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Multiple Choice Problems Related to the Duration of the Secretary Problem

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Abstract

A version of the multiple choice secretary problem called the multiple choice duration problem, in which the objective is to maximize the time of possession of relatively best objects, is treated. For the m choice duration problem with a known number of objects, there exists a sequence of critical numbers $(s_1,s_2,\dots,s_m)$ such that, whenever there remain k choices yet to be made, then the optimal strategy immediately selects a relatively best object if it appears after or on time s_k, $1\le k\le m$. A simple recursive formula for calculating the critical numbers when the number of objects tends to infinity will be given. It can be shown that the multiple choice duration problem with a known number of objects is related to the multiple choice (best-choice) secretary problem with an unknown number of objects having a uniform prior on the actual number of objects. Extensions to models involving an acquisition cost or a replacement cost are made.

1. Introduction and summary

Though Ferguson, Hardwick and Tamaki[2] considered the various duration models extensively, they confined themselves to the study of the one choice duration problems. In this paper, we attempt to extend the one choice problems to the multiple choice problems. For the m choice duration problem, we are allowed to choose at most m objects sequentially, and receive each time a unit payoff as long as either of the chosen objects remains a *candidate*(for simplicity we refer to a relatively best object as a candidate). Obviously only candidates can be chosen, the objective being to maximize the expected payoff.

It can be shown in Section 2 that, for the m choice duration problem, there exists a sequence of integer-valued critical numbers $(s_1,s_2,\dots,s_m)$ such that, whenever there remain k choices yet to be made, then the optimal strategy immediately selects a candidate if it appears after or on time s_k, $1\le k\le m$. It is also shown that s_k is non-increasing in k. s_k/n converges to some definite value s_k^* and a recursive formula for calculating s_k^* in terms of $s_1^*, s_2^*,\dots,s_{k-1}^*$ will be given by

$$s_k^* = \exp\left[-\left\{1+\sqrt{1-2\sum_{i=1}^{k-1}\frac{\left[(k-i+2)+(k-i+1)\log s_i^*\right]}{(k-i+2)!}\left(\log s_i^*\right)^{k-i+1}}\right\}\right]. \tag{1}$$

It is also shown that, as $n\to\infty$, (the maximum expected payoff)/n converges to $-\sum_{k=1}^{m} s_k^*\log s_k^*$.

The best-choice secretary problem is concerned with maximizing the probability of choosing the best object from among all. We show that the multiple choice duration problem with a known number of n objects is equivalent to the multiple choice best-choice secretary problem with an unknown number of objects having a uniform distribution on $\{1,2,...,n\}$.

In Sections 3 and 4, the multiple choice duration problem treated in Section 2 is generalized by introducing cost. In Section 3, we consider a problem in which a constant acquisition cost is incurred each time an object is chosen. Thus far we have assumed that the objects, once chosen, are possessed until the process terminates. We consider in Section 4 a problem which allows us to possess only one object at a time. A constant replacement cost is incurred each time we replace a previously chosen object with a new one. The objectives are, in Sections 3 and 4, to maximize the expected net payoff. It can be shown that, under an appropriate cost condition, the optimal strategies have the same structure as that for the problem involving no cost.

2. Multiple choice duration problem

We assume that all that can be observed are the relative ranks of the objects as they are presented. Thus if X_i denotes the relative rank of the i th object among those observed so far (i th object is a candidate if $X_i=1$), the sequentially observed random variables are $X_1,X_2,\ldots,X_n$. It is well known, under the assumption that the objects are put in random order with all n! permutations equally likely, that

(a) the X_i are independent random variables and

(b) $P\{X_i=j\} = 1/i$, for $1 \le j \le i$, $1 \le i \le n$.

2.1 The finite horizon problem

We consider the m choice duration problem as a Markovian decision process model. Since serious decision of either selection or rejection takes place only when a candidate appears, we describe the state of the process as (i,k), $1 \le i \le n$, $1 \le k \le m$ if the i th object is a candidate and there remain k more choices to be made.

Let $W_i^{(k)}$ be the expected additional payoff under an optimal strategy starting from state (i,k), $1 \le i \le n$, $1 \le k \le m$, and also let $U_i^{(k)}\left(V_i^{(k)}\right)$ be the expected additional payoff when we select(reject) the i th object and then continues search in an optimal manner. Then the principle of optimality yields, for $1 \le k \le m$

$$W_i^{(k)} = \max\left\{U_i^{(k)},\ V_i^{(k)}\right\}, \quad 1 \le i \le n, \tag{2}$$

where

$$U_i^{(k)} = \frac{1}{n}\sum_{j=i}^{n} \frac{i}{j} + \sum_{j=i+1}^{n} \frac{i}{j(j-1)} W_j^{(k-1)} \tag{3}$$

and

$$V_i^{(k)} = \sum_{j=i+1}^{n} \frac{i}{j(j-1)} W_j^{(k)} . \tag{4}$$

Equations (2)-(4), combined with the boundary condition $W_i^{(0)}=0$, $1\le i\le n$, can be solved recursively to yield the optimal strategy and the optimal value $W_1^{(m)}$.

Theorem 2.1

For the m choice duration problem, there exists a sequence of integer-valued critical numbers $(s_1, s_2, \ldots, s_m)$ such that, whenever there remain k choices yet to be made, i.e., we have already chosen m - k objects, then the optimal strategy immediately selects a candidate if it appears after or on time s_k. Moreover, s_k is non-increasing in k and determined by

$$s_k = \min\left\{ i : G_i^{(k)} \ge 0 \right\}, \tag{5}$$

where $G_i^{(k)}$, $1\le i\le n$, $1\le k\le m$, is defined recursively as

$$G_i^{(k)} = G_i^{(1)} + \sum_{j=\max(i+1,\, s_{k-1})}^{n} \frac{1}{j-1} G_j^{(k-1)} , \qquad k\ge 2 \tag{6}$$

starting with

$$G_i^{(1)} = \sum_{j=i}^{n} \frac{1}{j} - \sum_{j=i+1}^{n} \frac{1}{j-1} \sum_{t=j}^{n} \frac{1}{t}. \tag{7}$$

Let q_m, $m\ge 1$, be the expected payoff for the m choice problem, i.e., $q_m \equiv W_1^{(m)}$. Then, from the property of the optimal strategy, we have

$$q_m = \tilde{V}_{s_m-1}^{(m)} = \left(\frac{s_m-1}{n}\right) \sum_{j=s_m}^{n} \frac{1}{j-1} \sum_{t=j}^{n} \frac{1}{t} + \sum_{j=s_m}^{n} \frac{s_m-1}{j(j-1)} V_j^{(m-1)},$$

where $V_i^{(m-1)}$, $m\ge 2$, are calculated recursively as

$$V_i^{(m-1)} = \begin{cases} q_{m-1}, & i < s_{m-1}-1 \\ \frac{i}{n} \sum_{j=i+1}^{n} \frac{1}{j-1} \sum_{t=j}^{n} \frac{1}{t} + \sum_{j=i+1}^{n} \frac{i}{j(j-1)} V_j^{(m-2)}, & i \ge s_{m-1}-1 \end{cases}$$

with the interpretation that $V_i^{(0)} \equiv 0$.

2.2 Asymptotic results

It is of interest to investigate the asymptotic behaviors of s_k, $1\le k\le m$, and q_m as n tends to infinity. To do this, we here employ an intuitive approach of approximating the infinite sum by the corresponding integral. When m=1, $G_i^{(1)}$ is a Riemann approximation to the integral

$$G^{(1)}(x) = \int_x^1 \frac{dy}{y} - \int_x^1 \frac{dy}{y}\int_y^1 \frac{dz}{z} = -\frac{(2+\log x)\log x}{2}. \tag{8}$$

Thus, from (5), $s_1^* = \lim_{n\to\infty}\frac{s_1}{n} = e^{-2}$ is obtained as a unique root $x\in(0, 1)$ of the equation $G^{(1)}(x) = 0$.

Define in general $s_k^* = \lim_{n\to\infty}\frac{s_k}{n}$. Then, in a similar way, we can obtain s_k^* for $k\geq 2$ successively as a unique root $x\in(0, s_{k-1}^*)$ of the equation

$$G^{(k)}(x) = 0, \tag{9}$$

if $G^{(k)}(x)$, $0<x<1$, are defined recursively as

$$G^{(k)}(x) = G^{(1)}(x) + \int_{\max(x,\, s_{k-1}^*)}^1 \frac{1}{y} G^{(k-1)}(y)\, dy\ , \qquad k\geq 2 \tag{10}$$

starting with $G^{(1)}(x)$ (note that $G_i^{(k)}$ is a Riemann approximation to $G^{(k)}(x)$ if one lets $i/n\to x$ as $n\to\infty$).

From (9) and (10), s_k^* is a root of the equation

$$G^{(1)}(x) = -\int_{s_{k-1}^*}^1 \frac{1}{y} G^{(k-1)}(y)\, dy\,,$$

or equivalently, from (8)

$$s_k^* = \exp\left\{-\left(1+\sqrt{1+2\int_{s_{k-1}^*}^1 \frac{G^{(k-1)}(y)}{y}\,dy}\right)\right\}. \tag{11}$$

Lemma 2.1

Define, for a positive integer $k\geq 1$,

$$A_{k,\,i} = \int_{s_{k-i}^*}^1 \frac{(\log x)^i}{x} G^{(k-i)}(x)\, dx, \qquad 0\leq i\leq k-1$$

$$a_{k,\,i} = \int_{s_{k-i}^*}^1 \frac{(\log x)^i}{x} G^{(1)}(x)\, dx, \qquad 0\leq i\leq k-1.$$

Then $A_{k,i}$ satisfies the following recursive relation

$$A_{k,i} = a_{k,i} + \left(\frac{1}{i+1}\right)\left[A_{k,i+1} - \left(\log s^*_{k-i}\right)^{i+1} A_{k-i-1,0}\right], \tag{12}$$

with the interpretation that $A_{k,k} = 0$, $k \ge 0$.

For simplicity, let $A_{k,0}$ be denoted by A_k. Then the repeated use of (12) immediately gives the following recursive relation of A_k.

Lemma 2.2

A_k, $k \ge 1$ satisfies the following recursive relation

$$A_k = \sum_{i=1}^{k} \left[\frac{a_{k,k-i}}{(k-i)!} - \frac{\left(\log s^*_i\right)^{k-i+1}}{(k-i+1)!} A_{i-1}\right].$$

Let N_k, $k \ge 1$ be defined as

$$N_k = -\left(1 + \sqrt{1 + 2A_{k-1}}\right).$$

Then, from (11)

$$s^*_k = \exp(N_k)$$

and we have the following lemma.

Lemma 2.3

N_k, $k \ge 1$ satisfies the following recursive relation

$$N_k = -\left[1 + \sqrt{1 - 2\sum_{i=1}^{k-1} \frac{\{(k-i+2)+(k-i+1)N_i\}(N_i)^{k-i+1}}{(k-i+2)!}}\right]. \tag{13}$$

Recursive formula (1) is an immediate consequence from (13). From (1) we successively have

$$s^*_1 = \exp\{-2\} \approx 0.1353$$

$$s^*_2 = \exp\left\{-\left(1 + \sqrt{\frac{7}{3}}\right)\right\} \approx 0.0799$$

$$s_3^* = \exp\left\{-\left(1+\tfrac{1}{3}\sqrt{15+14\sqrt{\tfrac{7}{3}}}\right)\right\} \approx 0.0493$$

$$s_4^* = \exp\left\{-\left(1+\sqrt{\tfrac{31}{45}+\tfrac{2}{81}\left(15+14\sqrt{\tfrac{7}{3}}\right)^{3/2}}\right)\right\} \approx 0.0311.$$

See Table 1 for s_5^* and s_{10}^* (c=0).

Concerning the expected payoff, we have the following lemma.

Lemma 2.4

Let $q_m^* = \lim_{n\to\infty} q_m$ for $m\geq 1$. Then $q_m^* = -\sum_{k=1}^{m} s_k^* \log s_k^*$.

Numerical values of the first four q_m^* are $q_1^* = 0.2707$, $q_2^* = 0.4725$, $q_3^* = 0.6208$, $q_4^* = 0.7287$. See Table 2 for q_5^* and q_{10}^* (c=0).

2.3 Multiple choice secretary problem with a random number of objects

It can be shown that the multiple choice duration problem with a known number of n objects is equivalent to the multiple choice (best-choice) secretary problem with an unknown number of objects having a uniform distribution on {1,2,...,n} in the sense that the optimal strategies and the expected payoffs are the same.

3. Multiple choice duration problem with an acquisition cost

In this section, the multiple choice duration problem is generalized by imposing a constant acquisition cost c(>0) each time an object is chosen.

3.1 The finite horizon problem

We treat the m choice duration problem with an acquisition cost c. Let the state of the process be defined as in Section 2, and let also $W_i^{(k)}$, $U_i^{(k)}$ and $V_i^{(k)}$ be defined similarly as the expected additional net payoff under an optimal strategy starting from state (i,k), $1\leq i\leq n$, $1\leq k\leq m$. Then the optimality equations (2)-(4) still hold if (3) is replaced by

$$U_i^{(k)} = -c + \frac{1}{n}\sum_{j=i}^{n}\frac{i}{j} + \sum_{j=i+1}^{n}\frac{i}{j(j-1)}W_j^{(k-1)}, \qquad 1\leq k\leq m.$$

It is easy to see that

$$U_i^{(1)} = -c + \frac{1}{n}\sum_{j=i}^{n}\frac{i}{j}$$

is unimodal with mode at the value

$$K(n) = \min\left\{i : \sum_{j=i+1}^{n}\frac{1}{j} \leq 1\right\}$$

Then, if c is large so as to make $U^{(1)}_{K(n)} < 0$, we do not choose a candidate no matter when it appears. Thus we consider only the case $U^{(1)}_{K(n)} \geq 0$, namely,

$$c \leq \frac{K(n)}{n} \sum_{j=K(n)}^{n} \frac{1}{j}. \tag{14}$$

Let $b(n) = \max\left\{i : U_i^{(1)} \geq 0\right\}$. Then it goes without saying that the optimal strategy selects no object after time b(n), and hence our attention can be concentrated on the candidates that appear no later than b(n).

Theorem 3.1

For the m choice duration problem with the cost condition (14), there exists a sequence of integer-valued critical numbers $(s_1, s_2, \ldots, s_m)$ such that, whenever there remain k choices yet to be made, then the optimal strategy immediately selects a candidate if it appears after or on time s_k, but no later than b(n). Moreover, s_k is non-increasing in k and determined by

$$s_k = \min\left\{i \leq b(n) : G_i^{(k)} \geq 0\right\},$$

where $G_i^{(k)}$, $1 \leq i \leq b(n)$, $1 \leq k \leq m$, is defined recursively as

$$G_i^{(k)} = G_i^{(1)} + \sum_{j=\max(i+1,\, s_{k-1})}^{b(n)} \frac{1}{j-1} G_j^{(k-1)}, \qquad k \geq 2$$

starting with

$$G_i^{(1)} = \sum_{j=i}^{n} \frac{1}{j} - \sum_{j=i+1}^{b(n)} \frac{1}{j-1} \sum_{t=j}^{n} \frac{1}{t} - \frac{cn}{b(n)}.$$

3.2 Asymptotic results

Lemma 3.1

When $c \leq e^{-1}$, s_k^* satisfies the following recursive relation

$$s_k^* = \exp\left[-\left\{1 + \sqrt{(1+\log\beta)^2 - 2\sum_{i=1}^{k-1} \frac{[(k-i+2)B_{k+1,\,i} + (k-i+1)B_{k+2,\,i}]}{(k-i+2)!}}\right\}\right].$$

Let $\gamma = 1 + \log\beta$. Then from Lemma 3.1 we can calculate s_k^* successively as follows.

$$s_1^* = \exp\{-(1+\gamma)\}$$

$$s_2^* = \exp\left\{-\left(1 + \gamma\sqrt{1 + \frac{4}{3}\gamma}\right)\right\}$$

$$s_3^* = \exp\left\{-\left[1+\gamma\sqrt{1+\frac{2}{3}\gamma\left\{1+\left(1+\frac{4}{3}\gamma\right)^{3/2}\right\}}\right]\right\}.$$

Let q_m^*, $m\geq 1$, be the expected net payoff for the m choice duration problem when n tends to infinity. Then we have

Lemma 3.2

When $c\leq e^{-1}$, we have for $m\geq 1$ $q_m^* = -\left(\sum_{k=1}^{m} s_k^* \log s_k^* + mc\right)$.

Table 1

The asymptotic critical number s_m^* for some values of m and c

c	β	s_1^*	s_2^*	s_3^*	s_5^*	s_{10}^*	$s_\infty^*(=\beta')$
0.0	1.0000	0.1353	0.0799	0.0493	0.0199	0.0024	0.0000
0.1	0.8942	0.1513	0.0990	0.0698	0.0416	0.0281	0.0280
0.2	0.7717	0.1754	0.1294	0.1047	0.0839	0.0787	0.0787
0.3	0.6130	0.2208	0.1898	0.1761	0.1690	0.1684	0.1684

Table 2

The asymptotic expected net payoff q_m^* for some values of m and c

c	q_1^*	q_2^*	q_3^*	q_5^*	q_{10}^*	q_∞^*
0.0	0.2707	0.4725	0.6208	0.8066	0.9656	1.0000
0.1	0.1858	0.3147	0.4005	0.4871	0.5195	0.5197
0.2	0.1053	0.1700	0.2062	0.2322	0.2363	0.2363
0.3	0.0335	0.0489	0.0547	0.0569	0.0570	0.0570

Lemma 3.3

$$q_\infty^* = \left(\beta - \beta'\right)\left(1 - \frac{c^2}{\beta\beta'}\right)$$

4. Multiple choice duration problem with a replacement cost

Thus far we have implicitly assumed that the objects, once chosen, are possessed until the process terminates. Instead, in this section, we are allowed to possess only one object at a time and a constant cost d(>0) is incurred each time replacement takes place.

4.1 The finite horizon problem

We treat the m choice duration problem with a replacement cost d (the problem is here referred to as the m choice problem if we are allowed to make replacement of the objects up to m-1 times, m≥2). Let the state of the process be defined as in Section 2, and let also $W_i^{(k)}$, $U_i^{(k)}$ and $V_i^{(k)}$ be defined similarly as the expected additional net payoff under an optimal strategy starting from state (i,k), $1 \le i \le n$, $1 \le k \le m$. Then the optimality equations (2)-(4) still hold if (3) is replaced by

$$U_i^{(k)} = -d + \frac{1}{n}\sum_{j=i}^{n} \frac{i}{j} + \sum_{j=i+1}^{n} \frac{i}{j(j-1)} W_j^{(k-1)}, \qquad 1 \le k \le m-1$$

$$U_i^{(m)} = \frac{1}{n}\sum_{j=i}^{n} \frac{i}{j} + \sum_{j=i+1}^{n} \frac{i}{j(j-1)} W_j^{(m-1)}.$$

Observe that, once the first choice is made, our problem reduces to the m-1 choice problem with an acquisition cost d. Thus the main concern of this problem is to determine when to make the first choice.

If $d > \frac{K(n)}{n} \sum_{j=K(n)}^{n} \frac{1}{j}$, where K(n) is as defined in Section 3, no replacement takes place and hence the m choice problem reduces to the one choice problem treated in Section 2.

In the case

$$d \le \frac{K(n)}{n} \sum_{j=K(n)}^{n} \frac{1}{j}, \tag{15}$$

the optimal strategy can be summarized as follows

Theorem 4.1

For the m choice duration problem with the cost condition (15), there exists a sequence of integer-valued critical numbers $(s_1, s_2, \ldots, s_{m-1}, t_m)$ such that the optimal strategy first selects a candidate that appears after or on time t_m and then it replaces the previously chosen object with a new candidate that appears after or no time s_k, but no later than c(n) if k more replacements are available, $1 \le k \le m-1$, where

$$c(n) = \max\left\{ i : U_i^{(1)} \ge 0 \right\}.$$

Moreover, $t_m \le s_{m-1}$ and s_k is non-increasing in k and these values are determined by

$$t_m = \min\left\{ i \le c(n) : G_i^{(m)} \ge \sum_{j=c(n)+1}^{n} \frac{1}{j-1} \sum_{t=j}^{n} \frac{1}{t} - \frac{dn}{c(n)} \right\}$$

$$s_k = \min\left\{ i \le c(n) : G_i^{(k)} \ge 0 \right\}, \qquad 1 \le k \le m-1 \ ,$$

where $G_i^{(k)}$, $1 \le i \le c(n)$, $1 \le k \le m$, is defined recursively as

$$G_i^{(k)} = G_i^{(1)} + \sum_{j=\max(i+1,\, s_{k-1})}^{c(n)} \frac{1}{j-1} G_j^{(k-1)}, \qquad k \ge 2$$

starting with

$$G_i^{(1)} = \sum_{j=i}^{n} \frac{1}{j} - \sum_{j=i+1}^{c(n)} \frac{1}{j-1} \sum_{t=j}^{n} \frac{1}{t} - \frac{dn}{c(n)}.$$

4.2 Asymptotic results

The cost condition (15) is reduced, as $n \to \infty$, to

$$d \le e^{-1}. \tag{16}$$

Let $\delta = \lim_{n\to\infty} \frac{c(n)}{n}$. Then, under the condition (16), δ is a unique root $x \in [e^{-1}, 1)$ of the equation $-x\log x = d$. We have the following result concerning the limiting values $s_k^* = \lim_{n\to\infty} \frac{s_k}{n}$, $k \ge 1$ and $t_m^* = \lim_{n\to\infty} \frac{t_m}{n}$.

Lemma 4.1

When $d \le e^{-1}$, t_m^* is expressed in terms of s_m^* as

$$t_m^* = \exp\left[-\left\{ 1 + \sqrt{(1+\log s_m^*)^2 - (2 + \log \delta)\log \delta} \right\} \right],$$

where s_k^*, $1 \le k \le m$, satisfies the following recursive relation

$$s_k^* = \exp\left[-\left\{ 1 + \sqrt{ (1+\log \delta)^2 - 2\sum_{i=1}^{k-1} \frac{[(k-i+2)B_{k+1,\,i} + (k-i+1)B_{k+2,\,i}]}{(k-i+2)!} } \right\} \right],$$

where

$$B_{k,\,i} = (\log s_i^*)^{k-i} - (\log \delta)^{k-i}.$$

Let $\lambda = 1 + \log \delta$. Then from Lemma 4.1

$$t_2^* = \exp\left\{ -\left(1 + \sqrt{1 + \frac{4}{3}\lambda^3} \right) \right\}$$

$$t_3^* = \exp\left\{-\left[1 + \sqrt{1 + \frac{2}{3}\lambda^3\left\{1 + \left(1 + \frac{4}{3}\lambda\right)^{3/2}\right\}}\right]\right\}.$$

Let r_m^*, $m \geq 2$, be the expected net payoff for the m choice duration problem when n tends to infinity. Then we have

Lemma 4.2

(i) When $d > e^{-1}$, $r_m^* = 2e^{-2}$.

(ii) When $d \leq e^{-1}$, $r_m^* = -\left(\sum_{k=1}^{m-1} s_k^* \log s_k^* + t_m^* \log t_m^* + (m-1)d\right)$.

Table 3

The asymptotic critical number t_m^* for some values of m and d

d	t_2^*	t_3^*	t_5^*	t_{10}^*	t_∞^*
0.1	0.0916	0.0656	0.0397	0.0270	0.0268
0.2	0.1063	0.0885	0.0725	0.0684	0.0684
0.3	0.1243	0.1186	0.1154	0.1151	0.1151

Table 4

The asymptotic expected net payoff r_m^* for some values of m and d

d	r_2^*	r_3^*	r_5^*	r_{10}^*	r_∞^*
0.1	0.4047	0.4934	0.5828	0.6166	0.6168
0.2	0.3435	0.3845	0.4146	0.4198	0.4198
0.3	0.2927	0.3017	0.3056	0.3059	0.3059

Lemma 4.3

Let δ' be the unique root $x \in (0, e^{-1}]$ of the equation $-x \log x = d$ for $d \leq e^{-1}$. Then

$$r_\infty^* = (\delta - \delta')\left(1 - \frac{d^2}{\delta\delta'}\right) - t_\infty^* \log t_\infty^* ,$$

where

$$t_\infty^* = \exp\left\{-\left[1 + \sqrt{1 - 2d\left(\frac{\delta - \delta'}{\delta\delta'}\right) + (\delta - \delta')(\delta + \delta')\left(\frac{d}{\delta\delta'}\right)^2}\right]\right\}$$

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