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STOCHASTIC COHOMOLOGY AND HOCHSCHILD COHOMOLOGY

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INTRODUCTION

It is known since a long time that the Hochschild cohomology is equal to the cohomology of the smooth based loop space, if the manifold is simply connected ([Ad]). An extension to the smooth free loop space is performed in [Ch]. The purpose of this work is to give a generalization to the case of the Brownian bridge to this classical result of Adams, by using the theory of forms and of stochastic Chen forms of [J.L].

Namely, [J.L] have introduced a tangent space over the free Brownian bridge of the manifold. Let us recall namely that in infinite dimensional analysis, it is known since [Gr_1] that the tangent space of a Banach space is a smaller Hilbert space. [L_2] and [F.M] remark that the tangent space introduced by [J.L] is nothing else than the tangent space introduced by Bismut in [Bi], in order to get intrinsic integration by parts formulas over the Brownian motion of a compact Riemannian manifold. [J.L] performed an L^p theory of forms over the loop space such that the Bismut-Chern character belongs to all the L^p.

A scalar Sobolev Calculus over the free loop space is performed in [L_2] and [L_3]. [L_4] does a Sobolev Calculus over the algebraic space corresponding to the Chen forms: it is shown that the Hochschild coboundary is continuous over the intersection of Sobolev spaces. [L_4] produces a Sobolev Calculus over the set of forms of the free loop space such that the exterior derivative is continuous over this set of forms. It is strongly related to the theory of Stochastic anticipative integrals, because the Lie bracket of two vector fields is not a vector field. The map stochastic Chen iterated integral is continuous.

[L_5] had shown that this result is still true for the free Brownian bridge of an homogeneous manifold. [L_5] used deeply the Albeverio-Hoegh-Krohn quasi-invariance formula over loop group in order to give a stochastic interpretation of the path fibration property of the path space with fiber the based loop space, which is used in the deterministic context in order to prove this property.

The purpose of this communication is to give a stochastic interpretation of the result of Adams, by using Driver's flow (See [Cr], [Dr], [E.S], [H], [N], [L.N]). It is a short version of [L_8].

ALGEBRAIC PROPERTIES AND STATEMENT OF THE MAIN THEOREM

Let M be a compact Riemannian simply connected manifold of dimension d. Let Δ be the Laplace-Beltrami operator. Let $p_t(x,y)$ be the associated heat kernel. Let $dP_{1,x}$ be the law of the Brownian bridge starting from x and coming back in time 1 to x. Let dP_1^x be the law of the Brownian motion starting from x. The time interval is [0,1]. We consider 2 infinite dimensional spaces:

-)The path space $P_x(M)$: it is the space of continuous functions γ_s from [0,1] into M endowed with the measure dP_1^x such that $\gamma_0 = x$.

-)The based loop space $L_x(M)$: it is the space of continuous functions γ_s from the circle S^1 into M such that $\gamma_0 = x$ endowed with the measure $dP_{1,x}$.

Let τ_t be the parallel transport from γ_0 to γ_t for the Levi-Civita connection over M. It is almost surely defined. These 2 infinite dimensional curved spaces are endowed with different tangent bundles.

-)For the path space, a tangent vector is of the shape $X_s = \tau_s H_s$ where the path H_s takes its values in the linear space $T_{\gamma_0}(M)$, and has bounded energy. Moreover, since we consider the based loop space, we have $H_0 = 0$. We take as Hilbert structure

$$\|X\|^2 = \int_0^1 \|d/dsh_s\|^2 ds \tag{1.1}$$

-)For the based loop space, we suppose moreover that $X_1 = 0$, and we take the same Hilbert structure.

Over the path space, we define the connection ∇. For $X_s = \tau_s H_s$, we have

$$(\nabla X)_s = \tau_s \nabla H_s \tag{1.2}$$

where ∇H_s is the H-derivative of H_s. Let us recall briefly the notion of H-derivative. If H_s is deterministic over the path space or over the loop space, we get the integration by parts for a cylindrical formula:

$$E[< dF, X >] = E[F div X] \tag{1.3}$$

where

$$div X = \int_0^1 < \tau_S d/ds H_s, \delta\gamma_s > + 1/2 \int_0^1 < S_{X_s}, \delta\gamma_s > \tag{1.4}$$

S is the Ricci tensor of the Levi-Civita connection, and δ denotes the curved Itô integral. dF exists for cylindrical functionals and can be extended by continuity. dF can be seen as a one form over the path space or over the based loop space. $d^r_\nabla F$ is a r-cotensor defined by induction over the path space or over the loop space. We get:

$$\begin{aligned} d^{r+1}_\nabla F(X_1,..,X_{r+1}) &= < d(d^r_\nabla F(X_1,..,X_r)), X_{r+1} > \\ &- \sum d^r_\nabla F(X_1,..,X_{i-1}, \nabla_{X_{r+1}} X_i, X_{i+1},..,X_r > \end{aligned} \tag{1.5}$$

It can be extended continuously because we have integration by parts (See [L2] and [L3] for analoguous considerations). $d^r_\nabla F$ is given by a kernel $k^r(s_1,..,s_r)$:

$$d^r_\nabla F(X_1,..,X_r) = \int\int k^1(s_1,..,s_r)d/dsH^1_{s1}..d/dsH^r_{s_r}ds_1..ds_r \tag{1.6}$$

If we work over the loop space, we have the same formula with the extra-condition:

$$\int k^r(s_1,..,s_r)ds_i = 0 \tag{1.7}$$

Let σ be an r-form over the path space or over the loop space. σ is given by a kernel σ_r:

$$\sigma(X_1,..,X_r) = \int\int\int \sigma_r(s_1,..,s_r)d/dsH^1_{s_1}..d/dsH^r_{s_r}ds_1..ds_r \tag{1.8}$$

If we work over the loop space, we have moreover $\int \sigma_r(s_1,..,s_r)ds_i = 0$. We are now ready to define the Nualart-Pardoux spaces of forms. Let $\nabla^l\sigma$ be the covariant derivative of order l of a r-form. It has kernels $\sigma_r(s_1,..,s_r;t_1,..,t_l)$. We suppose that over all connected components of the complement subset of the diagonals of $[0,1]^r \times [0,1]^l$, we have:
(1.9)

$$\|\sigma_r(s_1,..,s_r;t_1,..,t_l') - \sigma_r(s_1',..,s_r';t_1',..,t_{l'}'\|_{L^p} \leq C_{p,l}(\sigma)(\sum\sqrt{|s_i - s_i'|} + \sum\sqrt{|t_j - t_j'|})$$

for $l' \leq l$ and we suppose that

$$\|\sigma_j(s_1,..,s_r;t_1,..,t_{l'})\|_{L^p} \leq C'_{p,l}(\sigma)$$

We call the Nualart-Pardoux norms of order l in L^p the quantity $C_{p,l}(\sigma)+C'_{p,l}(\sigma) = \|\sigma\|_{p,l}$. If $\|\sigma\|_{p,l} < \infty$, we say that $\sigma \in (N.P)_{p,l}(Path)$ or $\sigma \in (N.P)_{p,l}(Loop)$.

Definition I.1: We say that an r-form σ is smooth in the Nualart-Pardoux sense if $\sigma \in (N.P)_{p,l}$ for all p, l. We say in that sense that $\sigma \in (N.P)_{\infty-}(Path)$ or that $\sigma \in (N.P)_{\infty-}(Loop)$.

Let us recall what is the meaning of $\nabla\tau_t$: let X_t be a section of the pull-back bundle of the tangent bundle by the evaluation map $\gamma \to \gamma_t$. Let ∇^t be the pull-back of the Levi-Civita connection by this evaluation map. We get:

$$\nabla^t(\tau_t H_t) = (\nabla\tau_t)H_t + \tau_t\nabla H_t \tag{1.11}$$

Moreover, see [Bi1] and [L3], we have:

$$\nabla_X\tau_t = \tau_t\int_0^t \tau_s^{-1}R(d\gamma_s, X_s)\tau_s \tag{1.13}$$

where R is the curvature tensor.

Let $X_t = \tau_t H_t$ and let $X'_t = \tau_t H'_t$ be two vector fields over the path space or the loop space. We recall that:

$$(1.14) \qquad [X, X']_t = \tau_t \nabla_{X'} H_t + \tau_t \int_0^t \tau_s^{-1} R(d\gamma_s, X'_s)\tau_s H_t + antisymmetry$$

Therefore the tangent space of the path space or of the loop space is not stable by Lie bracket.

Let us recall that for a $n-1$ form, its exterior derivative is defined by:

$$(1.15) \qquad \begin{aligned} d\sigma(X_1,..,X_n) &= \sum(-1)^{i-1} < d\sigma(X_1,..,X_{i-1},X_{i+1},X_n), X_i > \\ &+ \sum_{i<j}(-1)^{i+j}\sigma([X_i,X_j],..,X_{i-1},X_{i+1},..,X_{j-1},X_{j+1},..,X_n) \end{aligned}$$

Let us recall ([L$_5$]):

Theorem I.2: d is continuous over the set of forms of finite degree smooth in the Nualart-Pardoux sense over the path space or over the loop space.

Let $H^p_{\infty-}(N.P)(Path)$ be the associated cohomology group of order p over the path space. We get by [L$_6$] the following theorem which reflects that the path space retracts over the constant paths.

Theorem I.3: If $p > 0$, $H^p_{\infty-}(N.P)(Path) = 0$. If $p = 0$, $H^0_{\infty-}(N.P)(Path) = C$.

We work of course in this paper with C-valued forms.

We get an inclusion from the based loop space into the path space. We deduce a restriction map i^* from the set of forms over the path space to the set of forms over the based loop space. Let us recall ([L$_4$]) that i^* is continuous from the set of forms smooth in the Nualart-Pardoux sense over the path space onto the set of forms smooth in the Nualart-Pardoux sense over the loop space.

One of the main tool of the proof of theorem which is the goal of this work is:

Theoem I.4: i^* is a surjection from $(N.P)_{\infty-}(Path)$ over $(N.P)_{\infty-}(Loop)$.

Let us recall the following definition which is related to the cobar construction of forms over the manifold. Let $\Omega(M)$ be the set of forms over the manifold and let $\Omega_.(M)$ be the set of differential forms over M of degree > 0. Let $\tilde{\omega}_n = \omega_1 \otimes .. \otimes \omega_n \otimes \omega_{n+1}$ an element of $\Omega_.(M)^{\otimes n} \otimes \Omega(M)$: we suppose it here to be a simple tensor product, but we can suppose that $\tilde{\omega}_n$ belongs to the tensor product of Hilbert spaces associated to the Hilbert Sobolev space of k^{th} order involved with the operator $d^*d + dd^* + 2$ over $\Omega(M)$. The total degree of $\tilde{\omega}_n$ is $\sum_{i=1}^n (deg\omega_i - 1) + deg\omega_{n+1}$, and we decide to keep the total degree of a given element. The corresponding Sobolev norm is denoted by $\|.\|_{2,k}$. We consider a series $\sum \tilde{\omega}_n = \tilde{\omega}$ with total degree l, and we introduce the family of semi-norms

$$(1.16) \qquad \phi_{k,z}(\tilde{\omega}) = \sum \frac{z^n}{\sqrt{n!}} \|\tilde{\omega}_n\|_{2,k}$$

We get an intersection of Banach spaces, when z or $k \to \infty$. If $\phi_{k,z}(\tilde{\omega}) < \infty$ for all k and $z > 0$, we say that $\tilde{\omega}$ is smooth. We can do the same considerations if we remove the last term $\Omega(M)$ in the tensor product.

The Hochschild boundary is given by $b_p = b_{0,p} + b_{1,p}$ where

$$b_{0,p} = d\omega_1 \otimes .. \otimes \omega_1 + \sum_{1<j\leq<n+1} (-1)^{\epsilon_{i-1}}\omega_1 \otimes .. \otimes d\omega_i \otimes .. \otimes \omega_{n+1} \tag{1.17}$$

if $\epsilon_i = \sum_{1\leq j\leq i}(deg(\omega_j) - 1)$ and

$$b_{1,p} = \omega_1 \wedge \omega_2 \otimes \omega_3 .. \otimes \omega_{n+1} + \sum_{1<i\leq n} (-1)^{\epsilon_i}\omega_0 \otimes \omega_1 .. \otimes \omega_i \wedge \omega_{i+1} \otimes ... \otimes \omega_{n+1} \tag{1.18}$$

We can do the same definition if we remove the last term in the considered tensor products $\Omega(M)$. We get two operations $b_{0,l}$ and $b_{1,l}$.

$S_i(f)$ is the following operation: in $\tilde{\omega}_n$, we introduce f between ω_i and ω_{i+1}, if f is a smooth function. $D^l_{\infty-}$ is the closure for the family of norms (1.15) of the linear space spanned by $[b_p, S_i(f)]\tilde{\omega}_n$ for element of fixed degree l. $N^l_{\infty-}(C, \Omega(M), \Omega(M))$ is the quotient of the spaces given by the family of norms (1.15) by $D^l_{\infty-}$.

We perform the same construction for the based loop space. We get a space called $N^l_{\infty-}(C, \Omega(M), C)$ and a Hochschild coboundary operator $b_l = b_{0,l} + b_{1,l}$ which is continuous.

Let us recall that we get the following commutative diagram:

$$\begin{array}{ccc} N^l_{\infty-}(C, \Omega(M), \Omega(M)) & \rightarrow & N^l_{\infty-}(C, \Omega(M), C) \\ \downarrow & & \downarrow \\ (N.P)^l_{\infty-}(Path) & \rightarrow & (N.P)^l_{\infty-}(Loop) \end{array} \tag{1.19}$$

The horizontal arrows are the restriction maps, which are continuous (See [L4]). The vertical arrows are the map Chen iterated integrals. Let us recall quickly their definitions.

Let $\tilde{\omega}_n = \omega_1 \otimes .. \otimes \omega_n \otimes \omega_{n+1}$. $\Sigma\tilde{\omega}_n$ is the form over the path space:

$$\Sigma\tilde{\omega}_n = \int_{0<s_1..<s_n<1} \omega_1(d\gamma_{s_1}, .) \wedge .. \wedge \omega_n(d\gamma_{s_n}, .) \wedge \omega_{n+1} \tag{1.20}$$

If ω_{n+1} is a form of degree r, ω_{n+1} is the form over the path space which to r vectors X_t over the path space associates $\omega_{n+1}(\gamma_1)(X^1_1, .., X^r_1)$. If ω_1 and ω_2 are two forms of degree 2, $\Sigma(\omega_1 \otimes \omega_2)$ is a 2 form over the path space given by the following formula:

$$\Sigma(\omega_1 \otimes \omega_2)(X^1, X^2) = \int_{0<s_1<s_2<1} \omega_1(d\gamma_{s_1}\tau_{s_1}H^1_{s_1})\omega_2(d\gamma_{s_2}, \tau_{s_2}H^2_{s_2}) + antisymmetry \tag{1.21}$$

We can do the same computations over the loop space, but the contribution of ω_{n+1} vanishes. The vertical maps of the commutative diagram (1.19), which are constituted of iterated Chen integrals, are continuous.

Let us recall the following theorem (See [L5]):

Theorem I.5: The first vertical map in the commutative diagram (1.19) induces an isomorphism in cohomology.

Let us denote by $H.H^l_{\infty-}(C, \Omega(M), C)$ the cohomology group in degree d of the Hochschild complex b_l which acts over $N^l_{\infty-}(C, \Omega, C)$. Let us denote by $H^l_{\infty-}(L_x(M)$ the cohomology group in degree l of $(N.P)_{\infty-}(Loop)$. Let us introduce a small neighborhood U of x which is contractible. Let us denote by $P_U(M)$ the space of continuous paths starting from x and arriving in U.

Let $\phi_{k,U}$ an increasing sequence of functions from U into [0,1] with compact support and tending to 1. We denote by $(N.P)^l_{\infty-,U}(Path)$ the space of forms of degree l over $P_U(M)$ such that $\phi_{k,U}(\sigma)$ belongs to $(N.P)^l_{\infty-}(Path)$ for all k. We can define the stochastic exterior derivative over $(N.P)_{\infty-}(Path)$ with cohomology groups $H^l_{\infty-,U}(x(M))$.

We will show the theorem:

Theorem I.6: If U is a small contractible ball, the stochastic cohomology groups $H^l_{\infty-,U}(P_x(M))$ are equal to the stochastic cohomology group $H^l_{\infty-}(L_x(M))$.

The proof uses Driver's flow in reversed time, as well as to justify some formal computations small time asymptotics of diffusions (See [K], [L1] and [Wa2] for surveys). The reader can find the proof of this statement in [L8]. This theorem allows, since M is simply connected, to repeat the spectral sequence argument of [G.J.P] and of [L5]. We get the following theorem, which is the goal of this paper:

Theorem I.7: The map which is constituted of stochastic Chen iterated integral induces an isomorphism in cohomology between $H.H^l_{\infty-}(C, \Omega(M), C)$ and $H^l_{\infty-}(L_x(M))$.

Remark: It should be possible to prove the full stochastic Chen theorem by using Driver's flow in the two sense of time, as we will see later. Let us indicate how to proof this generalization. We consider the free loop space $L(M)$ of continuous applications from the circle S_1 into M endowed with the B-H-K measure $p_1(x,x)dx \otimes dP_{1,x}$ where $p_t(x,y)$ is the heat kernel associated to the heat semi-group over the Riemannian manifold. We consider the free path space $P(M)$ of continuous applications from $[0,1]$ into M endowed with the measure $dx \otimes dP_1^x$. A vector field over the free path space is of the form $\tau_. H_.$ with H_0 arbitrarly chosen instead of being 0 as before. We take the Hilbert norm (1.1) of [L5]. For the free loop space, we have $\tau_0 H_0 = \tau_1 H_1$. Vector fields of the form $\tau_t((1-t)X_0 + \tau_1^{-1} t X_0)$ with Hilbert norm $\|X_0\|^2$ are by definition orthogonal to the based loop space tangent vector fields (See [L5] after (1.2)). We break the rotational symmetry over the free loop space, but it should be possible to take the Hibert norm of [J.L] p106, which leads to much more complicated algebraic computations. Over the free path space, we introduce a connection ∇. Over the free loop space, we introduce a obvious connection by taking the restriction of the connection over the free path space to the tangent space of a based loop ($H_0 = H_1 = 0$!) and by taking the covariant derivative of X_0 over the orthogonal complement defined before. The stochastic cohomology of the free path space is equal to the cohomology of the manifold (See [L5] Theorem I.2). We can introduce the Hochschild spaces to the free loop space and the free path space, such that we have a generalization of the commutative diagramm (1.19) to this context (See [L4], Théoreme II.7). By using Driver's flow as in [L8] Proof of Theorem I.4, instead of the Albeverio-Hoegh-Krohn quasi-invariance formulas as in [L5] Theorem II.1, we can show that the horizontal maps in the Theorem II.7 of [L4] are surjections. In others words, we get a generalization of the theorem I.4 to this new situation. The surjectivity from the space of smooth forms in the Nualart-Pardoux sense over the free path space to the space of forms smooth in the

Nualart-Pardoux sense over the free loop space holds by using Driver's flow in only one sense. The surjectivity from the space of forms smooth in the Nualart-Pardoux sense of the free path space to the space of forms smooth in the Nualart-Pardoux sense over the based path space holds by using Driver's flow in only one sense. The others statement of surjectivities are deduced from the two previous ones.

The fact that the Hochschild cohomology of the free loop space is equal to the stochastic cohomology of the free loop space of a spectral sequence argument as in [G.J.P] and in [L_5] and of the following 2 theorems:

Let U and V two contractible small open sets of M. Let $P_{U\times V}(M)$ be the set of continuous paths going from U and arriving in V. We can define the stochastic cohomology groups of $P_{U\times V}(M)$, called $H^l_{\infty-,U,V}(P(M))$. We get:

Theorem I.8: $H^l_{\infty-,U,V}(P(M))$ is equal to the cohomology group $H^l_{\infty-}(L_x(M))$.

We use Driver's flow in the two senses in order to get this theorem.

In the same manner, we consider the subset of loop $L_U(M)$ going from U. We get stochastic cohomology groups $H^l_{\infty-,U}(L(M))$.

Theorem I.9: $H^l_{\infty-,U}(L(M))$ is equal to the cohomology group $H^l_{\infty-}(L_x(M))$.

We use the fact that $H^l_{\infty-,U}(L(M))$ is equal to the cohomology group $H^l_{\infty-,U}(P_x(M))$ by using Driver's flow in direct sense, and theorem I.6, whose proof uses Driver's flow in the reversed sense.

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