数理解析研究所講究録
1090 巻 1999 年 100-109

Irregularities on hyperlanes of holonomic $\mathcal{D}$-module (especially defined by confluent hypergeometric partial differential equations)

お茶の水女子大学理学部数学科　真島　秀行
(Hideyuki Majima, Ochanomizu University)

1 Intoduction

In this expository paper, I will explain the irregularity at a singular point of differential equation. At first, I will give you a review of study on ordinary linear differential equations. Secondly, I will talk about holonomic $\mathcal{D}$-modules, especially, Humbert confluent hypergeometric differential modules in m variables.

2 Irregularity of holonomic $\mathcal{D}$-module defined by an ordinary differential operator.

Consider a linear ordinary differential operator with coefficients in holomorphic functions at the origin in the Riemann Sphere:

$$Pu = (\sum_{i=0}^{m} a_i(x)(d/dx)^i)u.$$

where a_m is supposed not to be identically zero. Let $\mathcal{O}$ and $\hat{\mathcal{O}}$ be the ring of convergent power-series and the ring of formal power-series in x, respectively. Then, we see the following isomorphism of linear spaces due to Deligne (cf. [23], etc.) :

$$H^1(S^1, \mathcal{K}er(P : \mathcal{A}_0)) \simeq \mathrm{Ker}(P; \hat{\mathcal{O}}/\mathcal{O}),$$

for, from the existence theorem of asymptotic solutions due to Hukuhara (cf. [26]) (and other many contributers), we have the short exact sequence

$$0 \to \mathcal{K}er(P : \mathcal{A}_0) \to \mathcal{A}_0 \xrightarrow{P} \mathcal{A}_0 \to 0,$$

from which, we get the exact sequence,

$$0 \to H^1(S^1, \mathcal{K}er(P : \mathcal{A}_0)) \to H^1(S^1, \mathcal{A}_0)(= \hat{\mathcal{O}}/\mathcal{O}) \xrightarrow{P} H^1(S^1, \mathcal{A}_0)(= \hat{\mathcal{O}}/\mathcal{O}) \to 0.$$

The dimension is finite and is equal to

$$\begin{aligned} i_0(P) &= \sup\{i - v(a_i) : i = 0, ..., m\} - (m - v(a_m)) \\ &= (v(a_m) - m) - \inf\{v(a_i) - i : i = 0, ..., m\}, \end{aligned}$$

which is called the irregularity by Malgrange [17], [18], the invariant of Fuchs by Gérard-Levelt [3], [4] or the irregular index by Komatsu (in a private communication), where,

$$v(a) = \sup\{p : x^{-p}a(x) \quad \textit{is} \quad \textit{holomorphic} \quad \textit{at} \quad \textit{the} \quad \textit{origin.}\}.$$

Remark 0: Let $\mathcal{K}$, $\hat{\mathcal{K}}$ and $\mathcal{E}$ be the ring of the ring of convergent Laurent series with finite negative order terms, the ring of formal, the ring of formal Laurent series with finite negative order terms and the ring of convergent Laurent series, respectively. Denote by F one of $\mathcal{O}$, $\hat{\mathcal{O}}$, $\mathcal{K}$, $\hat{\mathcal{K}}$ and $\mathcal{E}$. We consider P as an operator from F to itself. Then, $\mathrm{Ker}(P; F)$ and $\mathrm{Coker}(P; F)$ are finite dimensional, and has a index $\chi(P; F) = \dim_C \mathrm{Ker}(P; F) - \dim_C \mathrm{Coker}(P; F)$, which can be calculated as follows:

$$\begin{aligned} \chi(P; \mathcal{O}) &= m - v(a_m), \\ \chi(P; \hat{\mathcal{O}}) &= \sup\{i - v(a_i) : i = 1, ..., m\}, \\ \chi(P; \mathcal{K}) &= m - v(a_m) - \sup\{i - v(a_i) : i = 1, ..., m\}, \\ \chi(P; \hat{\mathcal{K}}) &= 0, \\ \chi(P; \mathcal{E}) &= 0. \end{aligned}$$

The quantity $i_0(P)$ is also equal to the followings [17], [18] :

$$\begin{aligned} &\chi(P; \hat{\mathcal{O}}) - \chi(P; \mathcal{O}), \\ &\chi(P; \hat{\mathcal{K}}) - \chi(\mathcal{K}), \\ &-\chi(P; \mathcal{K}), \\ &\chi(P; \hat{\mathcal{K}}/\mathcal{K}), \\ &\chi(P; \mathcal{E}) - \chi(P; \mathcal{K}), \\ &\chi(P; \mathcal{E}/\mathcal{K}), \\ &\chi(P; \mathcal{E}/\mathcal{O}) - \chi(P; \mathcal{K}/\mathcal{O}), \\ &\dim_C \mathrm{Ker}(P; \hat{\mathcal{O}}/\mathcal{O}), \\ &\dim_C \mathrm{Ker}(P; \hat{\mathcal{K}}/\mathcal{K}), \\ &\dim_C \mathrm{Ker}(P; \mathcal{E}/\mathcal{K}), \\ &\dim_C \mathrm{Ker}(P; (\mathcal{E}/\mathcal{O})/(\mathcal{K}/\mathcal{O})). \end{aligned}$$

Remark 1: If we consider a linear ordinary differential operator with coefficients in holomorphic functions at the infinity in the Riemann Sphere and we do not use the variable $t = \frac{1}{x}$, the quantity is equal to

$$\begin{aligned} i_\infty(P) &= \sup\{v'(a_i) - i : i = 0, ..., m\} - (v'(a_m) - m) \\ &= (m - v'(a_m)) - \inf\{i - v'(a_i) : i = 0, ..., m\}, \end{aligned}$$

where

$$v'(a) = \sup\{p : x^{-p}a(x) \quad is \quad holomorphic \quad at \quad the \quad infinity.\}.$$

Remark 2: We have also another important quantity associated with the linear ordinary differential operator $P = (\sum_{i=0}^{m} a_i(x)(d/dx)^i)$. At the origin, we set

$$k = \sup\{0, \frac{(v(a_m) - m) - (v(a_i) - i)}{m - i} : i = 0, ..., m - 1\},$$

and at the infinity, we set

$$k = \sup\{0, \frac{(m - v'(a_m)) - (i - v'(a_i))}{m - i} : i = 0, ..., m - 1\},$$

which is called the invariant of Katz by Gérard-Levelt [3], [4] or the order by Sibuya [28], and $k + 1$ is called the irregularity by Komatsu [9], [10]. In order to understand the importance of this quantity, see the above references and also Ramis [24], [25], Komatsu [11]. Malgrange [21]. In adding a word,

$$i_0(P) \geq k \geq \frac{i_0(P)}{m}, \quad mk \geq i_0(P) \geq k.$$

Let $\mathcal{D}_0$ be the stalk of germs of linear ordinary differential operators with holomorphic coefficients, and put $\mathcal{M}_0 = \mathcal{D}_0/\mathcal{D}_0 P$. Then, $\mathcal{M}_0$ has a projective resolution

$$0 \leftarrow \mathcal{M}_0 \leftarrow \mathcal{D}_0 \xleftarrow{P} \mathcal{D}_0 \leftarrow 0,$$

from which, by operating the functor $\mathcal{H}om_{\mathcal{D}_0}(\cdot, \mathcal{F}_0)$, we have the solution complex with values in $\mathcal{F}$ at the origin,

$$\mathcal{S}ol(\mathcal{M}_0, \mathcal{F}_0) : \mathcal{F}_0 \xrightarrow{P} \mathcal{F}_0 \rightarrow 0.$$

We have the isomorphism:

$$\mathrm{Ext}^0(\mathcal{M}_0, \mathcal{F}_0) \simeq \mathrm{Ker}(\mathcal{F}_0; P), \quad \mathrm{Ext}^1(\mathcal{M}_0, \mathcal{F}_0) \simeq \mathrm{Coker}(\mathcal{F}_0; P).$$

Therefore, the index as $\mathcal{D}$-module at the origin,

$$\chi(\mathcal{M}; \mathcal{F})_0 = \dim_C \mathrm{Ext}^0(\mathcal{M}_0, \mathcal{F}_0) - \dim_C \mathrm{Ext}^1(\mathcal{M}_0, \mathcal{F}_0),$$

is equal to the index $\chi(P;F)$, and the irregularity as $\mathcal{D}$-module at the origin,

$$\mathrm{Irr}(\mathcal{M})_0 = \chi(\mathcal{M}_0;\hat{\mathcal{O}}) - \chi(\mathcal{M}_0;\mathcal{O}),$$

is equal to the irregularity $\mathrm{Irr}(P)_0$ and

$$\mathrm{Irr}(\mathcal{M})_0 = \chi(\mathcal{M}_0;\hat{\mathcal{K}}) - \chi(\mathcal{M}_0;\mathcal{K}),$$

$$\mathrm{Irr}(\mathcal{M})_0 = \chi(\mathcal{M}_0;\mathcal{E}) - \chi(\mathcal{M}_0;\mathcal{K}),$$

$$\mathrm{Irr}(\mathcal{M})_0 = \chi(\mathcal{M}_0;\mathcal{E}/\mathcal{O}) - \chi(\mathcal{M}_0;\mathcal{K}/\mathcal{O}).$$

3 Indices of holonomic $\mathcal{D}$-modules and their irregularities

Let $\mathcal{D}$ be the sheaf of germs of linear partial differential operetors with coefficients of holomorphic functions on a manifold M and let $\mathcal{M}$ be a holonomic $\mathcal{D}$-module. The module $\mathcal{M}$ has a projective resolution

$$0 \leftarrow \mathcal{M} \leftarrow \mathcal{D}^{m_0} \xleftarrow{P_0} \mathcal{D}^{m_1} \xleftarrow{P_1} \mathcal{D}^{m_2} \xleftarrow{P_2} \cdots \xleftarrow{P_{2n-1}} \mathcal{D}^{m_{2n}} \leftarrow 0$$

from which, by operating the functor $\mathcal{H}om_{\mathcal{D}}(\cdot,\mathcal{F})$, we have the solution complex with values in $\mathcal{F}$,

$$\mathcal{S}ol(\mathcal{M},\mathcal{F}) : \mathcal{F}^{m_0} \xrightarrow{P_0^t} \mathcal{F}^{m_1} \xrightarrow{P_1^t} \cdots \xrightarrow{P_{2n-1}^t} \mathcal{F}^{m_{2n}} \to 0.$$

For a point p, the index of holonomic $\mathcal{D}$-module $\mathcal{M}$with values in $\mathcal{F}$ is defined by

$$\chi(\mathcal{M};\mathcal{F})_p = \sum_{i=0}^{2n} \dim_C (-1)^i \mathcal{E}xt^i(\mathcal{M},\mathcal{F})_p.$$

For the point p, the irregularity of holonomic $\mathcal{D}$-module $\mathcal{M}$ is defined by

$$\mathrm{Irr}(\mathcal{M})_p = \chi(\mathcal{M};\mathcal{O}_{M\hat{|}H}))_p - \chi(\mathcal{M};\mathcal{O}_{M|H})_p,$$

where $\mathcal{O}$ is the sheaf of germs of holomorphic functions on M , H is the set of singular points of $\mathcal{M}$, $\mathcal{O}_{M|H}$ is the zero-extension of the restriction of $\mathcal{O}$ on H and $\mathcal{O}_{M\hat{|}H}$ is the Zariski completion of $\mathcal{O}$ along H.

4 Holonomic $\mathcal{D}$-module defined by Humbert confluent hypergeometric partial differential equations Φ_D

In the sequel, we consider the solution complexes of holonomic $\mathcal{D}$-module defined by Humbert confluent hypergeometric partial differential equations Φ_D (derived from Lauricella F_D) and give the calculation of the cohomology groups.

We put $M=(P^1_C)^m$ and $H=\bigcup_{k=1}^m H_k$, where $H_k=P^1_C\times\cdots\times\{\infty\}\times\cdots\times P^1_C$. For a domain U included in H_k, we define

$$\mathcal{O}_{\widehat{M|H},s,A}(U)=\left\{\sum_{j\geq 0} f_j(y_k)(x_k)^{-j};\exists C>0,\forall n, s.t. \sup_{\hat{y}_k\in U}|f_n(y_k)|<CA^n\{(n-1)!\}^{s-1}\right\},$$

where $y_k=(x_1,\cdots,x_{k-1},x_{k+1},\cdots,x_m)$, $\hat{y}_k=(x_1,\cdots,x_{k-1},\infty,x_{k+1},\cdots,x_m)$. For a point $p\in H\backslash\bigcup_{k\neq l}(H_k\cap H_l)$, if $p\in H_k$ then we put

$$(\mathcal{O}_{\widehat{M|H},s,A})_p=\operatorname*{Ind\,lim}_{p\in U\subset H_k}\mathcal{O}_{\widehat{M|H},s,A}(U).$$

We define as follow:

$$\begin{aligned}(\mathcal{O}_{\widehat{M|H},s})_p &= \operatorname*{Ind\,lim}_{A>0}(\mathcal{O}_{\widehat{M|H},s,A})_p\ ,\\ (\mathcal{O}_{\widehat{M|H},(s)})_p &= \operatorname*{Proj\,lim}_{A>0}(\mathcal{O}_{\widehat{M|H},s,A})_p\ ,\\ (\mathcal{O}_{\widehat{M|H},s,A-})_p &= \operatorname*{Ind\,lim}_{0<B<A}(\mathcal{O}_{\widehat{M|H},s,B})_p\ ,\\ (\mathcal{O}_{\widehat{M|H},(s,A+)})_p &= \operatorname*{Proj\,lim}_{B>A}(\mathcal{O}_{\widehat{M|H},s,B})_p\ .\end{aligned}$$

The system of Humbert confluent hypergeometric partial differential equations Φ_D [1] is as follows:

$$\Phi_D:\ x_k\frac{\partial^2 u}{\partial x_k^2}+\sum_{l\neq k}x_l\frac{\partial^2 u}{\partial x_k\partial x_l}+(c-x_k)\frac{\partial u}{\partial x_k}-b_k u=0\quad(\text{denoted by}\, L_k u=0\,\text{for}\, k=1,\cdots,m)\ .$$

where $b_k(k=1,\cdots,m)$ and c are not non-negative integers. Note that L_k's commute with each other. We consider the $\mathcal{D}$-module $\mathcal{M}_D$ defined by Φ_D, namely we put

$$\mathcal{M}_D=\mathcal{D}/(\mathcal{D}L_1+\cdots+\mathcal{D}L_m).$$

We have a projective resolution like Koszul complex

$$0\longleftarrow\mathcal{M}_D\longleftarrow\mathcal{D}^{n_0}\xleftarrow{\nabla_0^m}\mathcal{D}^{n_1}\xleftarrow{\nabla_1^m}\cdots\xleftarrow{\nabla_{q-1}^m}\mathcal{D}^{n_q}\xleftarrow{\nabla_q^m}\cdots\xleftarrow{\nabla_{m-1}^m}\mathcal{D}^{n_m}\longleftarrow 0$$

and we have the solution complex $Sol(\mathcal{M}_D,\mathcal{F})$ with values in $\mathcal{F}$

$$\mathcal{F}^{n_0}\xrightarrow{\nabla_0^m}\mathcal{F}^{n_1}\xrightarrow{\nabla_1^m}\cdots\xrightarrow{\nabla_{q-1}^m}\mathcal{F}^{n_q}\xrightarrow{\nabla_q^m}\cdots\xleftarrow{\nabla_{m-1}^m}\mathcal{F}^{n_m}\longrightarrow 0,$$

where $n_q=\frac{m!}{q!(m-q)!}$ and ∇_q^m's are defined by the following recursive manner:

$$\nabla_0^1=L_1,\cdots,\nabla_0^m=\begin{pmatrix}L_1\\ \vdots\\ L_m\end{pmatrix},$$

$$\nabla_1^2 = ((-1)^{2-1}L_2, L_1), \cdots, \nabla_1^m = \begin{pmatrix} \nabla_1^{m-1} & 0 \\ (-1)^1 L_m I_{m-1} & \nabla_0^{m-1} \end{pmatrix}.$$

$$\nabla_q^m = \begin{pmatrix} \nabla_1^{m-1} & 0 \\ (-1)^q L_m I_{\frac{(m-1)!}{q!(m-1-q)!}} & \nabla_{q-1}^{m-1} \end{pmatrix}.$$

$$\cdots,$$

$$\nabla_{m-1}^m = ((-1)^{m-1}L_m, \nabla_{m-2}^{m-1}),$$

and we have the following

Theorem 1 . *Let $M = (P_C^1)^m$, $H = \bigcup_{k=1}^m H_k$, $p \in H \backslash \bigcup_{k,l}(H_k \cap H_l)$ be as above. The dimensions of chohomology groups of the solution complexes for the $\mathcal{D}$-module defined by Φ_D are as folow:*

(1) *If $1 \leq s < 2$,*

for $\mathcal{F} = \mathcal{O}_{\widehat{M|H},(s)}, \mathcal{O}_{\widehat{M|H},s,A-}, \mathcal{O}_{\widehat{M|H},(s,A+)}, \mathcal{O}_{\widehat{M|H},s},$

$$\dim_C \operatorname{Ext}^j((\mathcal{M}_D)_p, \mathcal{F}_p) = \begin{cases} 0, & (j = 0, 2, \cdots, m) \\ 1, & (j = 1) \end{cases}$$

(2) *If $s > 2$,*

for $\mathcal{F} = \mathcal{O}_{\widehat{M|H},(s)}, \mathcal{O}_{\widehat{M|H},s,A-}, \mathcal{O}_{\widehat{M|H},(s,A+)}, \mathcal{O}_{\widehat{M|H},s},$

$$\dim_C \operatorname{Ext}^j((\mathcal{M}_D)_p, \mathcal{F}_p) = 0, \quad (j = 0, 1, 2, \cdots, m).$$

(3) *In the case of $s = 2$,*

(i) *if $A > 1$,*

for $\mathcal{F} = \mathcal{O}_{\widehat{M|H},2,A-}, \mathcal{O}_{\widehat{M|H},(2,A+)},$

$$\dim_C \operatorname{Ext}^j((\mathcal{M}_D)_p, \mathcal{F}_p) = 0, \quad (j = 0, 1, 2, \cdots, m).$$

(ii) *if $0 < A < 1$,*

for $\mathcal{F} = \mathcal{O}_{\widehat{M|H},2,A-}, \mathcal{O}_{\widehat{M|H},(2,A+)},$

$$\dim_C \operatorname{Ext}^j((\mathcal{M}_D)_p, \mathcal{F}_p) = \begin{cases} 0, & (j = 0, 2, \cdots, m) \\ 1, & (j = 1) \end{cases}$$

(iii) *if $A = 1$,*

$$\dim_C \operatorname{Ext}^j((\mathcal{M}_D)_p, (\mathcal{O}_{\widehat{M|H},2,1-})_p) = \begin{cases} 0, & (j = 0, 2, \cdots, m) \\ 1, & (j = 1) \end{cases}.$$

$$\dim_C \mathrm{Ext}^j((\mathcal{M}_D)_p, (\mathcal{O}_{\widehat{M|H},(2,1+)})_p) = 0, \quad (j = 0, 1, 2, \cdots, m).$$

(iv) $\dim_C \mathrm{Ext}^j((\mathcal{M}_D)_p, (\mathcal{O}_{\widehat{M|H},(2)})_p) = \begin{cases} 0, & (j = 0, 2, \cdots, m) \\ 1, & (j = 1) \end{cases}.$

$\dim_C \mathrm{Ext}^j((\mathcal{M}_D)_p, (\mathcal{O}_{\widehat{M|H},2})_p) = 0, \quad (j = 0, 1, 2, \cdots, m).$

(4) $\dim_C \mathrm{Ext}^j((\mathcal{M}_D)_p, (\mathcal{O}_{\widehat{M|H}})_p) = 0, \quad (j = 0, 1, 2, \cdots, m).$

Corollary 1 . *The indexes of $\mathcal{D}$-module defined by Φ_2 are as follow:*

(1) *If* $1 \leq s < 2$,

for $\mathcal{F} = \mathcal{O}_{\widehat{M|H},(s)}, \mathcal{O}_{\widehat{M|H},s,A-}, \mathcal{O}_{\widehat{M|H},(s,A+)}, \mathcal{O}_{\widehat{M|H},s}$,

$$\mathcal{X}((\mathcal{M}_D)_p, \mathcal{F}_p) = -1.$$

(2) *If* $s > 2$,

for $\mathcal{F} = \mathcal{O}_{\widehat{M|H},(s)}, \mathcal{O}_{\widehat{M|H},s,A-}, \mathcal{O}_{\widehat{M|H},(s,A+)}, \mathcal{O}_{\widehat{M|H},s}$,

$$\mathcal{X}((\mathcal{M}_D)_p, \mathcal{F}_p) = 0.$$

(3) *In the case of* $s = 2$

(i) *if* $A > 1$,

for $\mathcal{F} = \mathcal{O}_{\widehat{M|H},2,A-}, \mathcal{O}_{\widehat{M|H},(2,A+)}$,

$$\mathcal{X}((\mathcal{M}_D)_p, \mathcal{F}_p) = 0.$$

(ii) *if* $0 < A < 1$,

for $\mathcal{F} = \mathcal{O}_{\widehat{M|H},2,A-}, \mathcal{O}_{\widehat{M|H},(2,A+)}$,

$$\mathcal{X}((\mathcal{M}_D)_p, \mathcal{F}_p) = -1.$$

(iii) *if* $A = 1$,

$$\mathcal{X}((\mathcal{M}_D)_p, (\mathcal{O}_{\widehat{M|H},2,1-})_p) = -1.$$

$$\mathcal{X}((\mathcal{M}_D)_p, (\mathcal{O}_{\widehat{M|H},(2,1+)})_p) = 0.$$

(iv) $\mathcal{X}((\mathcal{M}_D)_p, (\mathcal{O}_{\widehat{M|H},(2)})_p) = -1.$

$\mathcal{X}((\mathcal{M}_D)_p, (\mathcal{O}_{\widehat{M|H},2})_p) = 0.$

(4) $\mathcal{X}((\mathcal{M}_D)_p, (\mathcal{O}_{\widehat{M|H}})_p) = 0.$

Corollary 2 . *The irregularity* $\mathrm{Irr}((\mathcal{M}_D)_p) = 1.$

The essential parts for $k = 1$are as follow:

1. We have a formal solution

$$\hat{u}(x) = \sum_{n=1}^{\infty} \frac{(n-1)!(c-n)\cdots(c-2)}{(n-b_1)\cdots(1-b_1)} \Phi_D^{m-1}(b_2,\cdots,b_m;c-n;x_2,\cdots,x_m)(x_1)^{-n}$$

of the non-homogeneous system of partial differential equations

$$L_1\hat{u}(x) = \frac{\Phi_D^{m-1}(b_2,\cdots,b_m;c-1;x_2,\cdots,x_m)}{x_1}, \quad L_l\hat{u}(x) = 0 \ (l = 2,\cdots,m),$$

where $\Phi_D^{m-1}(b_2,\cdots,b_m;c-1;x_2,\cdots,x_m)$ is the Humbert confluent hypergeoetric function in $(m-1)$ variables with the parameter $(b_2,\cdots,b_m;c-1)$,

$$\Phi_D^{m-1}(b_2,\cdots,b_m;c-1;x_2,\cdots,x_m) = \sum_{j_2=0}^{\infty}\cdots\sum_{j_m=0}^{\infty} \frac{(b_2)_{j_2}\cdots(b_m)_{j_m}(x_2)^{j_2}\cdots(x_m)^{j_m}}{(c)_{j_2+\cdots+j_m}j_2!\cdots j_m!},$$

where $(b)_s = (b+1)\cdots(b+s-1)$.

2. If, for

$$v = \begin{pmatrix} \sum_{j=0}^{\infty} P_j^1(y_1)x_1^{-j} \\ \vdots \\ \sum_{j=0}^{\infty} P_j^m(y_1)x_1^{-j} \end{pmatrix}$$

$\nabla_1^m v = 0$, we have $\nabla_0^m(\sum_{j=0}^{\infty} f_j(y_1)x_1^{-j}) = v$, then

$$\frac{(1-b_1)\cdots(n+1-b_1)}{n!(c-n-1)\cdots(c-2)} f_{n+1}$$

$$= \sum_{j=1}^{n+1} \frac{(1-b_1)\cdots(j-1-b_1)}{(j-1)!(c--j)\cdots(c-2)} P_j^1 + \sum_{l=2}^{m} x_l \frac{\partial}{\partial x_l} \sum_{j=1}^{n} \frac{(1-b_1)\cdots(j-b_1)}{(j-1)!(c-j)\cdots(c-2)} f_j.$$

Put $F_{n+1} = \frac{(1-b_1)\cdots(n+1-b_1)}{n!(c-n-1)\cdots(c-2)} f_{n+1}$, then, for $l = 2,\cdots,m$,

$$\frac{\partial}{\partial x_l} F_{n+1} = \frac{1}{c-n-1}(x_l \frac{\partial}{\partial x_l} F_n + b_l F_n + P_n^l + x_l \frac{\partial}{\partial x_l} P_{n+1}^1),$$

and $\alpha(v) = \lim_{n\to\infty} F_n$ is well-defined as constant for $v \in (\mathcal{O}_{\widehat{M|H,s,A}}(U))^m$, where $U \subset H_1$ and, $0 < s < 2$ or ($s = 2$ and $0 < A < 1$).

参考文献

[1] Appell, P-E., and Kampé de Fériet, J.: Fonctions hypergéométriques et hypersphériques, Gauthier-Villars , (1926).

[2] Deligne, P.: Equations différentielles à points singuliers réguliers, Lecture Notes in Math., no. 163, Springer-Verlag, (1970).

[3] Gérard, R. and Levelt, A.: Mesure de l'irregularité en un point singulier d'un systeème d'equations différentielles linéaires, C.R. Acad. Sc. Paris, t. 274 (1972), pp.774-776, pp.1170-1172.

[4] Gérard, R. and Levelt, A.H.M.: Invariants mesurant l'irregularité en un point singulier des systeèmes d'equations différentielles linéaires, Ann. Inst. Fourier, Grenoble, t. 23 (1973), pp.157-195.

[5] Ishizuka, S. , On the solution complexes of $\mathcal{D}$-modules defined by confluent hypergeometric differential equations, Master's thesis Ochanomizu University, 1994.

[6] Ishizuka, S. and Majima, H. , On the solution complexes of confluent hypergeometric $\mathcal{D}$-modules, RIMS Kokyuroku 878(Singularities of Holomorphic Vector Fields and Related Topics, editedd by T. Suwa) 1994, pp10-19

[7] Kashiwara, M., Algebraic study for systems of partial differential equations, Master's thesis University of Tokyo, 1971.

[8] Komatsu, H.: On the index of ordinary differential operators, J. Fac. Sci. Univ. Tokyo, Sect. IA, Vol. 18, (1971), pp.379-398.

[9] Komatsu, H.: An introduction to the theory of hyperfunctions, Hyperfunctions and Pseudo-Differential Equatiions, in the Proceedings of a conference at Katata, 1971, edited by H. Komatsu, Lect. Note in Math. no. 287, Springer-Verlag (1973) pp.3-40.

[10] Komatsu, H.: On the regularity of hyperfunction solutions of linear ordinary differential equations with real analytic coefficients, J. Fac. Sci. Univ. Tokyo, Sect. IA, Vol. 20, (1973), pp.107-119.

[11] Komatsu, H.: Linear Ordinay Differential Equations with Gevrey Coefficients, J. Diff. Equat. Vol. 45, no. 2, (1982), pp.272-306.

[12] Majima, H.: Vanishing theorems in asymptotic analysis, Proc. Japan Acad., 59 Ser. A (1983), pp.150-153.

[13] Majima, H.: Vanishing theorems in asymptotic analysis II, Proc. Japan Acad., 60 Ser. A (1984), pp.171-173.

[14] Majima, H.: Asymptotic Analysis for Integrable Connections with Irregular Singular Points, Lect. Note in Math. no. 1075, Springer-Verlag (1984).

[15] Majima. H.: Resurgent Equations and Stokes Multipliyers for Generalized Confluent Hypergeometric Differential Equations of the Second Order, in the Proceedings of Hayashibara Forum'90 International Symposium on Special Functions, ICM Satellite Conference Proceedings, Springer-Verlag (1991), pp.222 – 233.

[16] Majima. H., Howls, C. J. and Olde Daalhuis, A. B.: Vanishing Theorem in Asymptotic Analysis III (to appear in "Structure of Solutions of Differential Equations" edited by T.Kawai and M. Morimoto, World Science, May 1996))

[17] Malgrange, B.: Remarques sur les points singuliers des équations différentielles, C.R. Acad. Sc. Paris, t. 273 (1971), pp.1136-1137.

[18] Malgrange, B.: Sur les points singuliers des équations différentielles, l'Enseignment Math.,Vol. 20, (1974), pp.147-176.

[19] Malgrange, B.: Remarques sur les Equations Différentielles à Points Singuliers Irréguliérs, in Equations Différentielles et Systèmes de Pfaff dans le Champ Complexe edited by R. Gérard and J.-P. Ramis, Lecture Notes in Math., No.712, Springer-Verlag, (1979), pp.77-86.

[20] Malgrange, B.: La classification des connexions irréguliérs à une variable, in Sém. Ec. Norm. Sup. 1979-1982, Progr. in Math., vol. 37, Birkhäuser (1983), pp.381-400.

[21] Malgrange, B.: Equations différentielles à coefficients polynomiaux, Progr. in Math., vol. 96, Birkhäuser (1991).

[22] Malgrange, B. and Ramis, J.-P.: Functions Multisommables, Ann Inst. Fourier, Vol. 42, no.1-2 (1992), pp.353-368.

[23] Ramis, J.-P.: Devissage Gevery, Astérisque, no. 59-60, (1978), pp.173-204.

[24] Ramis, J.-P.: Théorèmes d'indices Gevrey pour les équations différentielles ordinaires, Memoires, A.M.S., Vol. 296, (1984).

[25] Ramis, J.-P.: Les séries k-sommables et leurs applications, in Proceedings, Les Houches 1979, Complex Analysis, Microlocal Calculus and Relativeistic Quantum Theory, Lect. Notes in Phys. vol. 126, Springer-Verlag (1980), pp.178-199.

[26] Ramis, J.-P. and Sibuya, Y.: Hukuhara domains and fundamental existence and uniqueness theorems for asymptotic solutions of Geverey type, Asymptotic Anal., Vol. 2, (1989), pp.39-94.

[27] Sibuya, Y.: Linear Differential Equations in the Complex Domain: Problems of Analytic Continuation, Kinokuniya-shoten (1976) (in japanese); Trans. Math. Mono. Amer. Math. Soc, Vol.82 (1990).

[28] Sibuya, Y.: Stokes Phenomena, Bull. Amer. Math. Soc, Vol.83, (1977), pp.1075-1077.

[29] Takayama. N., Introduction to Kan virtual-machine (A system for computational algebraic analysis), Kobe university, (1992)