

An application of the Kripke sheaf semantics in intermediate predicate logics

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Abstract

An application of the Kripke sheaf semantics in intermediate predicate logics is presented. We deal with the pseudo-relevance property and the Halldén-completeness — and their weak versions — together with the disjunction property in intermediate predicate logics. We will determine the relationships between every combinations of the above properties by making use of the Kripke sheaf semantics.

Introduction

The Kripke sheaf semantics is an extended Kripke-type semantics, and was introduced by Shehtman and Skvortsov in [5]. We present here an application of this semantics in intermediate predicate logics.

Considering the results obtained here, this article is a sequel to Suzuki [7]. In [7], the author studied some syntactical properties — *pseudo-relevance property* (PRP), *Halldén-completeness* (H-completeness) and their weak versions (PRP* and H*-completeness) — in intermediate predicate logics, and determined the relationships between them. An intermediate predicate logic L is said to have PRP (PRP*, respectively), if for all formulas A and B which contain no predicate variables in common, $A \supset B \in L$ implies either $\neg A \in L$ or $B \in L$ (either $\neg A \in L$ or $\neg\neg B \in L$, respectively). An L is said to be H-complete (H*-complete, respectively), if for all formulas A and B which contain no predicate variables in common, $A \vee B \in L$ implies either $A \in L$ or $B \in L$ (either $\neg\neg A \in L$ or $\neg\neg B \in L$, respectively). We determined whether one property implies another or not.

In the present article, we will study the relationships between combinations of these properties together with the disjunction property by making use of the Kripke sheaf semantics.

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In section 1, we recall some definitions and results in Suzuki [7]. The main aim of this paper is proposed in a concrete fashion here; that is, to determine whether a combination of some properties implies another or not (see Figure 1). We prepare some tools in section 2. We repeat some basic definitions of Kripke sheaf semantics. To separate combinations, we need an easy sufficient condition for logics to have the disjunction property. We use the delta operation Δ on the set of all super-intuitionistic predicate logics. In [9], the author discussed some properties and proved that every fixed point of Δ has the disjunction property. In section 3 we achieve our aim by applying the Kripke sheaf semantics and the result on Δ .

1 Preliminaries

We fix a pure first-order language \mathcal{L} , which consists of logical connectives \vee (disjunction), \wedge (conjunction), \supset (implication), \neg (negation), and quantifiers \exists (existential quantifier) and \forall (universal quantifier), a denumerable list of individual variables and a denumerable list of m -ary predicate variables for each $m < \omega$. As usual, 0-ary predicate variables are identified with propositional variables. Note that \mathcal{L} contains neither individual constants nor function symbols.

A set \mathbf{L} of formulas of \mathcal{L} is said to be a *super-intuitionistic predicate logic*, if 1) \mathbf{L} contains all formulas provable in the intuitionistic predicate logic \mathbf{H}_* , 2) \mathbf{L} is closed under the rule of modus ponens, the rule of generalization, and the rule of substitution. A super-intuitionistic predicate logic \mathbf{L} is said to be an *intermediate predicate logic*, if every formula in \mathbf{L} is provable in the classical predicate logic \mathbf{C}_* . Following these terminologies, we identify \mathbf{H}_* and \mathbf{C}_* with the sets of formulas provable in them. For a set S of formulas of \mathcal{L} , we denote by $\mathbf{H}_* + S$ the smallest super-intuitionistic predicate logic containing $\mathbf{H}_* \cup S$. If $S = \{X_1, \dots, X_n\}$, we write $\mathbf{H}_* + X_1 + \dots + X_n$ instead of $\mathbf{H}_* + \{X_1, \dots, X_n\}$.

In this article, we are interested in some properties of intermediate predicate logics as follows.

Definition 1.1 Let \mathbf{L} be an arbitrary intermediate predicate logic.

- An \mathbf{L} is said to have the *pseudo-relevance property (PRP)*, if for all formulas A and B (in \mathcal{L}) which contain no predicate variables in common, $A \supset B \in \mathbf{L}$ implies either $\neg A \in \mathbf{L}$ or $B \in \mathbf{L}$.
- An \mathbf{L} is said to have *PRP**, if for all formulas A and B (in \mathcal{L}) which contain no predicate variables in common, $A \supset B \in \mathbf{L}$ implies either $\neg A \in \mathbf{L}$ or $\neg\neg B \in \mathbf{L}$.
- An \mathbf{L} is said to have the *disjunction property (DP)*, if for all formulas A and B (in \mathcal{L}) $A \vee B \in \mathbf{L}$ implies either $A \in \mathbf{L}$ or $B \in \mathbf{L}$.

- An \mathbf{L} is said to be *Halldén-complete* (*H-complete*), if for all formulas A and B (in \mathcal{L}) which contain no predicate variables in common, $A \vee B \in \mathbf{L}$ implies either $A \in \mathbf{L}$ or $B \in \mathbf{L}$.
- An \mathbf{L} is said to be *H*-complete*, if for all formulas A and B (in \mathcal{L}) which contain no predicate variables in common, $A \vee B \in \mathbf{L}$ implies either $\neg\neg A \in \mathbf{L}$ or $\neg\neg B \in \mathbf{L}$.

The following Proposition is easily seen from the definition.

Proposition 1.2 (Cf. Suzuki [7]) *Let \mathbf{L} be an intermediate predicate logic.*

- (1) *If \mathbf{L} has PRP, then \mathbf{L} has PRP*.*
- (2) *If \mathbf{L} has PRP*, then \mathbf{L} is H*-complete.*
- (3) *If \mathbf{L} has DP, then \mathbf{L} is H-complete.*
- (4) *If \mathbf{L} is H-complete, then \mathbf{L} is H*-complete.*

By this Proposition, we can illustrate the situation in Figure 1. Note that there are $31 (= 2^5 - 1)$ combinations of these properties. Each combination is equivalent to one of the combinations listed in Figure 1.

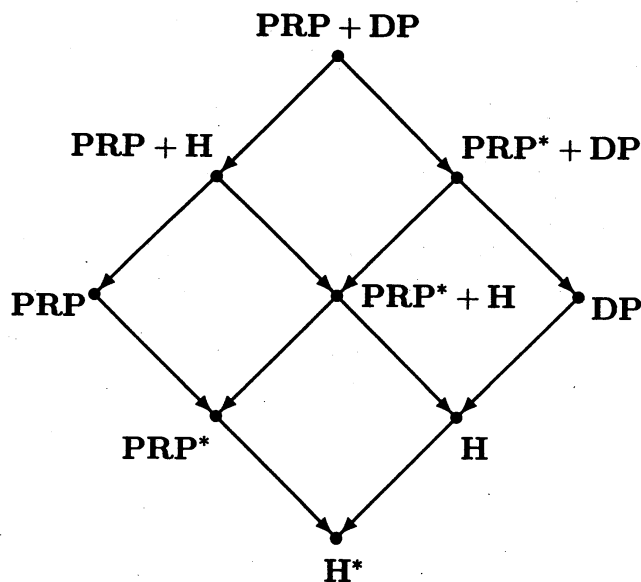


Figure 1

Theorem 1.3 *Figure 1 describes completely the situation. That is, we cannot add any additional \rightarrow 's (i.e., arrows of implication) in Figure 1.*

The main aim of this article is to show the above Theorem 1.3.

The following Proposition provides us an easy sufficient condition for intermediate predicate logics to have PRP*.

Proposition 1.4 (Suzuki [7]) *Let K be $\neg\neg\forall x(p(x) \vee \neg p(x))$, where p is a unary predicate variable. If K is provable in an intermediate predicate logic \mathbf{L} , then \mathbf{L} has PRP*.*

2 The Kripke sheaf semantics and the Delta operation

In this section we prepare two tools for proving Theorem 1.3. Basic definitions and properties of the Kripke sheaf semantics are stated here to make this article self-contained. Another tool is the delta operation Δ . One result on Δ provides us with an easy sufficient condition for an intermediate predicate logic to have DP.

We refer readers to [8] and [9] for details.

For each non-empty set U , we denote by $\mathcal{L}[U]$ the language obtained from \mathcal{L} by adding the name \bar{u} of each $u \in U$. In what follows, we will sometimes use the same letter u for the name of u . We sometimes identify $\mathcal{L}[U]$ with the set of all sentences of $\mathcal{L}[U]$.

Definition 2.1 A partially ordered set $\mathbf{M} = \langle M, \leq \rangle$ with the least element $0_{\mathbf{M}}$ is said to be a *Kripke base*. We can regard a Kripke base \mathbf{M} as a category in the usual way. Let \mathcal{S} denote the category of all non-empty sets. A covariant functor D from a Kripke base \mathbf{M} to \mathcal{S} is called a *domain-sheaf* over \mathbf{M} . That is,

- DS1) $D(a)$ is a non-empty set for every $a \in M$,
- DS2) for every $a, b \in M$ with $a \leq b$, there exists a mapping $D_{ab} : D(a) \rightarrow D(b)$,
- DS3) D_{aa} is the identity mapping $id_{D(a)}$ of $D(a)$ for every $a \in M$,
- DS4) $D_{ac} = D_{bc} \circ D_{ab}$ for every $a, b, c \in M$ with $a \leq b \leq c$.

A pair $\mathcal{K} = \langle \mathbf{M}, D \rangle$ of a Kripke base \mathbf{M} and a domain-sheaf D over \mathbf{M} is called a *Kripke sheaf*. If every D_{ab} ($a \leq b$) is the set-theoretic inclusion, $\langle \mathbf{M}, D \rangle$ is said to be a *Kripke frame*.

For each $d \in D(a)$ and each $b \in M$ with $a \leq b$, $D_{ab}(d)$ is said to be the *inheritor* of d at b . For each formula A of $\mathcal{L}[D(a)]$ and each $b \in M$ with $a \leq b$, the *inheritor* $A_{a,b}$ of A at b is a formula of $\mathcal{L}[D(b)]$ obtained from A by replacing occurrences of \bar{u} ($u \in D(a)$) by the name \bar{v} of the inheritor v of u at b . That is, $A_{a,b}$ is $A^{D_{ab}}$.

A binary relation \models between each $a \in M$ and each atomic sentence of $\mathcal{L}[D(a)]$ is said to be a *valuation* on $\langle \mathbf{M}, D \rangle$ if for every $a, b \in M$ and every atomic sentence A of $\mathcal{L}[D(a)]$, $a \models A$ and $a \leq b$ imply $b \models A_{a,b}$. We extend \models to a relation between each $a \in M$ and each sentence of $\mathcal{L}[D(a)]$ inductively as follows:

- $a \models A \wedge B$ if and only if $a \models A$ and $a \models B$,
- $a \models A \vee B$ if and only if $a \models A$ or $a \models B$,
- $a \models A \supset B$ if and only if for every $b \in M$ with $a \leq b$, either $b \not\models A_{a,b}$ or $b \models B_{a,b}$,
- $a \models \neg A$ if and only if for every $b \in M$ with $a \leq b$, $b \not\models A_{a,b}$,
- $a \models \forall x A(x)$ if and only if for every $b \in M$ with $a \leq b$ and every $u \in D(b)$, $b \models A_{a,b}(\bar{u})$,
- $a \models \exists x A(x)$ if and only if there exists $u \in D(a)$ such that $a \models A(\bar{u})$.

A pair (\mathcal{K}, \models) of a Kripke sheaf \mathcal{K} and a valuation \models on it is said to be a *Kripke-sheaf model*. A formula A of \mathcal{L} is said to be *true* in a Kripke-sheaf model (\mathcal{K}, \models) if $0_{\mathbf{M}} \models \bar{A}$, where \bar{A} is the universal closure of A . A formula A of \mathcal{L} is said to be *valid* in a Kripke sheaf \mathcal{K} if for every valuation \models on \mathcal{K} , A is true in (\mathcal{K}, \models) . The set of formulas of \mathcal{L} valid in $\mathcal{K} = \langle \mathbf{M}, D \rangle$ is denoted by $L(\mathcal{K})$ or $L(\mathbf{M}, D)$. The following propositions are fundamental properties of Kripke-sheaf semantics.

Proposition 2.2 *For every Kripke-sheaf model $(\langle \mathbf{M}, D \rangle, \models)$, every $a, b \in M$, and every sentence $A \in \mathcal{L}[D(a)]$, if $a \models A$ and $a \leq b$, then $b \models A_{a,b}$.*

Proposition 2.3 *For each Kripke-sheaf \mathcal{K} , the set $L(\mathcal{K})$ contains all formulas provable in \mathbf{H}_* , and is closed under the modus ponens, the rule of generalization and the rule of substitution. Namely, $L(\mathcal{K})$ is a super-intuitionistic predicate logic.*

Suppose that we have given a given formula A and an intermediate predicate logic $\mathbf{L} = \mathbf{H}_* + X_1 + \dots + X_n$. If we can construct a Kripke sheaf $\langle M, D \rangle$ such that 1) X_1, \dots, X_n are valid in $\langle M, D \rangle$, and 2) A is not valid in $\langle M, D \rangle$. Then, by the virtue of this Proposition, we have that $A \notin \mathbf{L}$.

Next, we introduce the delta operation Δ .

Definition 2.4 For each formula A , define

$$\Delta(A) \equiv p \vee (p \supset A),$$

where p is a propositional variable not occurring in A . Let \mathbf{L} be a super-intuitionistic predicate logic. We define a super-intuitionistic predicate logic $\Delta(\mathbf{L})$ by

$$\Delta(\mathbf{L}) = \mathbf{H}_* + \{\Delta(A) ; A \in \mathbf{L}\}.$$

The Δ is originally defined on the set of super-intuitionistic propositional logics (see [1]). The Δ for predicate logics was introduced in Komori [2] in a different way. It is important that for every super-intuitionistic predicate logic \mathbf{L} , it holds that $\Delta(\mathbf{L}) \subseteq \mathbf{L}$. Some properties of Δ on super-intuitionistic predicate logics can be found in Suzuki [9]. One of the most interesting results is the following.

Fact 2.5 (Suzuki [7]) (1) For every propositional logic \mathbf{J} , $\Delta(\mathbf{J}) = \mathbf{J}$ if and only if \mathbf{J} is the intuitionistic propositional logic. That is, the intuitionistic logic is the unique fixed point of Δ among super-intuitionistic propositional logics.

(2) There exists a super-intuitionistic predicate logic \mathbf{L} satisfying $\Delta(\mathbf{L}) = \mathbf{L}$ which is not identical to \mathbf{H}_* .

For example, $\mathbf{H}_* + K$ and $\mathbf{H}_* + W^*$ are fixed points of Δ , where W^* is Casari's formula $\forall x((p(x) \supset \forall y p(y)) \supset \forall y p(y)) \supset \forall x p(x)$. These non-trivial fixed points of Δ have interesting property from the logical point of view.

Lemma 2.6 ([9]) If $\Delta(\mathbf{L}) = \mathbf{L}$, then \mathbf{L} has DP.

We make a remark here that this Lemma is proved essentially by making use of the Kripke sheaf semantics (see Suzuki [9]).

We now have a sufficient condition for an intermediate predicate logic to have the DP.

Lemma 2.7 (1) K is provable in $\mathbf{H}_* + \Delta(K)$.

(2) For each sentence S , $S \supset W^*$ is provable in $\mathbf{H}_* + \Delta(S \supset W^*)$.

(3) For every set $\{X_i ; i \in I\}$ of sentences such that $\{X_i ; i \in I\} \subseteq \{S \supset W^* ; S \text{ is a sentence}\} \cup \{K\}$, $\mathbf{H}_* + \{X_i ; i \in I\}$ is a fixed point of Δ .

Proof. The proof of (1) can be found in Lemma 8 of [9].

(2): It is obvious that $\forall x(p(x) \vee (p(x) \supset (S \supset W^*))) \supset (S \supset W^*)$ is provable in \mathbf{H}_* . Since $\forall x(p(x) \vee (p(x) \supset (S \supset W^*)))$ is provable in $\mathbf{H}_* + \Delta(S \supset W^*)$, we have that $S \supset W^*$ is provable in $\mathbf{H}_* + \Delta(S \supset W^*)$.

(3): From (1) and (2). □

3 The proof of the theorem

We give the proof of Theorem 1.3 here.

In [7], the author constructed an intermediate predicate logic \mathbf{L}_1 which has PRP, but is not H-complete. The classical predicate logic \mathbf{C}_* has PRP and is H-complete, but has not DP. Hence, it remains to construct logics \mathbf{L}_2 and \mathbf{L}_3 such that

- \mathbf{L}_2 has both of PRP* and DP, but not PRP.
- \mathbf{L}_3 has DP but has not PRP*.

Lemma 3.1 Let \mathbf{L}_2 be $\mathbf{H}_* + K + \exists x q(x) \wedge \exists x \neg q(x) \supset W^*$, where q is a unary predicate variable distinct from p . Then \mathbf{L}_2 has both of PRP* and DP, but not PRP.

Proof. By Proposition 1.4, \mathbf{L}_2 has PRP*. By Lemma 2.7, \mathbf{L}_2 is a fixed point of Δ . By Lemma 2.6, \mathbf{L}_2 has DP.

We will show that neither $\neg(\exists xq(x) \wedge \exists x\neg q(x))$ nor W^* is in \mathbf{L}_2 . It is clear that $\mathbf{L}_2 \subseteq \mathbf{C}_*$. Since $\neg(\exists xq(x) \wedge \exists x\neg q(x))$ is not provable in \mathbf{C}_* , $\neg(\exists xq(x) \wedge \exists x\neg q(x))$ is not in \mathbf{L}_2 . Now we make use of the Kripke sheaf semantics. Let M be $\omega \cup \{\omega\} = \{i; i \leq \omega\}$. The M is a poset with the canonical order \leq . Define a domain sheaf D by:

$$D(i) = \begin{cases} \omega & \text{if } i < \omega, \\ \{0\} & \text{if } i = \omega, \end{cases}$$

$$D_{ij}(k) = \begin{cases} k & \text{if } i \leq j < \omega, \\ 0 & \text{if } j = \omega. \end{cases}$$

Then $\langle M, D \rangle$ is a Kripke sheaf.

Claim A. *In this Kripke sheaf $\langle M, D \rangle$, K and $\exists xq(x) \wedge \exists x\neg q(x) \supset W^*$ are valid.*

Proof of Claim A. It is clear that $\omega \models \forall x(p(x) \vee \neg p(x))$ for every valuation \models . Hence, we have $0 \models K$ for every valuation \models . Next, note that $i \not\models \exists xq(x) \wedge \exists x\neg q(x)$ for every valuation \models and every $i \in M$. Therefore, $0 \models \exists xq(x) \wedge \exists x\neg q(x) \supset W^*$ for every valuation \models . This completes the proof of Claim A.

Claim B. *W^* is not valid in $\langle M, D \rangle$.*

Proof of Claim B. Define a valuation \models on $\langle M, D \rangle$ by:

$$i \models p(k) \text{ in and only if } k < i,$$

for every $i \in M$ and every $k \in D(i)$. Then, we have $0 \not\models W^*$. This completes the proof of Claim B.

Hence, by Claim A, $\mathbf{L}_2 \subseteq L\langle M, D \rangle$. By Claim B, W^* is not in \mathbf{L}_2 . □

Definition 3.2 Let r be a binary predicate variable. Define F_1 , F_2 , F_3 and F as follows.

$$F_1 \equiv \forall x r(x, x) \wedge \forall x \forall y (r(x, y) \vee r(y, x)),$$

$$F_2 \equiv \forall x \forall y \forall z (r(x, y) \wedge r(y, z) \supset r(x, z)),$$

$$F_3 \equiv \forall x \exists y \neg r(y, x),$$

$$F \equiv F_1 \wedge F_2 \wedge F_3.$$

Lemma 3.3 *Let \mathbf{L}_3 be $\mathbf{H}_* + F \supset W^*$. Then \mathbf{L}_3 has DP but has not PRP*.*

Proof. Clearly, \mathbf{L}_3 is a fixed point of Δ , and has DP. It suffices to show that neither $\neg F$ nor $\neg\neg W^*$ is in \mathbf{L}_3 . We have $\mathbf{L}_3 \subseteq \mathbf{C}_* \not\supset \neg F$. We show that $\neg\neg W^* \notin \mathbf{L}_3$ by making use of the Kripke semantics. On the well-ordered set $\omega = \{i; i < \omega\}$, define a mapping U by

$$U(i) = \{0, 1, \dots, i\},$$

for each $i \in \omega$. Then $\langle \omega, U \rangle$ is a Kripke frame.

Claim C. $F \supset W^*$ is valid in $\langle \omega, U \rangle$.

Proof of Claim C. Suppose otherwise. There exist a valuation \models and $i \in \omega$ such that $i \models F$ and $i \not\models W^*$. On each $U(k)$ ($k \geq i$), we can define a binary relation R_k such that

- 1) R_k is a quasi-order on $U(k)$,
- 2) for every x and y in $U(k)$, $x R_k y$ or $y R_k x$,
- 3) for every $x \in U(k)$ there exists a $y \in U(k)$ such that $x R_k y$ but not $y R_k x$.

Let θ_k denote the equivalence relation induced by R_k , that is,

$$x \theta_k y \text{ if and only if } x R_k y \text{ and } y R_k x,$$

for every x and y in $U(k)$. For each $x \in U(k)$, we write x/θ_k for the equivalence class of x . Then, the set $U(k)/\theta_k$ of all equivalence classes is naturally a totally ordered set whose order \leq_k is defined by:

$$x/\theta_k \leq_k y/\theta_k \text{ if and only if } x R_k y,$$

for each x/θ_k and y/θ_k in $U(k)/\theta_k$. By the above 3), there exists a strictly increasing infinite sequence $x_1/\theta_k <_k x_2/\theta_k <_k \dots <_k x_n/\theta_k <_k \dots$ in $U(k)/\theta_k$. This contradicts that fact that $U(k)$ is finite. Therefore, $i \not\models F$ for every valuation \models and every $i \in \omega$. This completes the proof of Claim C.

Claim D. $\neg\neg W^*$ is not valid in $\langle \omega, U \rangle$.

Proof of Claim D. Define a valuation \models on $\langle \omega, U \rangle$ by:

$$i \models p(k) \text{ if and only if } k < i,$$

for every $i \in \omega$ and every $k \in U(i)$ ($= \{0, 1, \dots, i\}$). Then $i \not\models \forall x p(x)$ for every $i \in \omega$. Hence $i \models W^*$ if and only if $i \models \neg \forall x (\neg \neg p(x))$.

Suppose that $0 \models \neg \neg W^*$. There exists an $i \in \omega$ with $i \models W^*$. Then, we have $i \not\models \forall x (\neg \neg p(x))$. It follows that there exist j ($i \leq j$) and k ($k \leq j$) such that $j \not\models \neg \neg p(k)$. If $k < j$, then $j \models p(k)$, and hence $j \models \neg \neg p(k)$. Hence, it must hold that $k = j$. Note that for each $j \in \omega$, $j + 1 \models p(j)$. Hence, for each j , $j \models \neg \neg p(j)$. This is a contradiction. Thus, we have $0 \not\models \neg \neg W^*$. This completes the proof of Claim D.

By Claim C and Claim D, we have that $L_3 \subseteq L\langle \omega, U \rangle \not\models \neg \neg W^*$. This completes the proof of Lemma. \square

Therefore, we have our Theorem 1.3.

4 Concluding remark

The notion of Craig's interpolation property (CIP) is one of the most fascinating subjects in the study of intermediate predicate logics. (See e.g., Ono [4].)

Definition 4.1 An intermediate predicate logic L is said to have *Craig's interpolation property* (CIP), if for every $A \supset B \in L$, 1) if A and B contain no predicate variable in common, either $\neg A \in L$ or $B \in L$, 2) if A and B contain at least one predicate variable in common, there exists a C such that $A \supset C \in L$ and $C \supset B \in L$ and every predicate variable in C occurs both in A and in B .

Clearly, CIP implies PRP. One can add CIP (and its combinations) to Figure 1 as follows:

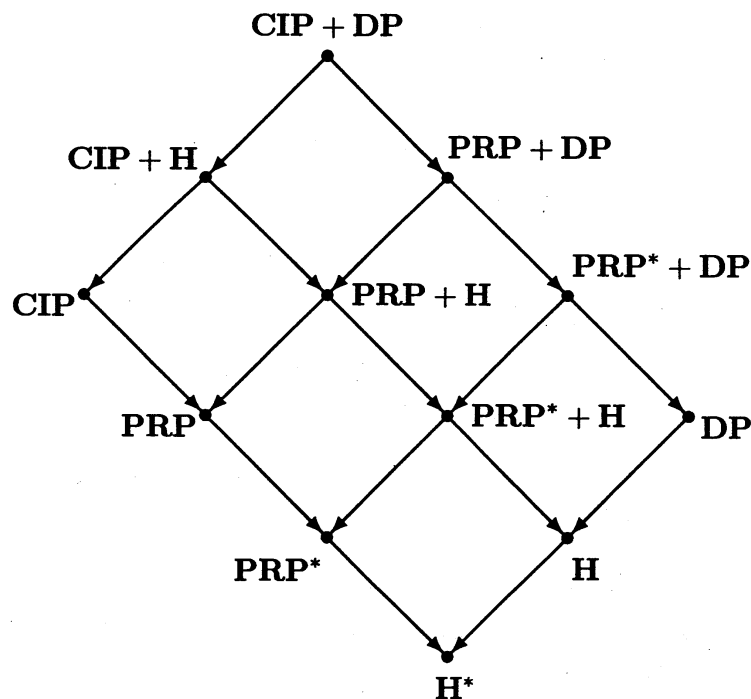


Figure 2

Now, a problem arises naturally. We state it as a conjecture here.

Conjecture. Figure 2 describes completely the situation. That is, we cannot add any additional \rightarrow 's (i.e., arrows of implication) in Figure 2.

Note that the classical predicate logic C_* has CIP and is H-complete, but has not DP. Hence, the problem is reduced to the following.

Problem. Are there exist logics L_4 and L_5 satisfying the following conditions?

- L_4 has both of PRP and DP, but not CIP,
- L_5 has CIP but is not H-complete,

If we can find them, we can see the above Conjecture to be true.

The above problem related to the following storied problem listed also in Ono [4].

Problem. Let LD be $H_* + \forall x(p(x) \vee s) \supset (\forall xp(x) \vee s)$ where s is a propositional variable. Has LD CIP?

If the answer is negative, LD can serve as L_4 in the previous problem. Not a few logicians believe that the answer is affirmative. But, at present, there are no *correct* proofs. Our knowledge about CIP in predicate logics is very limited.

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