数理解析研究所講究録
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Problems stated by Guang Feng Jiang (姜広峰)

Question 1 (with Dirk Siersma)

Let X be a hypersurface with isolated singularity defined in $n+1$ dimensional affine space $\mathbb{C}^{n+1}$ by $h=0$. Let $Y\subset X$ be a curve defined by an ideal I. Define

$$\mathcal{R}_{I}*=\{\phi\in\mathcal{R}\mid\phi^{*}(I)\subset I\},$$

which acts on mI, where m is the maximal ideal of $\mathcal{O}$. Define

$\tau_{I}(h):=\{\xi(h)\mid\xi(I)\subset I,\xi(m)\subset m\}$.
$c_{Y}:=\dim\frac{mI}{\tau(h)_{I}}$, called the $\mathcal{R}_{I}$-codimension of h.
$\lambda:=\dim\frac{\mathcal{O}}{\mathcal{J}(()+\mathcal{I}}$, called the torsion number of (Y, X), and $J(h)$ is the Jacobian ideal of h.

Conjecture If Y is smooth, then

$$c_{Y}=\mu+(n-2)\lambda-n+1,$$

where μ is the Milnor number of h.

The conjecture is true for $n=1$, and for examples known to us in case $n=2$ and 3.

Question 2 Let $Y\subset X$ be an analytic space germs defined by ideals $I\supset H$ respectively. In a paper joint with Simis (presented at the conference *Differential equations and singularity theory* in RIMS), we defined higher order relative primitive ideals of I relative to X. For the topology of the Milnor fibre of functions in the (first) primitive ideal was investigated in the thesis of Jiang. The questions is to study the same object for functions in the second, or higher relative primitive ideals.

Problem stated by Toshizumi Fukui (福井敏純)

Classification of lines of curvatures for generic surfaces in $\mathbf{R}^{3}$ of the form

$$z=\frac{k}{2}(x^{2}+y^{2})+\frac{1}{6}(px^{3}+3qx^{2}y+3rxy^{2}+sy^{3})+\text{h.o.t.}$$

has been known as Darboux classification. So I would like to ask the classification of the next generic case; Classify the lines of curvature for generic surfaces of the form:

$$z=\frac{k}{2}(x^{2}+y^{2})+\frac{1}{24}(px^{4}+4qx^{3}y+6rx^{2}y^{2}+4sxy^{3}+ty^{4})+\text{h.o.t..}$$

It would be interesting, because they are the generic umbilics with indices $+1$, 0, -1. I also ask the explicit description of the degenerate locus. The degenerate locus with respect to indices can be described explicitly and it is expressed by a quartic surface with a singular line under some generic assumption. What else we need to describe the degenerate locus for the configuration of line of curvatures?