, we will recall the definition of internal hom objects.

Definition 2.3. Let $(\mathcal{C}, \otimes)$ be a tensor category and X, Y be objects in $\mathcal{C}$. If the functor

$$T \mapsto Hom(T \otimes X, Y); \ \mathcal{C}^o \longrightarrow \text{(Sets)}$$

is representable, we denote the representing object by $\mathcal{H}om(X, Y)$ and call it an internal hom object of X and Y. We will write the object $\mathcal{H}om(X, \underline{1})$ simply by X^*.

Definition 2.4. A tensor category $(\mathcal{C}, \otimes)$ is called rigid when an internal hom object $\mathcal{H}om(X, Y)$ exists for any $X, Y \in \mathcal{C}$, and the canonical morphisms

$$X \longrightarrow X^{**}$$

and

$$\otimes_{i \in I} \mathcal{H}om(X_i, Y_i) \longrightarrow \mathcal{H}om(\otimes_{i \in I} X_i, \otimes_{i \in I} Y_i),$$

which are induced by definition of internal hom objects, are always isomorphic, where I is a finite index set and X, X_i's, and Y_i's are objects in $\mathcal{C}$.

Definition 2.5. A tensor category $(\mathcal{C}, \otimes)$ is called an abelian tensor category if $\mathcal{C}$ is an abelian category and $\otimes$ is a bi-additive functor.

Now we can define a Tannakian category as follows:

Definition 2.6. Let k be a field. A rigid abelian tensor category $(\mathcal{C}, \otimes)$ is called a Tannakian category over k if $End(\underline{1}) = k$ holds and there exists a field K and a fiber functor (:= an exact faithful tensor functor) $\omega : \mathcal{C} \longrightarrow Vec_K$, where Vec_K is the category of finite-dimensional vector spaces over K. If we can take $K = k$, $(\mathcal{C}, \otimes)$ is called a neutral Tannakian category.

Example 2.7. Let k be a field and G be a group scheme over k. Then the category $Rep_k(G)$ of finite-dimensional representations of G over k is a neutral Tannakian category over k.

Conversely, we have the following structure theorem for neutral Tannakian categories.

Theorem 2.8 (Saavedra, Deligne-Milne). Let k be a field, $\mathcal{C}$ be a neutral Tannakian category over k, and $\omega : \mathcal{C} \longrightarrow Vec_k$ be a fiber functor. Then the functor

$$(k\text{-algebras}) \longrightarrow (\text{Groups});\ R \mapsto Aut^{\otimes}(\mathcal{C} \xrightarrow{\omega} Vec_k \longrightarrow Mod_R)$$

is representable by a group scheme over k. Here, $Aut^{\otimes}$ is the group of automorphisms which preserves tensor structures and Mod_R is the category of R-modules. If we denote the representing group scheme by $G(\mathcal{C}, \omega)$, the functor ω induces an equivalence of categories

$$\mathcal{C} \longrightarrow Rep_k(G(\mathcal{C}, \omega)).$$

We can define several fundamental groups by using this theorem. Let us recall the definitions of (the unipotent quotient of) rational fundamental groups by Deligne:

Definition 2.9 (Deligne).

1. Let X be a topological space and $x \in X$. Let $\mathcal{C}$ be the category of nilpotent local systems of $\mathbb{Q}$-vector spaces on X. (Here, an object is called 'nilpotent' if it can be written as a successive extension by trivial local system $\mathbb{Q}$ on X.) And let $\omega : \mathcal{C} \longrightarrow Vec_{\mathbb{Q}}$ be the functor of taking the fiber at x. Then we define (the pro-unipotent completion of) Betti fundamental group $\pi_1^B(X, x)$ of X with base point x by
$$\pi_1^B(X, x) := G(\mathcal{C}, \omega).$$
2. Let X be a variety over an algebraically closed field k and $x \in X(k)$. Let $\mathcal{C}$ be the category of nilpotent smooth $\mathbb{Q}_l$-sheaves on X. And let $\omega : \mathcal{C} \longrightarrow Vec_{\mathbb{Q}_p}$ be the functor of taking the fiber at x. Then we define (the pro-unipotent completion of) l-adic fundamental group $\pi_1^l(X, x)$ of X with base point x by
$$\pi_1^l(X, x) := G(\mathcal{C}, \omega).$$

3. Let X be a smooth variety over a field k of characteristic zero and $x \in X(k)$. Let $\mathcal{C}$ be the category of coherent sheaves with integrable connections which are regular singular along boundaries and nilpotent. And let $\omega : \mathcal{C} \longrightarrow Vec_k$ be the functor of taking the fiber at x. Then we define de Rham fundamental group $\pi_1^{dR}(X, x)$ of X with base point x by
$$\pi_1^{dR}(X, x) := G(\mathcal{C}, \omega).$$

3. Crystalline Fundamental Groups

In this Section, we will define crystalline fundamental groups in several ways and state some properties of them.

Let k be a perfect field of characteristic $p > 0$, U be a smooth variety over k, and let x be a k-valued point of U. As we can see from the previous section, what we should do is to choose a Tannakian category and a fiber functor, but what category should we use? In view of Deligne's definition (Definition 2.9), We should use the category of 'p-adic' objects which correspond to smooth $\mathbb{Q}_l$-sheaves in l-adic theory. There are some candidates for this category, so we will consider some definitions.

First we will consider the definition by means of the category of nilpotent isocrystals on log crystalline site. Suppose we are given a compactification X of U such that $D := X - U$ is a normal crossing divisor in X. We will write the log structure on X defined by D also as D, and so regard a pair (X, D) as a log scheme naturally. In this report, the log structures are always in the sense of Fontaine-Illusie-Kato. For a basic definitions and properties on log structures, see [Ka]. First we will recall the definitions of log crystalline site and a crystal on it.

Definition 3.1. Let W be a Witt ring of k and γ be the canonical PD-structure on W. Denote $W/(p)^n$ by W_n. Then for a pair (X, D) as above, we define the log crystalline site $((X, D)/W)_{crys}$ of (X, D) over W as follows: Objects are 5-tuples (Y, T, L, i, δ), where Y is a scheme etale over X, (T, L) is a fine log scheme over W_n for some n, $i : (Y, D|_Y) \hookrightarrow (T, L)$ is an exact closed immersion over W_n, and δ is a PD-structure on the ideal of definition of Y in $\mathcal{O}_T$ which is compatible with γ. Morphisms are defined as morphisms of log schemes which are compatible with the above structures. And coverings are the ones induced by the etale topology of T. We will frequently denote a 5-tuple (Y, T, L, i, δ) simply by T.

And we will define the structure sheaf $\mathcal{O}_{X/W}$ of the site $((X,D)/W)_{crys}$ by $\mathcal{O}_{X/W}(T) := \Gamma(T, \mathcal{O}_T)$.

Definition 3.2. Let the notations be as above.

1. A sheaf of $\mathcal{O}_{X/W}$-modules $\mathcal{E}$ on $((X,D)/W)_{crys}$ is called a crystal if the morphism
$$f^*\mathcal{E}_T \longrightarrow \mathcal{E}_{T'}$$
induced by a morphism $T' \longrightarrow T$ in $((X,D)/W)_{crys}$ is isomorphic for any T, T'. Here, $\mathcal{E}_T$ is the sheaf on T induced by $\mathcal{E}$.
2. We define the category of isocrystals on $((X,D)/W)_{crys}$ as follows: Objects are the crystals on $((X,D)/W)_{crys}$. We will define morphisms by
$$Hom_{isoc}(\mathcal{E},\mathcal{F}) := K \otimes_W Hom_{crys}(\mathcal{E},\mathcal{F})$$
for crystals $\mathcal{E},\mathcal{F}$. We will denote the category of isocrystals on $((X,D)/W)_{crys}$ by $Kisoc((X,D)/W)$. For a crystal $\mathcal{E}$, we will write it as $K \otimes \mathcal{E}$ when we regard it as an isocrystal.
3. An object of $Kisoc((X,D)/W)$ is called nilpotent if it can be written as a successive extension by $K \otimes \mathcal{O}_{X/W}$. We will denote the full subcategory of $Kisoc((X,D)/W)$ which consists of nilpotent isocrystals by $\mathcal{N}Kisoc((X,D)/W)$.

In the above situation, we can prove the category $\mathcal{N}Kisoc((X,D)/W)$ is Tannakian ([Sh1], [Sh2]). So we can define a crystalline fundamental group as follows:

Definition 3.3 (Definition of π_1^{crys} (I)). Let the notations be as above. Then we define the crystalline fundamental group $\pi_1^{crys}((X,D)/W, x)$ of (X,D) over W with base point x by
$$\pi_1^{crys}((X,D)/W, x) := G(\mathcal{N}Kisoc((X,D)/W), \omega_x),$$
where ω_x is the fiber functor
$$\mathcal{N}Kisoc((X,D)/W) \longrightarrow \mathcal{N}Kisoc(x/W) \simeq Vec_{K_0}.$$
(Here $K_0 := Frac\, W$.)

Next we will give another definition by using the category of nilpotent isocrystals on log convergent site. Let (X, D) be as above. Then we define the log convergent site and isocrystals on it as follows:

Definition 3.4. Let V be a complete discrete valuation ring of mixed characteristic with residue field k and K be the fraction field of V. Then we define the convergent site $((X, D)/V)_{conv}$ of (X, D) over V as follows: Objects are triple (T, L, z), where (T, L) be a p-adic formal V-scheme over $Spf\, V$ and $z : (T_0, L) \longrightarrow (X, D)$ is a morphism over $Spf\, V$, where T_0 is the scheme $(Spec\, \mathcal{O}_T/(p))_{red}$. Morphism are morphism of formal log schemes which preserves the above structures. And coverings are the ones induced by the etale topology of T. We frequently write a triple (T, L, z) simply by T.

And we define the structure sheaf $\mathcal{O}_{X/V}$ by

$$\mathcal{O}_{X/V}(T) := \Gamma(T, \mathcal{O}_T).$$

Definition 3.5.

1. A sheaf $\mathcal{E}$ of $K \otimes_V \mathcal{O}_{X/V}$-modules on $((X, D)/V)_{conv}$ is called an isocrystal if the morphism
$$f^* \mathcal{E}_T \longrightarrow \mathcal{E}_{T'}$$
induced by a morphism $T' \longrightarrow T$ in $((X, D)/W)_{conv}$ is isomorphic for any T, T'. Here, $\mathcal{E}_T$ is the sheaf on T induced by $\mathcal{E}$. We will denote the category of isocrystals on $((X, D)/W)_{conv}$ by $Cisoc((X, D)/W)$.
2. An object of $Cisoc((X, D)/W)$ is called nilpotent if it can be written as a successive extension by $K \otimes_V \mathcal{O}_{X/V}$. We will denote the full subcategory of $Cisoc((X, D)/W)$ which consists of nilpotent isocrystals by $\mathcal{N}Cisoc((X, D)/W)$.

In the above situation, we can prove the category $\mathcal{N}Cisoc((X, D)/W)$ is also Tannakian ([Sh1], [Sh2]). So we can give the second definition of crystalline fundamental groups as follows:

Definition 3.6 (Definition of π_1^{crys} (II))**.** Let the notations be as above. Then we define the crystalline fundamental group $\pi_1^{crys}((X, D)/V, x)$ of (X, D) over V with base point x by

$$\pi_1^{crys}((X, D)/V, x) := G(\mathcal{N}Cisoc((X, D)/V), \omega_x),$$

where ω_x is the fiber functor

$$\mathcal{N}Cisoc((X,D)/V) \longrightarrow \mathcal{N}Cisoc(x/V) \simeq Vec_K.$$

(Here $K := Frac\, V$.)

Moreover, we can give the third definition by using the category of nilpotent overconvergent isocrystals. To explain this, we will review on rigid analytic geometry briefly.

Let k be a perfect field of characteristic $p > 0$, V be a complete discrete valuation ring of mixed characteristic with residue field k, and K be the fraction field of V. Let π be a uniformizer of V. For a p-adic affine formal scheme $P = Spf\ A$, we can introduce a structure of ringed space in the set of maximal ideals of $K \otimes A$ ([Be2], [BGR]). We denote it by $\tilde{P}$.

Let $X \hookrightarrow P$ be a closed immersion of a k-scheme X into P, and let $(\pi, f_1, \cdots, f_n)$ be the ideal of definition of X in P. Then we define the tubular neighborhood $]X[_{P,\lambda}$ of X in P with radius λ $(0 < \lambda \leq 1)$ by

$$]X[_{P,\lambda} := \{x \in \tilde{P} \mid |f_i(x)| < \lambda\, (1 \leq i \leq n)\}.$$

This definition is independent of the choices of a uniformizer π and a set of generators $f_1, \cdots, f_n$ if λ is sufficiently close to 1. We will denote $]X[_{P,1}$ simply by $]X[_P$.

Let U be a smooth variety over k and $X \supset U$ be a compactification of U. Set $Z := X - U$. Then, locally on X, there exists a p-adic affine formal scheme P and a closed immersion $X \hookrightarrow P$ such that P is formally smooth over $Spf\, V$ on a neighborhood of U. Set $U_\lambda :=]X[_P -]Z[_{P,\lambda}$ and let j_λ be an open immersion $U_\lambda \hookrightarrow]X[_P$. For a sheaf E of $\mathcal{O}_{]X[_P}$-modules (here $\mathcal{O}_{]X[_P}$ is the structure sheaf of $]X[_P$), we define $j^\dagger E$ by $j^\dagger E := \varinjlim_{\lambda \to 1} j_{\lambda,*} j_\lambda^* E$. Then for projections

$$p_i :]X[_{P^2} \longrightarrow]X[_P \ (i = 1, 2)$$

and

$$p_{ij} :]X[_{P^3} \longrightarrow]X[_{P^2} \ (1 \leq i < j \leq 3),$$

we can define the functors

$$p_i^* : (j^\dagger \mathcal{O}_{]X[_P}\text{-modules}) \longrightarrow (j^\dagger \mathcal{O}_{]X[_{P^2}}\text{-modules})$$

and

$$p_{ij}^* : (j^\dagger \mathcal{O}_{]X[_{P^2}}\text{-modules}) \longrightarrow (j^\dagger \mathcal{O}_{]X[_{P^3}}\text{-modules})$$

naturally.

Definition 3.7. Let the notations be as above. Then an overconvergent isocrystal on (U, X) over V with respect to P is a pair (E, ϵ), where E is a locally free $j^\dagger \mathcal{O}_{]X[_P}$-module and ϵ is an isomorphism $p_2^* E \xrightarrow{\sim} p_1^* E$ which satisfies the cocycle condition $p_{13}^*(\epsilon) = p_{12}^*(\epsilon) \circ p_{23}^*(\epsilon)$.

In particular, $j^\dagger \mathcal{O}_{]X[_P}$ is an overconvergent isocrystal on (U, X) with respect to P.

As for the choice of P as above, we have the following proposition, which is due to Berthelot:

Proposition 3.8 ([Be1], [Be2]). Let the notations be as above. Then the category of overconvergent isocrystals on (U, X) over V with respect to P is independent of the choice of P as above up to canonical equivalence.

In general, we do not have an embedding $X \hookrightarrow P$ globally, but we can define the notion of an overconvergent isocrystal on (U, X) over V by the above proposition. Moreover, we have the following proposition, which is also due to Berthelot:

Proposition 3.9 ([Be1], [Be2]). The category of overconvergent isocrystals on (U, X) over V is, up to canonical equivalence, independent of the choice of a compactification X of U.

So the notion of the category of overconvergent isocrystals on U over V is well-defined. We will denote this category by $Oisoc(U/V)$. And we will denote the object in $Oisoc(U/V)$ defined by recollecting $j^\dagger \mathcal{O}_{]X[_P}$'s by $\mathcal{O}_{U/V}$.

Definition 3.10. An overconvergent isocrystal on U over V is called nilpotent if is can be written as a successive extension by $\mathcal{O}_{U/V}$. We will denote the full subcategory of $Oisoc(U/V)$ which consists of nilpotent isocrystals by $\mathcal{N}Oisoc(U/V)$.

It is known that the category $\mathcal{N}Oisoc(U/V)$ is Tannakian (This is also due to Berthelot). So we can give the third definition of crystalline fundamental groups as follows:

Definition 3.11 (Definition of π_1^{crys} (III)). Let the notations be as above and let x be a k-valued point of U. Then we define the crystalline fundamental group $\pi_1^{crys}(U/V, x)$ of U over V with base point x by

$$\pi_1^{crys}(U/V,x) := G(\mathcal{N}Oisoc(U/V), \omega_x),$$

where ω_x is the fiber functor

$$\mathcal{N}Oisoc(U/V) \longrightarrow \mathcal{N}Oisoc(x/V) \simeq Vec_K.$$

We have defined the notion of crystalline fundamental groups in three ways. We can show that these three definitions are compatible in the following sense:

Theorem 3.12. 1. ([Sh1], [Sh2]) When $V = W$ holds, the first definition of $\pi_1^{crys}((X,D)/W,x)$ coincides with the second one.

2. ([Sh2]) Let U be a smooth variety over k and X be a compactification of U such that $D := X - U$ is a normal crossing divisor on X. Then the second definition of $\pi_1^{crys}((X,D)/V,x)$ is canonically isomorphic to the third definition of $\pi_1^{crys}(U/V,x)$.

Corollary 3.13. The first and second definitions of the crystalline fundamental group of (X,D) with base point x is independent of the choice of a compactification of $U := X - D$ as above.

To prove the above theorem , first we construct a functor between the categories considered above. Then we are reduced to the statement concerning cohomologies. We omit the details.

Now we state some properties of crystalline fundamental groups. In the statement of the following theorem, we will use the second definition of crystalline fundamental groups.

Let k be a perfect field, V be a complete discrete valuation ring of mixed characteristic with residue field k, and K be the fraction field of V. And let $W = W(k)$ be the Witt ring of k.

Theorem 3.14 ([Sh1], [Sh2]). Let U be a smooth variety over k and X be a smooth compactification of U such that $D := X - U$ is a normal crossing divisor. And let x be a k-valued point of U. Then:

1. $\pi_1^{crys}((X,D)/V,x)$ is a pro-unipotent algebraic group over K.
2. On $\pi_1^{crys}((X,D)/W,x)$, there is an action of crystalline Frobenius operator which is Frobenius-linear, and it induces an automorphism.

3. (Hurewicz isomorphism) There exists the following canonical isomorphism:

$$\pi_1^{crys}((X,D)/W,x)^{ab} \stackrel{\simeq}{=} (\mathbb{Q}\otimes_{\mathbb{Z}} H^1_{\text{log-crys}}((X,D)/W))^*,$$

where $H^1_{\text{log-crys}}$ on right hand side is the log crystalline cohomology.

4. (Base change) Let V' be a complete discrete valuation ring with residue field k' which is finite over V and K' be the fraction of V'. Then there exists an isomorphism

$$\pi_1^{crys}((X,D)/V,x)\times_K K' \stackrel{\simeq}{=} \pi_1^{crys}((X\times_k k', D\times_k k')/V', x\times_k k').$$

5. (Comparison with de Rham fundamental groups) Assume we are given the following diagram:

$$\begin{tikzcd}
x_0 \arrow[r] \arrow[d] & \tilde{x} \arrow[d] & x \arrow[l] \arrow[d] \\
(X_0,D_0) \arrow[r] \arrow[d] & (\tilde{X},\tilde{D}) \arrow[d] & (X,D) \arrow[l] \arrow[d] \\
Spec\, k \arrow[r] & Spec\, V & Spec\, K. \arrow[l]
\end{tikzcd}$$

Here $\tilde{X}$ is a proper, smooth scheme over V, $\tilde{D}\subset\tilde{X}$ is a relative normal crossing divisor, $\tilde{x}\in(\tilde{X}-\tilde{D})(V)$, and all the rectangles in the above diagram are Cartesian. Then there exists a canonical isomorphism

$$\pi_1^{crys}((X_0,D_0),x_0) \stackrel{\simeq}{=} \pi_1^{dR}(X-D,x).$$

We will comment on the proofs of the above theorem briefly. 1. is immediate from the definition. 2. and 3. are deduced from the first definition. 4. is proved by using the second definition and the base change of Tannakian categories ([Del]). 5. is shown by proving the equivalence of categories between $\mathcal{N}Cisoc((X_0,D_0)/V)$ and the category of coherent sheaves with integrable connections on $X-D$ which are regular singular along boundaries and nilpotent. We will omit the proofs.

Remark 3.15. 1. We have the theory of tangential base points and tangential maps for crystalline fundemental groups.

2. We can define crystalline fundamental groups for certain log schemes by using the category of nilpotent isocrystals on log crystalline site or log convergent site, and we can show the similar theorems to the above ones.

REFERENCES

[Be1] P. Berthelot, *Géométrie Rigide et Cohomologie des Variétés Algébriques de Caractéristique p*, Bull. Soc. Math. de France, Mémoire **23**(1986), 7–32.

[Be2] P. Berthelot, *Cohomologie rigide et cohomologie rigide à support propre*, preprint.

[BGR] S. Bosch, U. Güntzer, and R. Remmert, *Non-Archimedean Analysis*, Springer, 1984.

[De1] P. Deligne, *Le Groupe Fondamental de la Droite Projective moins Trois Points*, in Galois Groups over **Q**, Springer Verlag, New York, 1989.

[De2] P. Deligne, *Catégories Tannakiannes*, in Grothendieck Festschrift, Progress in Mathematics, Birkhäuser.

[De-Mi] P. Deligne and J. S. Milne, *Tannakian Categories*, in Hodge Cycles, Motives, and Shimura Varieties, Lecture Note in Math. 900, Springer Verlag, 1982, pp. 101–228.

[Ka] K. Kato, *Logarithmic Structures of Fontaine-Illusie*, in Algebraic Analysis, Geometry, and Number Theory, J-I.Igusa ed., 1988, Johns Hopkins University, pp. 191–224.

[Og1] A. Ogus, *F-isocrystals and de Rham Cohomology II — Convergent Isocrystals*, Duke Math., **51**(1984), 765–850.

[Og2] A. Ogus, *The Convergent Topos in Characteristic p*, in Grothendieck Festschrift, Progress in Math., Birkhäuser.

[Sh1] A. Shiho, *Crystalline Fundamental Groups*, preprint.

[Sh2] A. Shiho, *Theory of Crystalline Fundamental Groups*, preprint.

[Ta] J. Tate, *Rigid Analytic Spaces*, Invent. Math., **12**(1971), 257–289.