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SEVERAL VALUE DISTRIBUTION THEOREMS FOR THE LERCH ZETA-FUNCTION

A. Laurinčikas

Let $s = \sigma + it$ be a complex variable. In 1887 M. Lerch [12] considered the function $L(\lambda, \alpha, s)$ defined for $\sigma > 1$ by the following Dirichlet series

$$L(\lambda, \alpha, s) = \sum_{m=0}^{\infty} \frac{e^{2\pi i \lambda m}}{(m+\alpha)^s}.$$

Here $\lambda \in \mathbb{R}$, $0 < \alpha \leq 1$ are fixed parameters where, as usual, $\mathbb{R}$ denotes the set of all real numbers. When λ is an integer number $L(\lambda, \alpha, s)$ reduces to the Hurwitz zeta-function. Suppose $0 < \lambda < 1$. Then M. Lerch proved [12] that $L(\lambda, \alpha, s)$ is analytically continuable to an entire function. Moreover, he obtained that $L(\lambda, \alpha, s)$ satisfies the following functional equation

$$\begin{aligned} L(\lambda, \alpha, 1-s) = (2\pi)^{-s}\Gamma(s)\Big(& \exp\Big\{\frac{\pi i s}{2} - 2\pi i \alpha \lambda\Big\} L(-\alpha, \lambda, s) \\ & + \exp\Big\{-\frac{\pi i s}{2} + 2\pi i \alpha(1-\lambda)\Big\} L(\alpha, 1-\lambda, s)\Big), \end{aligned} \tag{1}$$

where $\Gamma(s)$ stands for the Euler gamma-function. Two new simple proofs of this functional equation were given by B. C. Berndt [2]. The first of them uses contour integration, the second the Euler-Maclaurin summation formula. Once one proof of (1) was found by M. Mikolás [11].

D. Klusch in [7] obtained the asymptotic formulae for

$$\int_0^{\infty} \left|L(\lambda, \alpha, \sigma + it)\right|^2 e^{-\delta t}\, dt, \qquad \delta > 0,$$

$$\int_0^{T} \left|L(\lambda, \alpha, \sigma + it)\right|^2 dt$$

in the strip $\frac{1}{2} \leq \sigma < 1$. In [8] he found a version of the Atkinson formula for $L(\lambda, \alpha, s)$. W. Zhang in [19] proved an asymptotic formula for

$$I(\lambda, s) = \int_0^1 \left|L(\lambda, \alpha, s) - \alpha^{-s}\right|^2 d\alpha.$$

Asymptotic expansions for $I(\lambda, s)$ were given by M. Katsurada [6].

Denote by $\mathcal{B}(S)$ the class of Borel sets of the space S, and let, for $T > 0$,

$$\nu_T^t(\dots) = \frac{1}{T}\mathrm{meas}\,\{t \in [0, T], \dots\},$$

where $\mathrm{meas}\,\{A\}$ stands for the Lebesgue measure of the set A, and in place of dots we write a condition satisfied by t. Denote by $\mathbb{C}$ the complex plane.

Theorem 1. *Suppose $\sigma > \frac{1}{2}$. Then there exists a probability measure P on $(\mathbb{C}, \mathcal{B}(\mathbb{C}))$ such that the measure*

$$P_T(A) = \nu_T^t\big(L(\lambda, \alpha, \sigma + it) \in A\big), \qquad A \in \mathcal{B}(\mathbb{C}),$$

converges weakly to P as $T \to \infty$.

Proof of the theorem is given in [4].

Now let $D = \{s \in \mathbb{C} : \sigma > \frac{1}{2}\}$, and let $H(D)$ denote the space of analytic on D functions equipped with the topology of uniform convergence on compacta.

Theorem 2. *Let α be a transcendental number. Then there exists a probability measure Q on $\big(H(D), \mathcal{B}(H(D))\big)$ such that the measure*

$$\nu_T^t\big(\lambda, \alpha, s + i\tau) \in A\big), \qquad A \in \mathcal{B}(H(D)),$$

converges weakly to Q as $T \to \infty$.

Proof of the theorem is given in [9].

It was observed by B. Bagchi [1] that functional limit theorems for Dirichlet series have serious applications, however, in these applications the explicit form of the limit measure is necessary. For this reason in [10] the explicit form of the measure Q was found.

Denote by γ the unit circle on $\mathbb{C}$, i.e. $\gamma = \{s \in \mathbb{C} : |s| = 1\}$, and let

$$\Omega = \prod_{m=0}^{\infty} \gamma_m$$

where $\gamma_m = \gamma$ for all $m = 0, 1, 2 \ldots$. With the product topology and pointwise multiplication Ω is a compact topological Abelian group. Therefore there exists the unique probability Haar measure m_H on $\big(\Omega, \mathcal{B}(\Omega)\big)$. Thus we obtain the probability space $\big(\Omega, \mathcal{B}(\Omega), m_H\big)$. Let $\omega(m)$ stand for the projection of $\omega \in \Omega$ to the coordinate space γ_m. Then we have that $\{\omega(m), m = 0, 1, 2, \ldots\}$ is a sequence of independent complex random variables uniformly distributed on γ. Let

$$L(\lambda, \alpha, s, \omega) = \sum_{m=0}^{\infty} \frac{e^{2\pi i \lambda m} \omega(m)}{(m + \alpha)^s}, \qquad \omega \in \Omega.$$

Then it is not difficult to show that for almost all $\omega \in \Omega$ the latter series converges uniformly on compact subsets of D, and therefore $L(\lambda, \alpha, s, \omega)$ is an $H(D)$-valued random element defined on $\big(\Omega, \mathcal{B}(\Omega), m_H\big)$. Denote by P_L the distribution of the random element $L(\lambda, \alpha, s, \omega)$, i.e.

$$P_L(A) = m_H\big(\omega \in \Omega : L(\lambda, \alpha, s, \omega) \in A\big), \qquad A \in \mathcal{B}(H(D)).$$

Theorem 3. *Let α be a transcendental number. Then the limit measure Q in Theorem 3 coincides with P_L.*

This theorem also gives the explicit form of the limit measure P in Theorem 1. Let the function $h : H(D) \to \mathbb{C}$ be given by the formula $h(f) = f(\sigma)$ where $f \in H(D)$ and $\sigma > \frac{1}{2}$ is fixed. Clearly, the function h is continuous, therefore we have that $P_T h^{-1}$ converges weakly to Ph^{-1} as $T \to \infty$. Therefore the measure P in Theorem 1 equals to

$$m_H\big(L(\lambda, \alpha, \sigma, \omega) \in A\big), \qquad A \in \mathcal{B}(\mathbb{C}).$$

Now suppose α is a rational number. In this case the system $\{\log(m+\alpha),\ m = 0, 1, \ldots\}$ is not linearly independent over the field of rational numbers Q, and we must consider the system $\{\log p,\ p \text{ is a prime}\}$ which is linearly independent over Q. Let

$$\Omega_1 = \prod_p \gamma_p,$$

where $\gamma_p = \gamma$ for all primes p. Denote by m_{1H} the probability Haar measure on $\big(\Omega_1, \mathcal{B}(\Omega_1)\big)$, and by $\omega_1(p)$ the projection of $\omega_1 \in \Omega_1$ to the coordinate space γ_p. Then we have that $\{\omega(p),\ p \text{ is a prime}\}$ is a sequence of independent random variables defined on the probability space $\big(\Omega_1, \mathcal{B}(\Omega_1), m_{1H}\big)$. We take, for naturals m,

$$\omega_1(m) = \prod_{p^\alpha \| m} \omega_1^\alpha(p),$$

where $p^\alpha \| m$ means that $p^\alpha | m$ but $p^{\alpha+1} \nmid m$. Thus, $\omega_1(m)$ is a completely multiplicative function.

Let $\alpha = \frac{a}{q}$, $1 \le a \le q$, $(a, q) = 1$. Define on $\big(\Omega, \mathcal{B}(\Omega_1), m_{1H}\big)$ an $H(D)$-valued random element $L_1(\lambda, \alpha, s, \omega_1)$ by the equality

$$L_1(\lambda, \alpha, s, \omega_1) = \omega_1(q)\, q^s e^{-2\pi i \lambda a/q} \sum_{\substack{m=1 \\ m \equiv a \pmod q}}^{\infty} \frac{e^{2\pi i \lambda m/q} \omega_1(m)}{m^s}, \qquad \omega_1 \in \Omega, s \in D.$$

Let P_{L_1} be the distribution of $L_1(\lambda, \alpha, s, \omega_1)$.

Theorem 4. *The probability measure*

$$P_{1T}(A) = \nu_T^t\big(L(\lambda, \alpha, s + i\tau) \in A\big), \qquad A \in \mathcal{B}(H(D)),$$

converges weakly to P_{L_1} *as* $T \to \infty$.

All limit theorems stated above can be generalized in the following manner. Let T_0 be a fixed number, and let $w(t)$ be a positive function of bounded variation on $[T_0, \infty)$ such that its variation $V_a^b w$ on $[a, b]$ satisfies the inequality $V_a^b w \le c w(a)$ for all $b > a \ge T_0$ with some constant $c > 0$. Let

$$U = U(T, w) = \int_{T_0}^{T} w(t)\, dt,$$

and suppose that $\lim_{T\to\infty} U(T,w) = \infty$. Then we can consider the weak convergence of the measure

$$\frac{1}{U}\int_{T_0}^{T} w(\tau) I_{\{\tau:\dots\}} d\tau$$

instead of that of the measure $\nu_T^t(\dots)$. Here I_A denotes the indicator function of the set A. The mentioned generalizations were done in [3], [5].

Theorem 3 can be applied to derive the universality property for the Lerch zeta-function. Note that the universality of the Riemann zeta-function $\zeta(s)$ was discovered by S. M. Voronin in 1975. The contemporary statement of universality theorem for $\zeta(s)$ is the following.

Let K be a compact subset of the strip $\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\}$ with connected complement. Let $f(s)$ be a nonvanishing continuous function on K which is analytic in the interior of K. Then for every $\varepsilon > 0$

$$\liminf_{T\to\infty} \nu_T^\tau\Big(\sup_{s\in K} \big|\zeta(s+i\tau) - f(s)\big| < \varepsilon\Big) > 0.$$

Later S. M. Gonek, B. Bagchi, A. Reich and the author proved the universality for some classes of Dirichlet series. There exists an hypothesis that all Dirichlet series have the universality property.

Theorem 5. *Let α be a transcendental number, K be a compact subset of the strip $\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\}$ with connected complement, and let $f(s)$ be a continuous function on K which is analytic in the interior of K. Then for every $\varepsilon > 0$*

$$\liminf_{T\to\infty} \nu_T^\tau\Big(\sup_{s\in K} \big|L(\lambda,\alpha,s+i\tau) - f(s)\big| < \varepsilon\Big) > 0.$$

Note that in Theorem 5 $f(s)$ is not necessary nonvanishing function on K.

Unfortunately, in the case of rational α the random variables $\omega_1(m)$ are not independent with respect to m_{1H}, and therefore Theorem 4 is not useful for the proof of the universality of $L(\lambda,\alpha,s)$. For this reason we suppose that λ is also a rational number. Let $\lambda = l/r \le l < r$, $(l,r) = 1$. Denote, for brevity, $k = rq$, $\beta_m = lm/k$, and let

$$\eta_v = \sum_{\substack{m=1\\ m=a(\mathrm{mod}\ q)}} e^{2\pi i\beta_m}\bar{\chi}_v(m),$$

where $\bar{\chi}_v(m)$, $v = 0,1,\dots,\varphi(k)-1$, are Dirichlet characters modulo k, and $\varphi(k)$ is the Euler function. Then at least two numbers η_v are distinct from zero.

Theorem 6. *Suppose that there exist at least two primitive characters modulo k, such that the corresponding numbers η_v are distinct from zero. Let $0 < R < \frac{1}{4}$, and let $f(s)$ be a function continuous in the disc $|s| \le R$ and analytic in the interior of this disc. Then for every $\varepsilon > 0$*

$$\liminf_{T\to\infty} \nu_T^\tau\Big(\max_{|s|\le R} \big|q^{-s-3/4-i\tau}L(l/r,a/q,s+3/4+i\tau) - f(s)\big| < \varepsilon\Big) > 0.$$

Theorems 5 and 6, and an application of the Cauchy integral formula lead to the following results.

Theorem 7. *Suppose α is a transcendental number. Let the map $h : \mathbb{R} \to \mathbb{C}^N$ be defined by the formula*

$$h(t) = \big(L(\lambda, \alpha, \sigma + it),\, L'(\lambda, \alpha, \sigma + it), \ldots, L^{(N-1)}(\lambda, \alpha, \sigma + it)\big), \qquad \frac{1}{2} < \sigma < 1.$$

Then the image of $\mathbb{R}$ is dense in $\mathbb{C}^N$.

Theorem 8. *Let $\lambda = l/r$ and $\alpha = a/q$ be rational numbers. Suppose there exist at least two primitive characters modulo $k = rq$ such that the corresponding numbers η_v are distinct from zero. Let the function $h : \mathbb{R} \to \mathbb{C}^N$ be defined by the formula*

$$h(t) = \Big(q^{-\sigma-it}L(\lambda, \alpha, \sigma + it),\, \big(q^{-\sigma-it}L(\lambda, \alpha, \sigma + it)\big)', \ldots,$$
$$\big(q^{-\sigma-it}L(\lambda, \alpha, \sigma + it)\big)^{(N-1)}\Big), \qquad \frac{1}{2} < \sigma < 1.$$

Then the image of $\mathbb{R}$ is dense in $\mathbb{C}^N$.

Theorem 7 and 8 allow to obtain the functional independence of the Lerch zeta-function. Note that during the International Congress of Mathematicians in 1990 D. Hilbert formulated the problem of algebraic-differential independence for Dirichlet series. He noted that an algebraic-differential independence of $\zeta(s)$ can be proved using the algebraic-differential independence of the Euler gamma-function and the functional equation for $\zeta(s)$. D. Hilbert also conjectured that there is no algebraic-differential equation with partial derivatives which could be satisfied by the function

$$\zeta(s, x) = \sum_{m=1}^{\infty} \frac{x^m}{m^s}.$$

This conjecture was proved independently by D. D. Mordukhai-Boltovskoi [13] and by A. Ostrovski [14]. A. G. Postnikov [15] generalized the Hilbert problem for a system of Dirichlet series considering their differential independence. In [16] he investigated the function

$$L(x, s, \chi) = \sum_{m=1}^{\infty} \frac{\chi(m)}{m^s}\, x^m,$$

where $\chi(m)$ is a Dirichlet character, and proved that the equation

$$P\left(x, s, \frac{\partial^{l+r} L(x, s, \chi)}{\partial x^l \partial s^r}\right) = 0$$

can not be satisfied for any polynomial P. S. M. Voronin [17], [18] obtained the functional independence of the Riemann zeta-function proving that if F_m, $m = 0, 1, \ldots, n$, are continuous functions and the equality

$$\sum_{m=0}^{n} s^m F_m\big(\zeta(s), \zeta'(s), \ldots, \zeta^{(N-1)}(s)\big) = 0$$

is valid identically for s, then $F_m \equiv 0$ for $m = 0, 1, \ldots, n$.

Theorem 9. *Suppose α is a transcendental number. Let F_m, $m = 0, 1, \dots, n$, be continuous functions, and let the equality*

$$\sum_{m=0}^{n} s^m F_m\big(L(\lambda, \alpha, s),\ L'(\lambda, \alpha, s), \dots, L^{(N-1)}(\lambda, \alpha, s)\big) = 0$$

be valid identically for s. Then $F_m \equiv 0$ for $m = 0, 1, \dots, n$.

Theorem 10. *Let $\lambda = l/r$ and $\alpha = a/q$ be rational numbers. Suppose there exist at least two primitive characters modulo $k = rq$ such that the corresponding number η_v are distinct from zero. Let F_m, $m = 0, 1, \dots, n$, be continuous functions, and let the equality*

$$\sum_{m=0}^{n} s^m F_m\Big(q^{-s}L(\lambda, \alpha, s),\ \big(q^{-s}L(\lambda, \alpha, s)\big)', \dots, \big(q^{-s}L(\lambda, \alpha, s)\big)^{(N-1)}\Big) = 0$$

be valid identically for s. Then $F_m \equiv 0$ for $m = 0, 1, \dots, n$.

Proof of Theorem 9. It is sufficient to prove that $F_n \equiv 0$. Let, on the contrary, $F_n \not\equiv 0$. Then there exists a bounded region $\mathcal{G}$ in $\mathbb{C}^N$ such that the inequality

$$\big|F_n(s_0, s_1, \dots, s_{N-1})\big| > c > 0 \tag{2}$$

holds for all points $(s_0, s_1, \dots, s_{N-1}) \in \mathcal{G}$. By Theorem 7 there exists a sequence $\{t_k\}$, $t_k \to \infty$, such that

$$\Big(L(\lambda, \alpha, \sigma + it_k),\ L'(\lambda, \alpha, \sigma + it_k),\ \dots, L^{(N-1)}(\lambda, \alpha, \sigma + it_k)\Big) \in \mathcal{G}.$$

However this and (2) contradict the hypothesis of the theorem. Hence $F_n \equiv 0$.

Proof of Theorem 10 is similar to that of Theorem 9, and it uses Theorem 8.

Now we present some results on the zeros of the Lerch zeta-function. They were obtained by my student R. Garunkštis.

Theorem 11. *If $\sigma \geq 1 + \alpha$, then $L(\lambda, \alpha, s) \neq 0$.*

Let $L_\varepsilon(l) = \{s \in \mathbb{C} : \varrho(s, l) < \varepsilon\}$, where l is a line on the complex plane $\mathbb{C}$, and $\varrho(s, l)$ stands for the distance of s from l.

Theorem 12. *Suppose $\lambda \neq \frac{1}{2}$. Then there exist constants $\sigma_0 \leq 0$ and $\varepsilon > 0$ such that $L(\lambda, \alpha, s) \neq 0$ for $\sigma < \sigma_0$ and*

$$s \notin L_{\varepsilon_0}\left(\sigma = \frac{\pi t}{\log \frac{1-\lambda}{\lambda}} + 1\right).$$

Theorem 13. *Suppose $\lambda \neq \frac{1}{2}$. For any $\varepsilon > 0$, $L(\lambda, \alpha, s)$ has infinitely many zeros lying in*

$$L_\varepsilon\left(\sigma = \frac{\pi t}{\log \frac{1-\lambda}{\lambda}} + 1\right).$$

Theorem 14. *If* $|t| \geq 1$ *and* $\sigma < -\frac{1}{2}$, *then* $L\left(\frac{1}{2}, \alpha, s\right) \neq 0$.

We say that zero s_0 of $L(\lambda, \alpha, s)$ is trivial if

$$s_0 \in L_{\varepsilon_0}\left(\sigma = \frac{\pi t}{\log \frac{1-\lambda}{\lambda}} + 1\right)$$

for $\lambda \neq \frac{1}{2}$, or s_0 lies on the real axis if $\lambda = \frac{1}{2}$. Here ε_0 is defined in Theorem 12.

Let $[u]$ denote the integer part of u.

Theorem 15. *If* $\sigma \leq -\left(2\alpha + 1 + 2\left[\frac{3}{4} - \alpha\right]\right)$ *and* $|t| \leq 1$, *then* $L(\frac{1}{2}, \alpha, s) \neq 0$, *except for trivial zeros on the negative real axis, one in each interval* $(-2m - 2\alpha - 1, -2m - 2\alpha + 1)$, $m \geq \frac{3}{4} - \alpha$.

Denote by $N^+(\lambda, \alpha, T)$ and $N^-(\lambda, \alpha, T)$ the number of nontrivial zeros of the function $L(\lambda, \alpha, s)$ in the regions $0 < t < T$ and $-T < t < 0$, respectively.

Theorem 16. *We have*

$$N^+(\lambda, \alpha, T) = \frac{T}{2\pi} \log T - \frac{T}{2\pi} \log(2\pi\alpha\lambda) + O(\log T),$$
$$N^-(\lambda, \alpha, T) = N^+(1 - \lambda, \alpha, T).$$

Now we give some results on zeros of the Lerch zeta-function in the half-plane $\sigma > 1$ as well as in the strip $0 \leq \sigma \leq 1$.

Theorem 17. *Let* α *is a non-rational number. Then there exists a constant* $c = c(\lambda, \alpha) > 0$ *such that, for sufficiently large* T, *the function* $L(\lambda, \alpha, s)$ *has more than* cT *zeros lying in the region* $\sigma > 1$, $|t| \leq T$.

Theorem 18. *Let* α *be a transcendental number. Then for any* σ_1, σ_2, $\frac{1}{2} < \sigma_1 < \sigma_2 < 1$, *there exists a constant* $c = c(\lambda, \alpha, \sigma_1, \sigma_2) > 0$ *such that, for sufficiently large* T, *the function* $L(\lambda, \alpha, s)$ *has more than* cT *zeros lying in the rectangle* $\sigma_1 < \sigma < \sigma_2$, $|t| < T$.

Theorem 19. *Let* $\lambda = l/r$ *and* $\alpha = a/q$ *be rational numbers. Suppose there exist at least two primitive characters modulo* $k = rq$ *such that the corresponding numbers* η_v *are distinct from zero. Then for the function* $L(\lambda, \alpha, s)$ *the assertion of Theorem 18 is true.*

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Department of Mathematics
Vilnius University
Naugarduko, 24
2006 Vilnius
Lithuania

e-mail: antanas.laurincikas@maf.vu.lt

Department of Mathematics and Physics
Šiauliai University
P. Višinskio 25
5419 Šiauliai
Lithuania