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Nonlinear Ship Motion in Waves

波浪中の船体運動にみられる非線形現象

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Abstract

Water flooding into ships led to some tragic capsizing accidents. We experimentally found nonlinear motion, including chaotic one, of a flooded ship in waves. The experimental results suggested that coupling of roll motion and flooded water dominates this nonlinear phenomenon. We derive a mathematical model for this coupled motion and study bifurcation of periodic solutions. The results show that the nonlinearly coupled system can produce complicated bifurcation phenomena and that chaotic solutions are generated after periodic solutions become unstable with the Neimark-Sacker bifurcation.

1 Introduction

The passenger ferry *Estonia* capsized due to water flooding onto the vehicle deck in the Baltic sea in September, 1994, with heavy loss of lives. The ferry *Estonia* met the existing safety rule which is based on a static stability analysis. Why did such a safe ship capsize?

We carried out some experiments to examine dynamic stability of a flooded ship in waves[1]. The experimental results showed that a flooded ship could exhibit nonlinear roll motion in regular waves, and that coupling of roll motion and flooded water dominates this nonlinear response. In addition, we experimentally investigated variation of roll motion of a flooded ship with the wave height using a box-shaped model. The results demonstrated complicated bifurcation phenomena including chaotic motion.

Nonlinear dynamical approaches have been applied to a mathematical model for nonlinear ship motion[2-5]. We derive a mathematical model of the 1st order ODE form for the coupled motion of roll and flooded water. In order to understand mechanism of nonlinear response of a flooded ship in waves, we examine bifurcation of solutions of this model.

2 Experiments

Figure 1 shows a general view of a box-shaped model (the length L=0.92m, the breadth B=0.45m, the draft d=0.11m, the freeboard f_r=0.02m, the displacement W=45.54kg, the natural period of roll motion T_{nr}=1.54sec.) used in experiments[1]. We measured roll, sway and heave motion in waves of constant height and period using potentiometers.

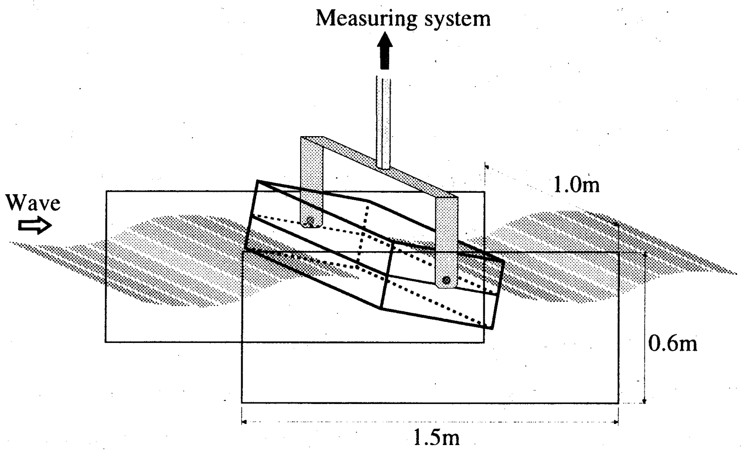
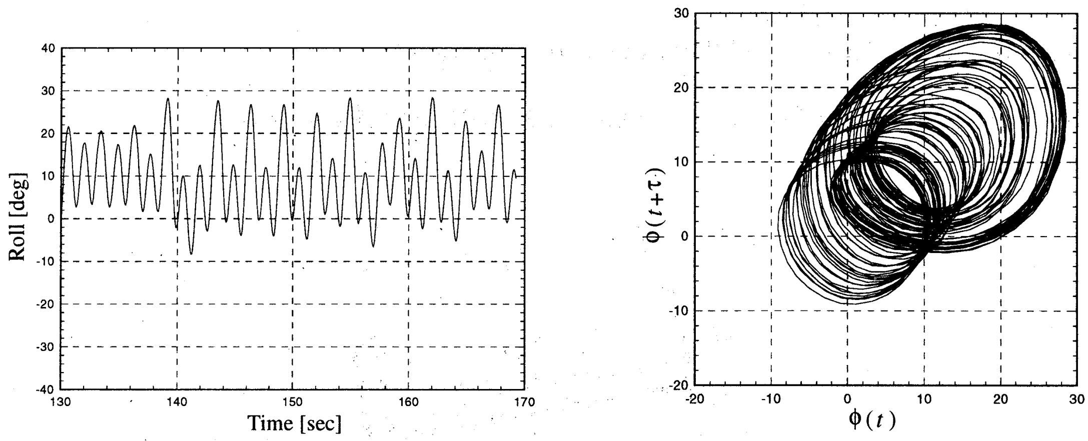


Fig.1 Experiments using a box-shaped model

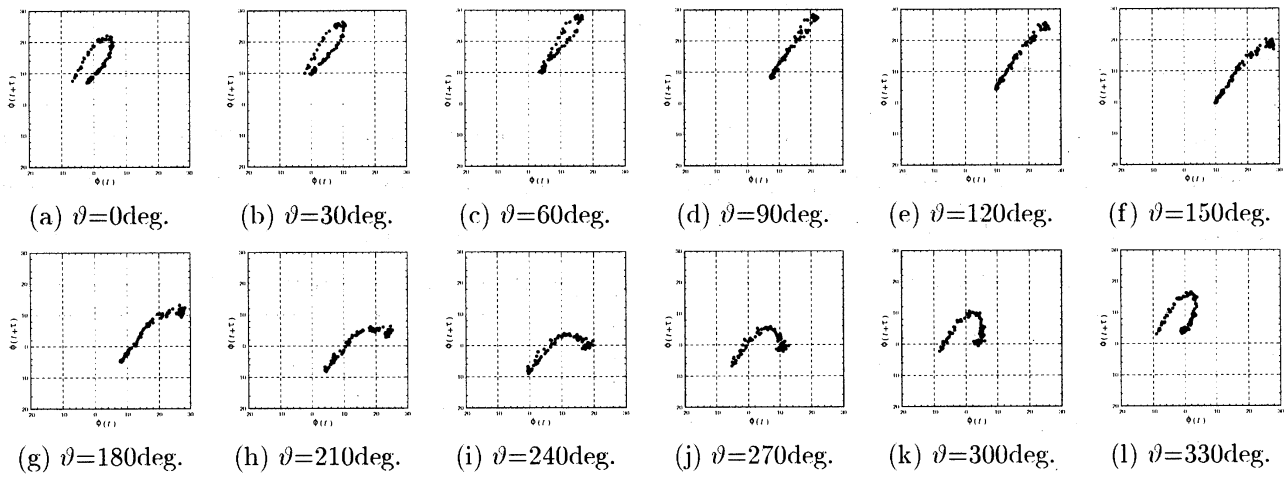


(a) Time series (b) Reconstructed attractor

Fig.2 Measured roll motion of the box-shaped model

(Wave height H=18.2cm, wave freq. f=0.7Hz, amount of water inside the ship w=5kg)

Figure 2 shows an example of measured roll angle ϕ under the conditions of the wave frequency f=0.7Hz, the wave height H=18.2cm, and the amount of water inside the ship w=5kg. The attractor in fig.2(b) is reconstructed by using delay coordinates $(\phi(t), \phi(t+\tau))$ with the delay time $\tau = T/4$ (T: wave period).



(a) ϑ=0deg. (b) ϑ=30deg. (c) ϑ=60deg. (d) ϑ=90deg. (e) ϑ=120deg. (f) ϑ=150deg.

(g) ϑ=180deg. (h) ϑ=210deg. (i) ϑ=240deg. (j) ϑ=270deg. (k) ϑ=300deg. (l) ϑ=330deg.

Fig.3 Stroboscopic plots of fig.2

(ϑ; phase of incident waves)

Figure 3 displays the stroboscopic plots of $(\phi(t),\ \phi(t+\tau))$ on Poincaré sections with ϑ=0, 30, 60, $\cdots$, 330 deg. where ϑ denotes the phase of the incident waves. We can clearly see stretching, folding, and compressing process. In addition, we calculated Liapunov exponents of this data, and found that the maximum one is positive. These results indicate that the measured roll motion in fig.2 is chaotic. We also found complicated bifurcation phenomena with changing wave heights.

3 Mathematical Model and Bifurcation Analyses

3.1 Modelling of the Coupled Motion of Roll and Flooded Water

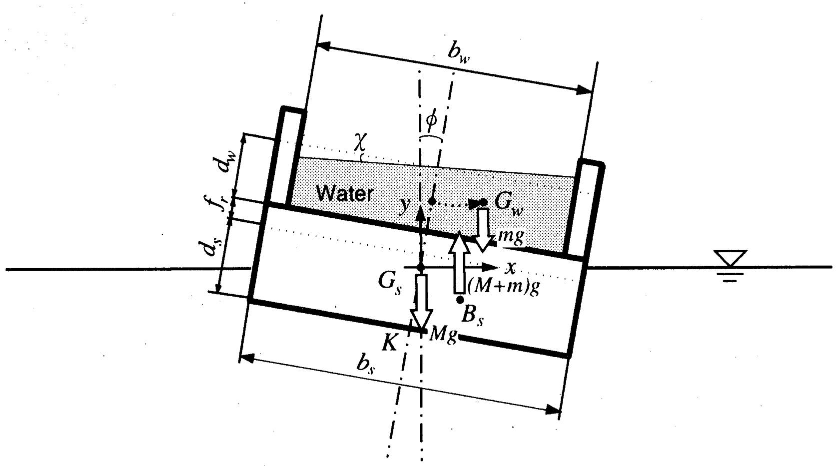


Fig.4 Illustration of two-dimensional motion of a flooded box-shaped ship in regular waves

(ϕ; roll angle of a ship, χ; slope of the surface of flooded water, b_s; breadth of a ship, b_w; breadth of a vehicle deck, d_s; draft, f_r; freeboard, d_w; depth of flooded water, G_s; center of gravity of a ship, G_w; center of gravity of flooded water, B_s; center of buoyancy of a ship)

From observations of the experiments, it was found that the nonlinear phenomena were dominated by coupled motion of roll and flooded water in waves. Then we derived a mathematical model for the coupled motion in regular waves, as shown in fig.4, assuming (1) coupling of roll motion and flooded water is dominant, and sway and heave modes can be neglected, (2) the surface of flooded water is flat with the slope χ, (3) the motion of flooded water can be approximated by that of a material particle located at the center of gravity G_w, (4) the forcing roll moment varies sinusoidally with the same angular frequency as the incident waves Ω, and (5) the damping moments on the ship and flooded water vary linearly with $\dot{\phi}$ and $\dot{\chi}$ ($\dot{}$ $=d/dt$), respectively. Take the coordinates, as shown in fig.4, of which the x- and the y- axis are defined to the horizontal and the vertically upward directions, respectively, and the origin is set at the center of gravity of the ship G_s. On the above assumptions, the kinetic energy K, the potential energy P, and the rate of energy dissipation D can be expressed as

$$
\begin{aligned}
&K_s = \frac{1}{2} M \kappa^2 \dot{\phi}^2 \ , \ K_w = \frac{1}{2} m \left(\dot{x_{G_w}}^2 + \dot{y_{G_w}}^2 \right) \ , \\
&P_s = -(M+m) g y_{B_s} \ , \ P_w = m g y_{G_w} \ , \\
&P_e(\phi, t) = -\phi \{ A_0 + A_1 \sin(\Omega t + \psi) \} \ , \\
&D = \frac{1}{2} \nu_s \dot{\phi}^2 + \frac{1}{2} \nu_w \dot{\chi}^2 \ ,
\end{aligned}
\tag{1}
$$

where the subscripts $_s$, $_w$, and $_e$ denote the ship, the flooded water, and the forcing roll moment, M and m the masses of the ship and of the flooded water, κ the radius of gyration, g the gravitational acceleration, $\boldsymbol{x}_{G_w} = (x_{G_w}, y_{G_w})$ the location of the center of gravity of the flooded water G_w, $\boldsymbol{x}_{B_s} = (x_{B_s}, y_{B_s})$ the location of the center of buoyancy of the ship B_s, $A_0 + A_1 \sin(\Omega t + \psi)$ the forcing roll moment, and ν the damping coefficent, respectively. We can obtain model equations for the coupled motion by sutstituting eq.(1) into Lagrange's equations of motion

$$
\begin{aligned}
&\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) - \frac{\partial L}{\partial \phi} + \frac{\partial D}{\partial \dot{\phi}} = 0 \ , \\
&\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\chi}}\right) - \frac{\partial L}{\partial \chi} + \frac{\partial D}{\partial \dot{\chi}} = 0 \ ,
\end{aligned}
\tag{2}
$$

where the Lagrangian $L = K - P$, $K = K_s + K_w$, and $P = P_s + P_w + P_e$, respectively. On the above assumptions, $\boldsymbol{x}_{B_s}$ and $\boldsymbol{x}_{G_w}$ correspond to the center of the cross-section of the ship under the still water surface and that of flooded water, respectively. Thus the necessary coordinates in eq.(1) can be geometrically determined. In addition, we can express the model equations of the first order form $d\boldsymbol{x}/dt = \boldsymbol{F}(t, \boldsymbol{x}, \boldsymbol{\sigma})$ where $\boldsymbol{x} = (\phi, \dot{\phi}, \chi, \dot{\chi})$ and $\boldsymbol{\sigma}$ denotes a set of parameters.

3.2 Bifurcation analyses

The mathematical model produces complicated bifurcation phenomena. Then we obtained some 2-parameter bifurcation diagrams in the (Ω, A_1) plane using the Newton method[6]. Figure 5 shows bifurcation curves of the 'period 1' and 'period 2' solutions when the ratio of the amount of flooded water to the total weight of the ship w/W=0.19 and the geometrical conditions of the ship are set to almost the same as a ferry model used in the experiments[1]. We can see that the 'period 1' and 'period 2' solutions coexist in some regions. Figure 6 shows the phase portraits of them when Ω=0.9 and A_1=0.02. This coexistence was also found in the experiments[1].

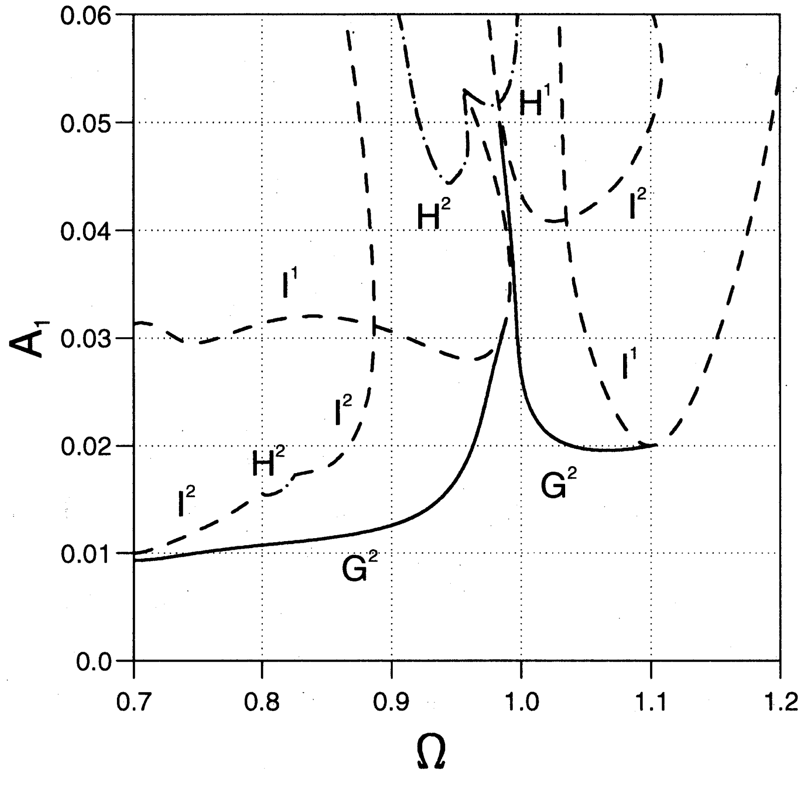
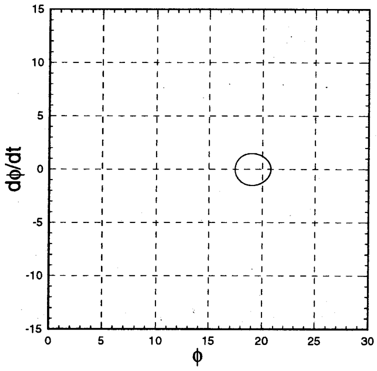
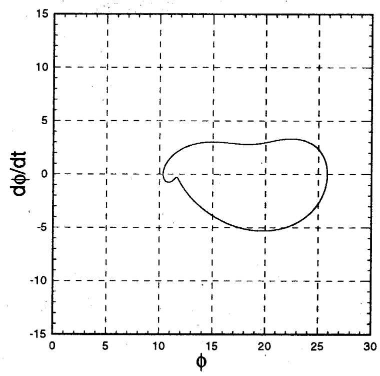


Fig.5 Bifurcation diagram, Case 1
(Geometrical conditions of the ship are close to the same as the ferry model[1]. Ω and A_1 are represented in the real ship scale. G^N: saddle-node (solid line), I^N: period doubling (dashed line), and H^N: Neimark-Sacker (dash-dotted line). N: 'period N'.)

Figure 7 shows bifurcation curves of the 'period 1', 'period 2' and 'period 3' solutions when the ratio of the amount of flooded water to the total weight of the ship w/W=0.15 and the geometrical conditions of the ship are set to almost the same as the box-shaped model used in the experiments[1]. Figure 8 shows variation of the stroboscopic plots of the roll angle ϕ at $\vartheta(=\Omega t+\psi)$=0 and variation of the Liapunov exponents, and an example of the phase portrait of the chaotic solution, when Ω is fixed to 0.45 and A_1 is increased. In fig.7, chaotic solutions were found in the closed curves of the Neimark-Sacker bifurcation of the 'period 1' and 'period 2'.



(a) Period 1



(b) Period 2

Fig.6 Coexistence of 'period 1' and 'period 2'
(Case 1. Ω=0.9, A_1=0.02 in fig.5)

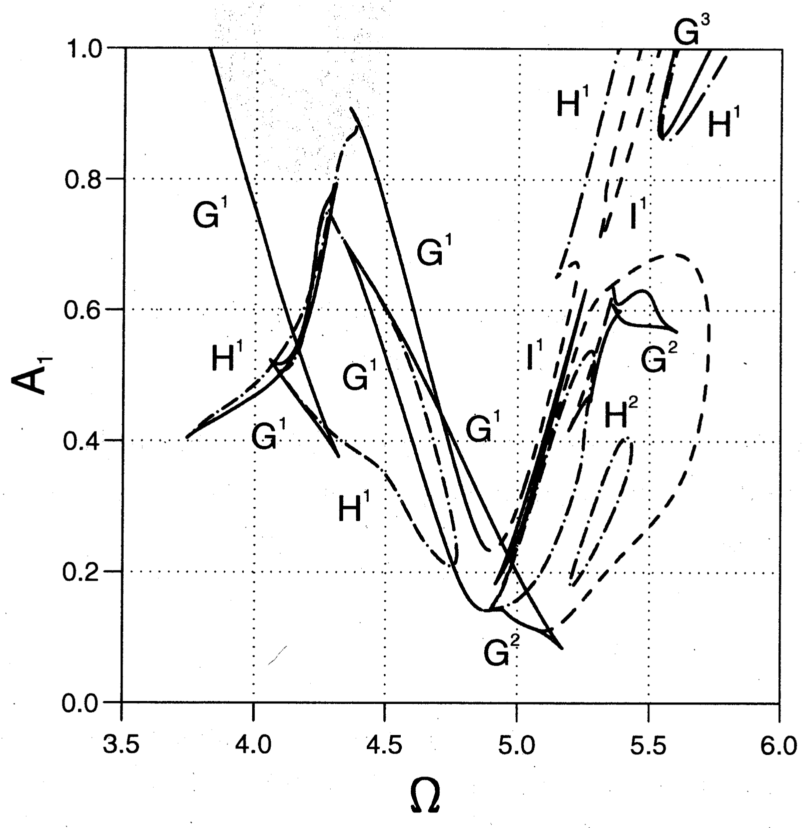
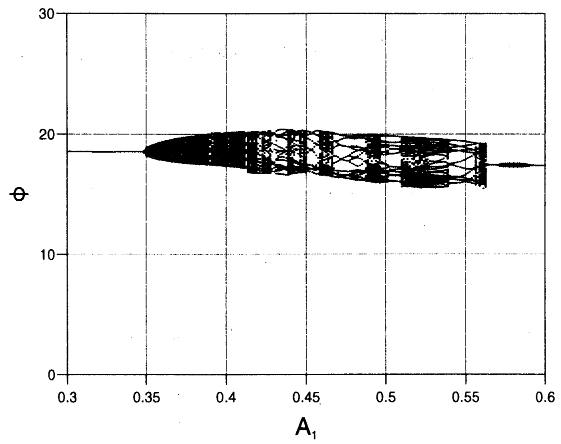
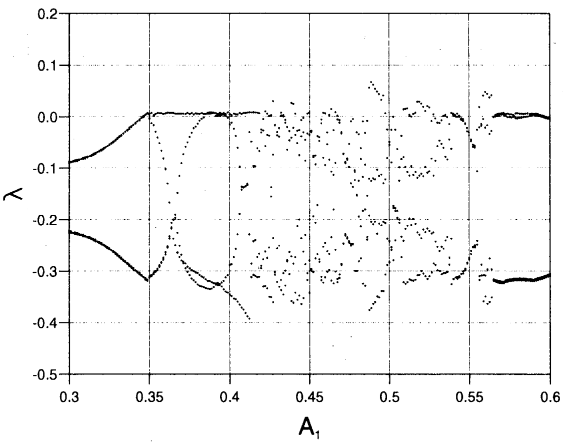


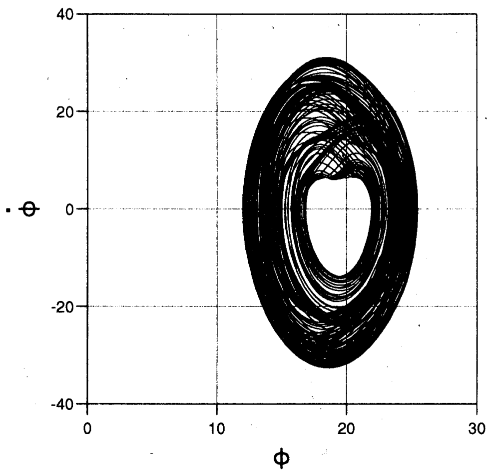
Fig.7 Bifurcation diagram, Case 2
(Geometrical conditions of the ship are close to the box-shaped model[1]. Ω and A_1 are represented in the experimental scale. G^N: saddle-node (solid line), I^N: period doubling (dashed line), and H^N: Neimark-Sacker (dash-dotted line). N: 'period N'.)



(a) Variation of stroboscopic plots of the roll angle $\phi(t_0+mT)$ (T: wave period, m=1,2,$\cdots$,100)



(b) Variation of the Liapunov exponents



(c) Phase portrait of the chaotic solution at A_1=0.51875

Fig.8 Variation of the stroboscopic plots of the roll angle ϕ, the Liapunov exponents λ, and the phase portrait of the chaotic solution (Case 2. Ω=4.5 in fig.7)

4 Conclusions

Some experimental works have demonstrated that a flooded ship can exhibit nonlinear roll motion, including chaotic one, in waves. Observations of the experiments suggested that nonlinearly coupled motion of roll and flooded water dominates these complicated phenomena. Then we derived the mathematical model for the coupled motion and investigated bifurcation of periodic solutions. The results showed that chaotic solutions were found in the closed curves of the Neimark-Sacker bifurcaiton. In order to understand mechanism of nonlinear response found in experiments, we need further study on bifurcation using this model.

References

[1] Murashige, S. and Aihara, K. : Experimental Study on Chaotic Motion of a Flooded Ship in Waves, Proc. R. Soc. Lond. A, [to be published].

[2] Virgin, L.N. : The Nonlinear Rolling Response of a Vessel including Chaotic Motions leading to Capsize in Regular Seas, Applied Ocean Research, Vol.9, No.2, 1987, pp.89–95.

[3] Soliman, M.S. and Thompson, J.M.T. : Transient and Steady State Analysis of Capsize Phenomena, Applied Ocean Research, Vol.13, No.2, 1991, pp.82–92.

[4] Thompson, J.M.T. : Designing against Capsize in Beam Seas: Recent Advances and New Insights, Appl. Mech. Rev., Vol.50, 1997, pp.307–325.

[5] Falzarno, J.M., Shaw, S.W., and Troesch, A.W. : Application of Global Methods for Analyzing Dynamical Systems to Ship Rolling Motion and Capsizing, Intl. J. Bifurcation and Chaos, Vol.2, 1992, pp.101–115.

[6] Kawakami, H. : Bifurcation of Periodic Responses in Forced Dynamic Nonlinear Circuits: Computation of Bifurcation Values of the System Parameters, IEEE Trans. Circuits and Systems, Vol.CAS-31, 1984, pp.248–260.