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近似有効解集合の性質について

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Abstract

Several concepts for ε-efficiency have been investigated. In general, the set of ε-efficient points is too large. We are interested in the concepts defined by White. Selecting the smaller kind of the sets, we discuss the relationship between the efficient set and the ε-efficient set.

1 Introduction

Let X be a linear topological space ordered by a non-trivial cone C. This cone is asumed to be convex, pointed, and $intC \neq \emptyset$. Let Z be a non-empty subset of X. For $x, y \in X$, we write $x \geq_C y$ if $x - y \in C$.

An element $v \in Z$ is called efficient w.r.t. C iff $x \geq_C v$ if there is $x \in Z$ such that $v \geq_C x$.

An element $v \in Z$ is called weak efficient w.r.t. C iff there is no $x \in Z$ such that $v - x \in intC$.

We denote the set of all efficient points resp. weak efficient poins by E resp. E^w. We define the set of monotone resp. strictly monotone maps by $H^m = \{h : X \to R | h(x) \leq h(y) \ \text{ if } \ x \leq_C y\}, H^s = \{h : X \to R | h(x) < h(y) \ \text{ if } \ x \leq_C y, x \neq y\}$.

2 Several concepts of ε-efficiency

Let ε be positive.

Defintion 2.1. (Loridan [4], Helbig and Pateva [2]) An element $v \in Z$ is called L-ε-efficent w.r.t. C and q iff

$$(v - \varepsilon q - C \backslash \{\theta\}) \cap Z = \emptyset.$$

Defintion 2.2.(Helbig et al. [2]) An element $v \in Z$ is called H-ε-efficent w.r.t. C and h iff

$$h(v) \leq h(x) + \varepsilon$$

if there is $x \in Z$ such that $x \leq_C v$.

We denote the set of all L-ε-efficent points resp. H-ε-efficent points by $E_L(\varepsilon)$ resp. $E_H(\varepsilon)$. We define new concept of ε-efficiency which is modified one of [5] and [6].

Defintion 2.3. An element $v \in Z$ is called W-ε-efficent w.r.t. C and $q \in C$ iff

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$v \in x + \varepsilon q - C$

if there is $x \in Z$ such that $x \leq_C v$.

We denote the set of all $W - \varepsilon$-efficent points by $E_W(\varepsilon)$.

Remark. If $\varepsilon = 0$, then $E_L(\varepsilon) = E_W(\varepsilon) = E$.
White [6] introduced six types of ε-approximate solution set for vector maximization problems. This ε-efficient set is the extension of the smallest set in the six sets.
If $X = R^n$ with max-norm and $C = R^n_+$, This set coincides with the another ε-efficinet set defined by Tanaka [5].

Example. Let $X = R^2, C = R^2_+, Z = \{x \in R^2 | x_1 \geq 0, x_2 \geq 0\}, \varepsilon = 1/2, q = (1,1)$. Then, $E = \{(0,0)\}, E^w = \{(0,x_2)|x_2 \geq 0\} \cup \{(x_1,0)|x_1 \geq 0\}, E_L = \{(x_1,x_2)|0 \leq x_1 \leq 1/2, x_2 \geq 0\} \cup \{(x_1,x_2)|x_1 \geq 0, 0 \leq x_2 \leq 1/2,\}$.
If $h(x) = x_1 + x_2$, $f \in H^s$, $E_H = \{(x_1,x_2)|x_1 \geq 0, x_2 \geq 0, x_1 + x_2 \leq 1/2\}$. If $h(x) = x_1$, $f \in H^m$, $E_H = \{(x_1,x_2)|0 \leq x_1 \leq 1/2, x_2 \geq 0\}$.
Also, we have $E_W = \{(x_1,x_2)|0 \leq x_1 \leq 1/2, 0 \leq x_2 \leq 1/2\}$.

Proposition 2.4. $E_W(\varepsilon) \subset E_L(\varepsilon)$.

Proposition 2.5. [1] [2]

(1)Let $h \in H^m$ and $\varepsilon \leq \delta$. Then $E \subset E_H(\varepsilon) \subset E_H(\delta)$.

(2)If $h \in H^s$, then $E = \cap_{\varepsilon>0} E_H(\varepsilon)$.

(3)Let $\varepsilon \leq \delta$. Then $E \subset E_L(\varepsilon) \subset E_L(\delta)$.

(4)Let $q \in intC$, then $E^w = \cap_{\varepsilon>0} E_L(\varepsilon)$.

Proposition 2.6.

(1)Let $\varepsilon \leq \delta$. Then $E \subset E_W(\varepsilon) \subset E_W(\delta)$.

(2) Let C be closed. $E = \cap_{\varepsilon>0} E_W(\varepsilon)$.

Proposition 2.7. [2]

(1)Assume that Z is closed. Let $v_\varepsilon \in E_H(\varepsilon)$ such that $v_\varepsilon \to v$ as $\varepsilon \to 0$. If $h \in H^s$ is continuous and $v \leq_C v_\varepsilon$ for each $\varepsilon > 0$. Then, $v \in E$.

(2)Assume that $q \in intC$ and Z is closed. If $v_\varepsilon \in E_L(\varepsilon)$ such that $v_\varepsilon \to v$ as $\varepsilon \to 0$. Then, $v \in E^w$.

Proposition 2.8. Assume that $q \in intC$ and Z is closed. If $v_\varepsilon \in E_W(\varepsilon)$ such that $v_\varepsilon \to v$ as $\varepsilon \to 0$. Then, $v \in E$.

In the following, X is assumed to be normed space.

Defintion 2.9. [5] An element $v \in Z$ is called $T-\varepsilon$-efficent w.r.t. C iff

$$(v-C)\cap(Z\backslash B_\varepsilon(v))=\emptyset$$

where $B_\varepsilon(v)=\{x|\parallel x-v\parallel\leq\varepsilon\}$

We denote the set of all $T-\varepsilon$-efficent points by $E_T(\varepsilon)$.

Remark. [7] If $X=R^n$ with max-norm and $C=R^n_+$, $E_W(\varepsilon)=E_T(\varepsilon)$.

Proposition 2.10. Let $q\in C$ be $q\geq_C x$ for any $x\in B_1(\theta)$: unit ball. Then, $E_T(\varepsilon)\subset E_W(\varepsilon)$.

Proposition 2.11. [2] Assume that there is $\alpha>0$ such that for any $v\in E_H(\alpha)$,

$$S(v)=\{x\in Z|x\geq_C v\}\cap E\neq\emptyset,$$

and that $B=\{c\in C|h(c)=1\}$ is bounded base for C with $K=sup_{b\in B}\parallel b\parallel$ where $h\in\{h\in X^*|h(c)\geq 0$ for each $c\in C$ and $h(c)>0$ for each $c\in C\backslash\{\theta\}$.

Then, for $\varepsilon\leq\alpha$,

$$Haus(E_H(\varepsilon),E)\leq\varepsilon K$$

where $Haus$ means Hausdorff distance.

Remark. If the vevtor optimization problem is externally stable i.e., $Z\subset E+C$, the first assumption of the above proposition holds.

Proposition 2.12. Assume that there is $\alpha>0$ such that for any $v\in E_T(\alpha)$,

$$S(v)\cap E\neq\emptyset.$$

Then,

$$Haus(E_T(\varepsilon),E)\leq\varepsilon.$$

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