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Structure of the stationary solution to Keller-Segel equation in one dimension

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I. Introduction

In the life cycle of cellular slime molds interesting phenomenon can be observed: slime mold first tend to distribute themselves uniformly over the space where a source of food (bacteria) is present. After exhausting their food supply, they begin to aggregate in a number of collecting points. The colony becomes slug form and it migrates to a source of food and forms a multicellar fruting body which is like plant. Eventually the fruting body spreads spores from its top, the spores grow into cellular slime molds.

In 1970 E. F. Keller and L. A. Segel [KS] proposed the equation which describes the aggregation process above:

$$\begin{cases} \dfrac{\partial u}{\partial t}=a\Delta u-\nabla\cdot\{u\nabla \mathrm{B}(\rho)\} & \text{in } \Omega\times(0,\infty),\\ \dfrac{\partial\rho}{\partial t}=d\Delta\rho+fu-g\rho & \text{in } \Omega\times(0,\infty),\\ \dfrac{\partial u}{\partial n}=\dfrac{\partial\rho}{\partial n}=0 & \text{on } \partial\Omega\times(0,\infty),\\ u(x,0)=u_0(x),\quad \rho(x,0)=\rho_0(x) & \text{in } \Omega. \end{cases}$$

Here, $\Omega\subset\mathbf{R}^{\mathrm{N}}$, $\mathrm{N}\geq 1$ is a bounded domain with smooth boundary, $u(x,t)$ denotes the population density of slime mold and $\rho(x,t)$ denotes the concentration of chemotactic substance at the position $x\in\Omega$ and the time $t\in[0,\infty)$. a, d, f and g are positive constants. n denotes the outer normal vector of $\partial\Omega$. u_0 and ρ_0 are the initial functions of slime mold and chemotactic substance respectively.

The equation of u_t is derived from the following fact by experiments: slime molds have a nature that they tend to migrate to chemotactic substance (this migration is called chemotaxis), these substance is secreted by themselves, and another slime molds aggregate to the substance. By this fact flux of u is presented as $\overrightarrow{\mathrm{F}}=\overrightarrow{\mathrm{F}}_{\text{diffusion}}+\overrightarrow{\mathrm{F}}_{\text{chemotaxis}}$; $\overrightarrow{\mathrm{F}}_{\text{diffusion}}=-a\nabla u$, $\overrightarrow{\mathrm{F}}_{\text{chemotaxis}}=u\nabla\mathrm{B}(\rho)$, the equation of u_t is nonlinear one which includes diffusion term which is derived from Fick's law and chemotaxis term. B is the smooth function of ρ with $\mathrm{B}'(\rho)>0$ for $\rho>0$ which is called sensitivity function. Several forms have been suggested (see e.g. [S]): $b_0\rho$, $b_0\log\rho$, $\frac{b_0\rho}{1+\rho}$; $b_0>0$. The equation of ρ_t is linear one which includes diffusion, production and decrese term. Neumann condition as boundary condition describe reflecion one which means for example experiment by considering Ω as schale.

Non-constant solution u means that slime mold do not distribute uniformly. If $u(x_0)$ has much larger value than $u(x)$, x in around x_0, we can interpret it as aggregation happens at x_0.

There are several results for each B like above. As for stationary problem, existence and non-existence of non-constant solution is studied by bifurcation theory(B is general like above) [S], existence of non-constant solution and position of aggregation part by variational methods ($\mathrm{B}(\rho) = b_0 \log \rho$) [NT]. As for evolutional problem, existence of time local and global solution is studied (B is general like above) [Y], blow up problem (mainly $\mathrm{B}(\rho) = b_0 \rho$) [HV] [NSY]. Approximation method is also studied in [NY].■

In this report, we consider the case when $\mathrm{N} = 1$, $\Omega = (\alpha,\ \beta)$, $-\infty < \alpha < \beta < \infty$:

$$\begin{cases} \dfrac{\partial u}{\partial t} = a\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial}{\partial x}\Big(u\dfrac{\partial}{\partial x}\mathrm{B}(\rho)\Big) & \text{in } \Omega \times (0,\infty), \\ \dfrac{\partial \rho}{\partial t} = d\dfrac{\partial^2 \rho}{\partial x^2} + fu - g\rho \quad \text{in } \Omega \times (0,\infty), & \\ \dfrac{\partial u}{\partial x}(\alpha,t) = \dfrac{\partial u}{\partial x}(\beta,t) = \dfrac{\partial \rho}{\partial x}(\alpha,t) = \dfrac{\partial \rho}{\partial x}(\beta,t) = 0 & \text{for } t \in (0,\infty), \\ u(x,0) = u_0(x), \quad \rho(x,0) = \rho_0(x) \quad \text{in } \Omega. & \end{cases}$$

We assume following condition on B. Let $B'(\rho) = b(\rho)$,

$$|b^{(i)}(\rho)| \le b_0\Big(1 + \frac{1}{\rho^k}\Big) \text{ for positive constant } b_0,$$

where $i = 0,\ 1,\ k = k(i) \ge 0$. Under this assumption B contains the all functions which are presented above.

For this system, we investigate the asymptotic behavior of solution by constructing attractor.■

II. Main theorems

We can get the following two theorems about global solution and global attractor:

Theorem 1 (global solution)
If

$$u_0 \in L^2(\Omega),\ u_0(x) \ge 0 \text{ in } \Omega,\ \rho_0 \in H^{\frac{1}{2}+\varepsilon}(\Omega),\ \rho_0(x) > 0 \text{ in } \Omega,\ 0 < \varepsilon < \frac{1}{2}, \textit{fixed},$$

then there exists a unique solution u, ρ such that

$$u \in C((0,\infty); H^1(\Omega)) \cap C([0,\infty); L^2(\Omega)) \cap C^1((0,\infty); \{H^1(\Omega)\}'),$$

$$\rho \in C((0,\infty); H^2(\Omega)) \cap C([0,\infty); H^{\frac{1}{2}+\varepsilon}(\Omega)) \cap C^1((0,\infty); L^2(\Omega)).■$$

Theorem 2 (global attractor)
Let l an arbitrary positive constant and

$$X_l = \left\{(u,\ \rho) \in L^2(\Omega) \times H^{\frac{1}{2}+\varepsilon}(\Omega);\ u(x) \geq 0,\ \rho(x) > 0 \text{ in } \Omega, \int_\Omega u(x)dx = l\right\},$$

where $0 < \varepsilon < 1/2,$ *fixed. Then there exists a global attractor of* X_l.■

Global attractor $\mathcal{A}_l$ is defined as follows (see e.g. [Te]):

Definition (global attractor)
We say that $\mathcal{A}_l$ of X_l is a global attractor for the semigroup $\{S(t)\}_{t\geq 0}$ if $\mathcal{A}_l$ is compact attractor that attracts the bounded sets of X_l, that is,

$$d(S(t)\mathcal{B}, \mathcal{A}_l) \to 0 \text{ as } t \to 0$$

for all bounded set $\mathcal{B}$ in X_l.
Here, we define d as semidistance of two sets $\mathcal{B}_0,\ \mathcal{B}_1$:

$$d(\mathcal{B}_0, \mathcal{B}_1) = \sup_{x\in\mathcal{B}_0} \inf_{y\in\mathcal{B}_1} \|x-y\|_{L^2\times H^{\frac{1}{2}+\varepsilon}}.■$$

Remark
In N $= 2$, for B$(\rho) = b_0\rho,\ b_0 > 0$ if l is sufficiently small and $\rho_0 \in H^{1+\varepsilon}(\Omega),\ 0 < \varepsilon < 1/2,$ *fixed*, then theorem1 and theorem2 hold.■

III. Proof of theorems

We give the skech of proof. Proof consists of several steps as following:

i) Existence of local solution.

ii) Smoothing effect.

iii) A priori estimate from below.

iv) A priori estimate from above.

v) Existence of attractor.

i) Existence of local solution

We first note the theorem on abstract evolution equation.
Let $V,\ H$ are separable Hilbert spaces such that V is dense and compactly embedded in H and their norms are $\|\cdot\|_V$ and $|\cdot|_H$ respectively. We also set space V' such that H is compactly embedded in V' by identifying H and H'. We denote the norm of V' by $\|\cdot\|_{V'}$.

We consider equation:

$$\text{(AP)} \qquad \begin{cases} \dfrac{dU}{dt} + AU = F(U), & 0 < t < \infty, \\ U(0) = U_0. \end{cases}$$

We define assumptions on A and F.

(Ai) A is a bounded linear operator from V to V' and satisfying

$$\langle AU,\ U\rangle_{V'\times V} \geq \delta \|U\|_V^2$$

for some positive constant δ.

(Fi) There exists constant θ such that $0 \leq \theta < 1$ and function ϕ from $\mathbf{R} \to \mathbf{R}$ which is smooth, increasing and satisfying

$$\|F(U)\|_{V'} \leq C(\|U\|_V^\theta + 1)\phi(|U|_H)$$

for all $U \in V$ and some positive constant C.

(Fii) There exists function ψ from $\mathbf{R} \to \mathbf{R}$ which is smooth, increasing and satisfying

$$\langle F(U) - F(\widetilde{U}),\ U - \widetilde{U}\rangle_{V'\times V} \leq C(\|U\|_V + \|\widetilde{U}\|_V)\psi(|U|_H + |\widetilde{U}|_H)|U - \widetilde{U}|_H \|U - \widetilde{U}\|_V$$

for all $U,\ \widetilde{U} \in V$ and some positive constant C.

Under these assumptions, we can get the theorem:

Theorem 3
Assume (Ai),(Fi),(Fii) for (AP). Then for all $U_0 \in H$ there exists $T(U_0) > 0$ such that there exists unique weak solution and satisfying

$$U \in L^2(0, T(U_0); V) \cap C([0, T(U_0)]; H) \cap H^2(0, T(U_0); V'). \blacksquare$$

Proof is derived by a priori estimate and Galerkin method [OY], [RY].

Let us apply this theorem to (KS). We formulate (KS) as (AP) with

$$A, F : H^1(\Omega) \times H_n^{\frac{3}{2}+\varepsilon}(\Omega) \to H^1(\Omega)' \times H^{\frac{1}{2}-\varepsilon}(\Omega)',$$

$$A = \begin{bmatrix} -a\dfrac{\partial^2}{\partial x^2} + a & 0 \\ 0 & -d\dfrac{\partial^2}{\partial x^2} + g \end{bmatrix}, \quad F(U) = \begin{bmatrix} \dfrac{\partial}{\partial x}\Big(ub(\rho)\dfrac{\partial \rho}{\partial x}\Big) \\ fu \end{bmatrix}, \quad U = \begin{bmatrix} u \\ \rho \end{bmatrix},$$

where $H_n^s = \{u \in H^s(\Omega);\ u'(\alpha) = u'(\beta) = 0\}$, for $s > 3/2$.

Then, if $u_0 \in L^2(\Omega)$, $u_0(x) \geq 0$ in Ω, $\rho_0 \in H^{\frac{1}{2}+\varepsilon}(\Omega)$, $\rho_0(x) > 0$ in Ω, then (KS) has a unique weak solution u and ρ locally in time, that is, there exist $T = T_{u_0,\rho_0} > 0$, such that u and ρ satisfy (KS), $u \geq 0$, $\rho > 0$ for $t \in [0, T]$ and

$$u \in L^2(0, T; H^1(\Omega)) \cap C([0, T]; L^2(\Omega)) \cap H^1(0, T; H^1(\Omega)'),$$
$$\rho \in L^2(0, T; H_n^{\frac{3}{2}+\varepsilon}(\Omega)) \cap C([0, T]; H^{\frac{1}{2}+\varepsilon}(\Omega)) \cap H^1(0, T; H^{\frac{1}{2}-\varepsilon}(\Omega)'). \blacksquare$$

ii) Smoothing effect

In this step also we note the following theorem for (AP) about regularity of solution. We define another assumptions on A and F.

(Aii) A is a linear operator from $\mathcal{D}(A)$ to V' which is genarator of analytic semigroup $\{e^{-tA}\}_{t\geq 0}$.

(Fiii) There exists constants γ, η such that $0<\eta<\gamma<1$ and function μ from $\mathbf{R}\to\mathbf{R}$ which is smooth, increasing and satisfying

$$\|F(U)-F(\widetilde{U})\|_{V'}\leq \mu(\|A^{\gamma}U\|_{V'}+\|A^{\gamma}\widetilde{U}\|_{V'})\|A^{\eta}(U-\widetilde{U})\|_{V'}$$

for all $U,\ \widetilde{U}\in\mathcal{D}(A),\mathcal{D}(A^{\gamma}),\mathcal{D}(A^{\eta})$.

Under these assumptions, we can get the following theorem:

Theorem 4
Assume (Aii),(Fii),(Fiii) for (AP). If $U_0\in\mathcal{D}(A^{\gamma})$, then there exists unique strict solution and satisfying

$$U\in C((0,T(U_0)];\mathcal{D}(A))\cap C([0,T(U_0)];\mathcal{D}(A^{\gamma}))\cap C^1((0,T(U_0)];V').\blacksquare$$

Proof is derived by semigroup method [OY].

Applying this theorem to local solution in i) with $\mathcal{D}(A^{\gamma})=H^{2\gamma-1}\times H_n(\Omega)^{2\gamma+1}(\Omega)$, we get the result of regularity of solution to (KS).$\blacksquare$

iii) A priori estimate from below

We get a priori estimate from below by virtue of the second equation of (KS) which has production and decrese term:

Let $(u_0,\ \rho_0)\in X_l$ and $(u,\ \rho)$ be solution to (AP). Then $u\geq 0,\ \ \rho>0$ for $t>0$ and

$$\text{there exists } T_l>0,\ \ \delta_l>0 \text{ such that } \rho>\delta_l,\ \text{ for } t\geq T_l,$$

where $T_l,\ \delta_l$ is independent on $u_0,\ \rho_0$.$\blacksquare$

iv) Apriori estimate from above

Step1 Integrating the first equation of (KS) in Ω gives

$$\frac{d}{dt}\int_{\Omega}u(x)dx=0$$

by Neumann condition, then $\|u(t)\|_{L^1}=l,\quad t\geq 0.$ $\blacksquare$

Step2 Integrating the second equation of (KS) in Ω gives differential equation

$$\frac{d}{dt}\|\rho(t)\|_{L^1}=f\|u\|_{L^1}-g\|\rho\|_{L^1}.$$

Solving this in $\|\rho\|_{L^1}$,

$$\|\rho(t)\|_{L^1} = \Big(\|\rho_0\|_{L^1} - \frac{fl}{g}\Big)e^{-gt} + \frac{fl}{g}, \quad t \geq 0. \blacksquare$$

Step3 From Gagliardo-Nirenberg inequality:

$$\|u\|_{L^2(\Omega)} \leq C\|u\|^{\frac{2}{3}}_{L^1(\Omega)}\|u\|^{\frac{1}{3}}_{H^1(\Omega)} \text{ for all } u \in H^1(\Omega),$$

we get

$$\|u\|^2_{L^2} \leq \varepsilon_1\|u\|^2_{L^2} + \varepsilon_1\Big\|\frac{\partial u}{\partial x}\Big\|^2_{L^2} + \frac{C_l}{\varepsilon_1}.$$

Hence,

$$\|u\|^2_{L^2} \leq \varepsilon_1\Big\|\frac{\partial u}{\partial x}\Big\|^2_{L^2} + \frac{C_l}{\varepsilon_1}. \blacksquare \tag{1}$$

Step4 Multiplying the first equation of (KS) by u and integrating the product in Ω, and noting $|b(\rho)| \leq b_0(1 + \frac{1}{\delta_l^k}) \leq C_l$ and $\int_\Omega u\frac{\partial u}{\partial x}\frac{\partial \rho}{\partial x}dx = -\frac{1}{2}\int_\Omega u^2\frac{\partial^2 \rho}{\partial x^2}dx$, then

$$\frac{1}{2}\frac{d}{dt}\|u(t)\|^2_{L^2} \leq -a\Big\|\frac{\partial u}{\partial x}\Big\|^2_{L^2} + \varepsilon_2 C_l\|u\|^4_{L^4} + \frac{C_l}{\varepsilon_2}\Big\|\frac{\partial^2 \rho}{\partial x^2}\Big\|^2_{L^2},$$

for positive constant ε_2.

From Gagliardo-Nirenberg inequality:

$$\|u\|_{L^4(\Omega)} \leq C\|u\|^{\frac{1}{2}}_{L^1(\Omega)}\|u\|^{\frac{1}{2}}_{H^1(\Omega)} \text{ for all } u \in H^1(\Omega),$$

we get

$$\|u\|^4_{L^4} \leq Cl^2\Big(\|u\|^2_{L^2} + \Big\|\frac{\partial u}{\partial x}\Big\|^2_{L^2}\Big).$$

Taking ε_2 so small as $-a + \varepsilon_2 C_l C l^2 < 0$, then

$$\frac{1}{2}\frac{d}{dx}\|u(t)\|^2_{L^2} \leq \varepsilon_2 C_l\|u\|^2_{L^2} - a_{\varepsilon_2}\Big\|\frac{\partial u}{\partial x}\Big\|^2_{L^2} + \frac{C_l}{\varepsilon_2}\Big\|\frac{\partial^2 \rho}{\partial x^2}\Big\|^2_{L^2}. \tag{2}$$

Next, multiplying the second equation by $\frac{\partial^2 \rho}{\partial x^2}$ and integrating in Ω, thanks to Schwartz inequality,

$$\frac{1}{2}\frac{d}{dt}\Big\|\frac{\partial \rho}{\partial x}(t)\Big\|^2_{L^2} \leq \frac{f}{\varepsilon_3}\|u\|^2_{L^2} - g\Big\|\frac{\partial \rho}{\partial x}\Big\|^2_{L^2} - d_{\varepsilon_3}\Big\|\frac{\partial^2 \rho}{\partial x^2}\Big\|^2_{L^2}, \tag{3}$$

where $-d_{\varepsilon_3} = -d + \varepsilon_3 f$ by taking ε_3 so small as $-d_{\varepsilon_3} < 0$.

Multiplying (3) by positive constant K and add the inequality to (2) and using the inequality (1), then we get differential inequality:

$$\frac{1}{2}\frac{d}{dt}\Big(\|u(t)\|^2_{L^2} + K\Big\|\frac{\partial \rho}{\partial x}(t)\Big\|^2_{L^2}\Big) \leq -g\Big(\|u\|^2_{L^2} + K\Big\|\frac{\partial \rho}{\partial x}\Big\|^2_{L^2}\Big) + C_l$$

by taking K as $-d_{\varepsilon_3}K + \frac{C_l}{\varepsilon_2} = 0$, and ε_2 as $-a_{\varepsilon_2} + \varepsilon_1(\varepsilon_2 C_l Cl^2 + \frac{fK}{\varepsilon_3} + g) = 0$.
Solving this,

$$\|u\|_{L^2}^2 + K\Big\|\frac{\partial\rho}{\partial x}\Big\|_{L^2}^2 \leq \Big(\|u_0\|_{L^2}^2 + K\Big\|\frac{\partial\rho_0}{\partial x}\Big\|_{L^2}^2 - \frac{C_l}{2g}\Big)e^{-2gt} + \frac{C_l}{2g}.\blacksquare \tag{4}$$

Step5 Multiplying the second equation by ρ and integrating the product in Ω, then

$$\frac{1}{2}\frac{d}{dt}\|\rho(t)\|_{L^2}^2 \leq \frac{f}{\varepsilon_4}\|u\|_{L^2}^2 - g_{\varepsilon_4}\|\rho\|_{L^2}^2 - d\Big\|\frac{\partial\rho}{\partial x}\Big\|_{L^2}^2.$$

From (4),

$$\frac{d}{dt}\|\rho(t)\|_{L^2}^2 \leq -g_{\varepsilon_4}\|\rho\|_{L^2}^2 + \frac{2f}{\varepsilon_4}\Big(C_{u_0,\rho_0}e^{-2gt} + \frac{C_l}{2g}\Big).$$

Solving this,

$$\|\rho(t)\|_{L^2}^2 \leq C_{u_0,\rho_0}e^{-2g_{\varepsilon_4}t} + C_l.\blacksquare$$

Step6 First we note the next proposition.

Proposition
For solutions u, ρ of (KS)

$$\frac{1}{2}\frac{d}{dt}\Big\|\frac{\partial u}{\partial x}(t)\Big\|_{L^2}^2 \leq L(t)\Big\|\frac{\partial^2\rho}{\partial x^2}\Big\|_{L^2}^{\frac{5}{3}} + L(t)\Big\|\frac{\partial^3\rho}{\partial x^3}\Big\|_{L^2}^{\frac{3}{2}} + L(t), \tag{5}$$

where $L(t) = C_{u_0,\rho_0}e^{-2\widetilde{g}t} + C_l$, $\widetilde{g} > 0$ for $t \geq T_l$.$\blacksquare$

Skech of proof is following. Multiplying the first equation of (KS) by $\frac{\partial^2 u}{\partial x^2}$,

$$\frac{1}{2}\frac{d}{dt}\Big\|\frac{\partial u}{\partial x}\Big\|_{L^2}^2 \leq -a\Big\|\frac{\partial^2 u}{\partial x^2}\Big\|_{L^2}^2 + C_l\Big(\Big\|\frac{\partial^2 u}{\partial x^2}\frac{\partial u}{\partial x}\frac{\partial\rho}{\partial x}\Big\|_{L^1} + \Big\|\frac{\partial^2 u}{\partial x^2}\,u\Big|\frac{\partial\rho}{\partial x}\Big|^2\Big\|_{L^1} + \Big\|\frac{\partial^2 u}{\partial x^2}\,u\,\frac{\partial^2\rho}{\partial x^2}\Big\|_{L^1}\Big).$$

We can estimate the latter three terms by Hölder's inequality and next inequality [Ta]

$$\|u\|_{W^{j,r}} \leq C\Big(\|u\|_{L^p}^{1-a}\Big\|\frac{\partial^m u}{\partial x^m}\Big\|_{L^p}^a + \|u\|_{L^p}\Big)$$

for $u \in W^{m,p}(\Omega)$, where m and j are integers satisfying $0 \leq j < m$, $1 \leq p < \infty$, $m - j - \frac{1}{p}$ is not a nonnegative interger, $\frac{j}{m} \leq a \leq 1$, $\frac{1}{r} = j + \frac{1}{p} - am \geq 0$, and C is constant. Then we get the estimate in proposition.

Next we introduce another differential inequality. Operating $\frac{\partial^2}{\partial x^2}$ and multiplying $\frac{\partial^2\rho}{\partial x^2}$ to the second equation of (KS), and integrating the product in Ω, then

$$\frac{1}{2}\frac{d}{dt}\Big\|\frac{\partial^2\rho}{\partial x^2}\Big\|_{L^2}^2 \leq \frac{f}{\varepsilon_5}\Big\|\frac{\partial u}{\partial x}\Big\|_{L^2}^2 - g\Big\|\frac{\partial^2\rho}{\partial x^2}\Big\|_{L^2}^2 - d_{\varepsilon_5}\Big\|\frac{\partial^3\rho}{\partial x^3}\Big\|_{L^2}^2. \tag{6}$$

Multiplying positive constant M by (4) and add (5) and (6), then we get differntial inequality and solving it,

$$
\begin{aligned}
&M\|u\|_{L^2}^2+\left\|\frac{\partial\rho}{\partial x}\right\|_{L^2}^2+KM\left\|\frac{\partial\rho}{\partial x}\right\|_{L^2}^2+\left\|\frac{\partial^2\rho}{\partial x^2}\right\|_{L^2}^2\\
&\quad\leq\left(M\|u_0\|_{L^2}^2+\left\|\frac{\partial\rho_0}{\partial x}\right\|_{L^2}^2+KM\left\|\frac{\partial\rho_0}{\partial x}\right\|_{L^2}^2+\left\|\frac{\partial^2\rho_0}{\partial x^2}\right\|_{L^2}^2-\frac{L(t)}{2\widetilde{M}}\right)e^{-2\widetilde{M}t}+\frac{L(t)}{2\widetilde{M}}.\blacksquare
\end{aligned}
$$

v) Existence of attractor

Let us apply the known theorem about existence of attractor ([Te], theorem1.1.).

Theorem 5 (Existence of global attractor)
We assume that H is a metric space and semigroup $\{S(t)\}_{t\geq 0}$ on H is continuous and uniformly compact for large t.

If there exists a bounded set $\mathcal{B}$ of H such that $\mathcal{B}$ is absorbing in H, then the ω-limit set of $\mathcal{B}$, $\mathcal{A}=\omega(\mathcal{B})$ is a global attractor.

By applying this theorem with $H=\bigcup_{t\geq T_l} S(t)X_l$, we get the existence of global attractor $\mathcal{A}$ of X_l.$\blacksquare$

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