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Component mode synthesis method for large scale coupled structure-acoustic fluid interaction problem and its supercomputing

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A component mode synthesis method capable of estimating vibration responses efficiently in a high-frequency range is presented. In order to analyze the response in a high-frequency rañge, a high- and low-order truncatable mode superposition method is incorporated in the reduction process included in both the formulation of component mode synthesis and the solving of the superimposed global equation of motion. By truncating lower modes, it becomes possible to analyze a system with fewer degrees of freedom than by the existing modal reduction methods. Frequency response analyses of several test problems of damped structures are performed and compared with the multipurpose versatile code, MSC/NASTRAN, to demonstrate the efficiency and versatility of the proposed method.

Key Words: Component Mode Synthesis Method, Mode Superposition Techniques, Structural Dynamics, Finite Element Method

1. Introduction

Computer simulations are widely used as design tools for vibration problems ; in particular, finite element analysis is one of the most powerful tools for dealing with them. In automobile industries, it is applied to the analysis of the vibration of various components of the vehicle, i. e., vehicle body, engine and power train, and chassis equipments. However, even though their problems are analyzed using supercomputers and can be assumed to be linear, the applications of finite element simulations are still limited. For example, the finite element analysis results for vehicle bodies can be successfully applied to frequency phenomena up to 100 Hz, but they are not applicable in the higher frequency range, because the vehicle bodies have very complex vibrations when their frequencies become high. In order to simulate such vibrations, very fine meshes are required. This makes them impossible to compute using computers with standard storage memory and computing time.

The component mode synthesis method is very promising for dealing with such large complex structures, because, a very large number of degrees of freedom of the model can be reduced to a problem of smaller size by using the component synthesis technique, which can be computed using computer with less storage capacity. Since Hurty[1] and Craig and Bampton[2] presented their papers on the component mode synthesis method, a number of researches have been carried out on this field. The existing methods have been applied with success to the analyses for the low-frequency ranges. However, the analysis for the high-frequency range are yet to be accomplished. Reduction of the number of degrees of freedom using component mode synthesis is usually based on the mode superposition or Rayleigh-Ritz method. The number of degrees of freedom that can be eliminated are reduced in the case of the analysis for the high-frequency range, and it leads to a loss of efficiency.

We have developed a mode superposition method that is able to truncate the lower modes as well as the higher ones without loss of accuracy. Hence, the response in the high-frequency range can be calculated with fewer degrees of freedom. In this work, it is incorporated in the reduction process of component mode synthesis. The efficiency of the method is verified through numerical study, and it is applied to the vibration of a vehicle body.

2. Mode Superposition Method Adopted to the Reduction Process

In order to improve the accuracy of the mode superposition technique, an effective procedure is to add extra terms in order to compensate for the error caused by the truncation of the modes. Hansteen[3] proposed the addition of static response to improve accuracy. This is effective for the compensation of the truncated higher modes. However, it is not applicable for the compensation of lower modes. Hence, the number of degrees of freedom inevitably increases in the analysis of the high-frequency range. We have developed an alternative compensation procedure[4], i.e, the addition of the direct frequency response to the mode superposition. In this section, we briefly summarize the formulation of the mode superposition method.

Finite element equations of the motion of a system can be written as:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{f\}, \qquad (1)$$

where, $\{u\}$, $\{f\}$, $[K]$, $[C]$, and $[M]$ are the displacement vector, the external force vector, and the stiffness, damping and mass matrices with respect to the physical coordinate system. Assuming modal damping or proportional damping, the corresponding equations of motion can be transformed to uncoupled equations using modal coordinates.

$$\ddot{q}_i+2\xi_i\omega_i\dot{q}_i+\omega_i^2 q_i=\langle\phi_i^T\rangle\{f\} \quad (i=1,2,\cdots,N) \qquad (2)$$

Where, $\{\phi_i\}$ and q_i are the eigenvector and modal displacement of the i-th mode. N is the total number of degrees of freedom. If the external force is a monotonous sine wave and its circular frequency is ω_c, the displacement response of the i-th mode can be derived from Eq. (2) as

$$q_i=\frac{\langle\phi_i^T\rangle\{f\}}{\omega_i^2+2i\xi_i\omega_i\omega_c-\omega_c^2} \qquad (3)$$

The displacement vector $\{u\}$ with respect to the physical coordinate system is expressed as the superposition of the modal displacements q_i multiplied by the corresponding eigenvector ϕ_i.

$$\{u\}=\sum_{i=1}^{N}\{\phi_i\}q_i \qquad (4)$$

Now, superposing only m-th (>1) to n-th ($<N$) modes, and approximating the remaining truncated part of modal displacements q_i by Eq. (4), the displacement vector $\{u\}$ with respect to the physical coordinate system can be approximated as follows.

$$\{u\}\cong\sum_{i=m}^{n}\{\phi_i\}q_i+\sum_{i=1}^{m-1}\frac{\{\phi_i\}\langle\phi_i^T\rangle}{\omega_i^2+2i\xi_i\omega_i\omega_c-\omega_c^2}\{f\} + \sum_{i=n+1}^{N}\frac{\{\phi_i\}\langle\phi_i^T\rangle}{\omega_i^2+2i\xi_i\omega_i\omega_c-\omega_c^2}\{f\} \qquad (5)$$

If all the N eigenvectors and eigenvalues are used, the following relations hold with respect to the physical coordinate system.

$$([K]+i\omega_c[C]-\omega_c^2[M])^{-1}=\sum_{i=1}^{N}\frac{\{\phi_i\}\langle\phi_i^T\rangle}{\omega_i^2+2i\xi_i\omega_i\omega_c-\omega_c^2} \qquad (6)$$

By substituting Eq. (6) into Eq. (7), an approximation based on the mode superposition is finally obtained:

$$\{u\}\cong\sum_{i=m}^{n}\{\phi_i\}q_i+\left(([K]+i\omega_c[C]-\omega_c^2[M])^{-1} - \sum_{i=m}^{n}\frac{\{\phi_i\}\langle\phi_i^T\rangle}{\omega_i^2+2i\xi_i\omega_i\omega_c-\omega_c^2}\right)\{f\}, \qquad (7)$$

where ω_c is a parameter of the mode superposition method. By setting this parameter to the range of frequency in which the analyses are carried out, improved accuracy is expected.

3. Formulation of Component Mode Synthesis

If a system can be modeled as the assembly of j components, as shown Fig. 1, the corresponding finite element equations of motion of the system can be expressed as

$$\begin{bmatrix} M_{p1} \cdots & 0 & 0 \\ \vdots \ \ddots & \vdots & \vdots \\ 0 \ \cdots & M_{pj} & 0 \\ 0 \ \cdots & 0 & M_b \end{bmatrix}\begin{Bmatrix}\ddot{u}_{p1}\\ \vdots \\ \ddot{u}_{pj} \\ \ddot{u}_b\end{Bmatrix}+[C]\begin{Bmatrix}\dot{u}_{p1}\\ \vdots \\ \dot{u}_{pj} \\ \dot{u}_b\end{Bmatrix} + \begin{bmatrix} K_{p1} \cdots & 0 & K_{p1b} \\ \vdots \ \ddots & \vdots & \vdots \\ 0 \ \cdots & K_{pj} & K_{pjb} \\ K_{pb1} \cdots & K_{pbj} & K_b \end{bmatrix}\begin{Bmatrix}u_{p1}\\ \vdots \\ u_{pj} \\ u_b\end{Bmatrix}=\begin{Bmatrix}0\\ \vdots \\ 0 \\ f_b\end{Bmatrix}, \qquad (8)$$

where subscripts pj, and b indicate quantities of component j and their boundaries, respectively. Masses

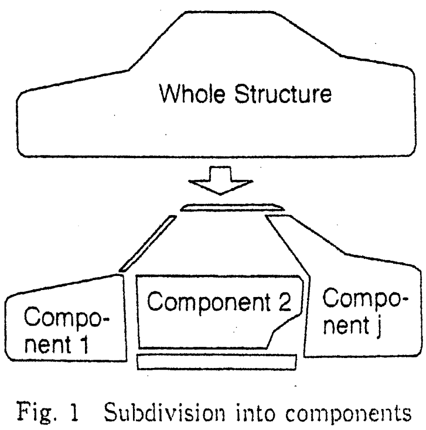


Fig. 1 Subdivision into components

are assumed to be lumped, and external force is considered only on boundaries of the substructures in Eq. (8).

Neglecting the damping effect, from Eq. (8), the equation of motion about the j-th components is derived as

$$[M_{pj}]\{\ddot{u}_{pj}\}+[K_{pj}]\{u_{pj}\}=-[K_{pjb}]\{u_b\}. \qquad (9)$$

We then apply the mode superposition method described in the previous section. From Eqs. (7) and (9), the following transforming relation for reduction of the number of degrees of freedom inside the components is obtained:

$$\begin{Bmatrix} u_{p1} \\ \vdots \\ u_{pj} \\ u_b \end{Bmatrix} = \begin{bmatrix} \phi_{p1} \cdots & 0 & G_{p1} \\ \vdots \ \ddots & \vdots & \vdots \\ 0 \ \cdots & \phi_{pj} & G_{pj} \\ 0 \ \cdots & 0 & I \end{bmatrix} \begin{Bmatrix} q_{p1} \\ \vdots \\ q_{pj} \\ u_b \end{Bmatrix} = [T]\{\bar{u}\} \qquad (10)$$

where

$$[G_{pj}]=\Big(([K_{pj}]-\omega_c^2[M_{pj}])^{-1} - \sum_{i=m}^{n}\frac{\{\phi_i\}\langle\phi_i^T\rangle}{(\omega_i^{pj})^2-(\omega_c)^2}\Big)[K_{pjb}] \qquad (11)$$

$$[\phi_{pj}]=[\phi_m^{pj}\cdots\phi_n^{pj}] \qquad (12)$$

$$\{q_{pj}\}=\langle q_m^{pj}\cdots q_n^{pj}\rangle^T. \qquad (13)$$

Substituting the transformation relation (Eq.(10)) into the global equation of motion (Eq.(8)), the following equations with reduced degrees of freedom are obtained:

$$[\bar{M}]\{\ddot{\bar{u}}\}+[\bar{C}]\{\dot{\bar{u}}\}+[\bar{K}]\{\bar{u}\}=\{\bar{f}\}, \qquad (14)$$

where

$$[\bar{M}]=[T]^T[M][T] \qquad (15)$$

$$[\bar{C}]=[T]^T[C][T] \qquad (16)$$

$$[\bar{K}]=[T]^T[K][T] \qquad (17)$$

$$\{\bar{f}\}=[T]^T\{f\}. \qquad (18)$$

In Eq. (14), the degrees of freedom with respect to the physical coordinate system inside the components are transformed into modal coordinates, but, the degree of freedom of the boundary of the components still remains. When the model is very large, the number of degrees of freedom of the boundary are also large. Therefore, the mode superposition method described in section 2 is applied again in order to reduce the number of degrees of freedom of the boundary.

Assuming modal damping, equations of motion of Eq. (14) can be rewritten, with respect to the modal coordinate, as

$$\ddot{\hat{q}}_i+2\hat{\xi}_i\hat{\omega}_i\dot{\hat{q}}_i+\hat{\omega}_i^2\hat{q}_i=\langle\hat{\phi}_i^T\rangle\{\bar{f}\} \ (i=1,2,\cdots,\hat{N}), \qquad (19)$$

where $\hat{N}$ is the total number of degrees of freedom at this stage. $\hat{q}_i$, $\hat{\omega}_i$, $\hat{\xi}_i$, and $\hat{\phi}_i$ are the displacement of modal coordinates, the circular eigenfrequency, the modal damping ratio, and the eigenvector, respectively. Then, applying the mode superposition method described in the section 2 again, and superposing m- to n-th modes of Eq. (19), the following relation with reduced degrees of freedom can be derived:

$$\{\bar{u}\}=[\hat{\phi}]\{\hat{q}\}+[\hat{G}]\{\bar{f}\}, \qquad (20)$$

where

$$[\hat{\phi}]=[\hat{\phi}_m\cdots\hat{\phi}_n] \qquad (21)$$

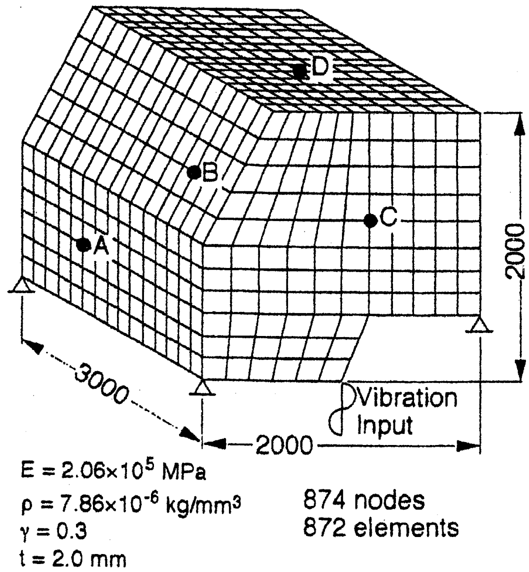
$$\{\hat{q}\}=\langle\hat{q}_m\cdots\hat{q}_n\rangle \qquad (22)$$

$$[\hat{G}]=([\bar{K}]+i\omega_c[\bar{C}]-\omega_c^2[\bar{M}])^{-1} - \sum_{i=m}^{n}\frac{\{\hat{\phi}_i\}\langle\hat{\phi}_i^T\rangle}{\hat{\omega}_i^2+2i\hat{\xi}_i\hat{\omega}_i\omega_c-\omega_c^2}. \qquad (23)$$

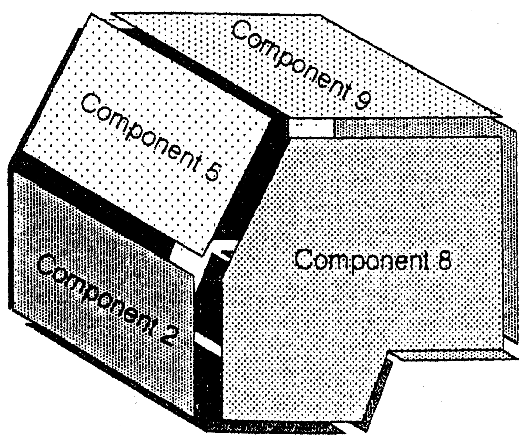
4. Numerical Examples

4.1 Analysis of simplified vehicle cabin

The component mode synthesis method mentioned above is implemented to MSC/NASTRAN (ver.67)[5] by modifying the NASTRAN source (DMAP; Direct Matrix Programming) code. In order to verify the efficiency of the proposed method, numerical study using a simplified model was carried out. Figure 2 (a) shows the finite element mesh of the model. Each panel of the cabin is modeled as a



(a) Finite element mesh



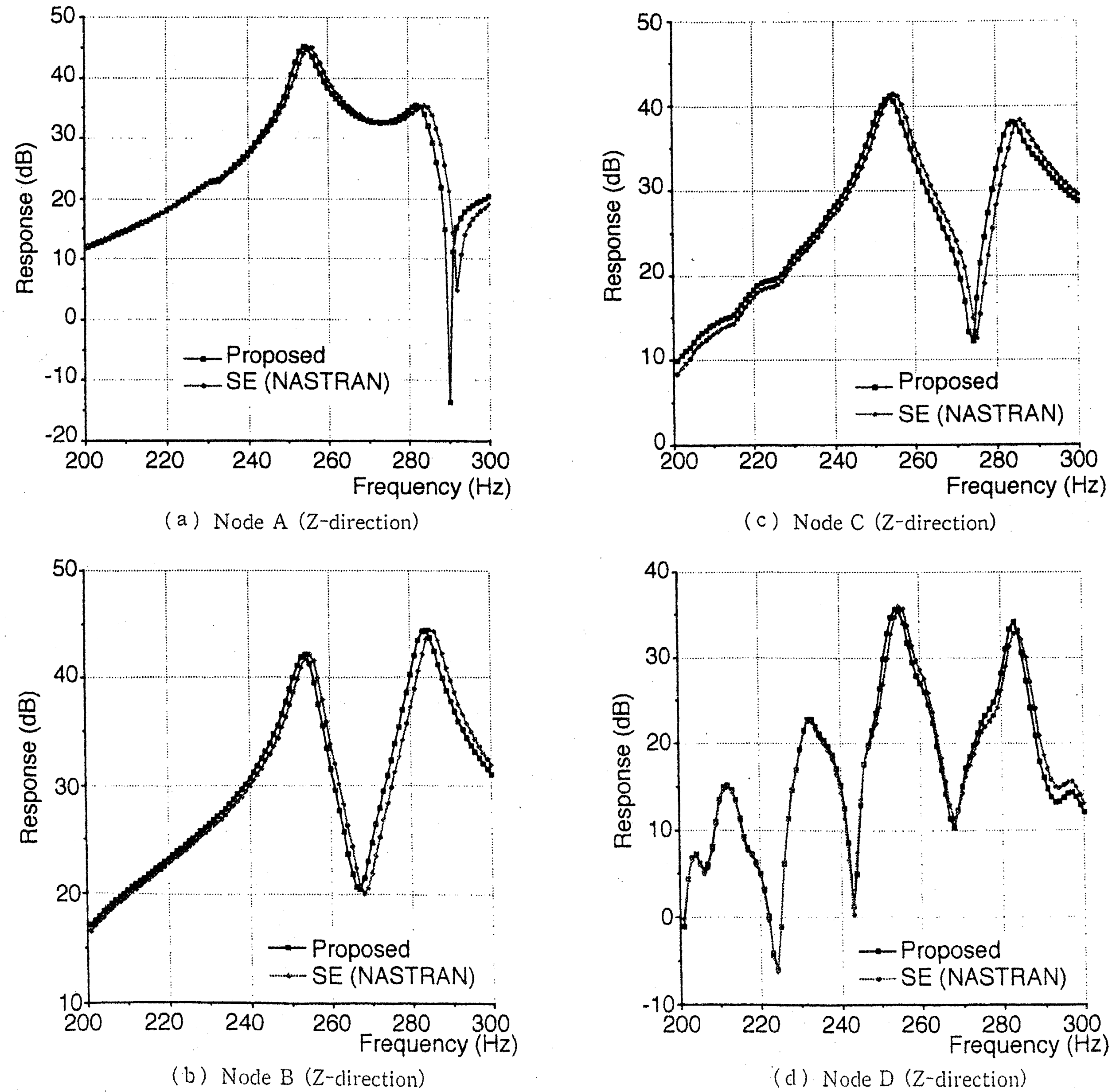
(b) Subdivision into components

Fig. 2 Simplified truck cabin model

component, as shown in Fig. 2(b). Numerical analysis using the MSC/NASTRAN (ver.67) superelement method is also carried out for comparison with the proposed method. For both analyses computed by the proposed method and NASTRAN, the component modes with frequencies over 600 Hz are truncated in the process of mode superposition. In the case of using the proposed method, component modes below 100 Hz are also truncated, and the frequency parameter of the mode superposition method is set to 250 Hz. Figure 3 shows the frequency responses on several nodal points. Table 1 shows the number of degrees of freedom for both cases. Comparison of CPU times required in this study is summarized in Table 2. In this simple case study, CPU time required in the proposed method in the process of reducing the

Table 1 Comparison of the number of degrees of freedom

		Proposed Method	Superelement (NASTRAN)
Frequency Range of the Used Eigenvectors (Hz)		100 - 600	0 - 600
Number of DOFs with respect to Modal Coordinates	Component 1	40	70
	Component 2	40	70
	Component 3	17	28
	Component 4	34	56
	Component 5	47	98
	Component 6	23	56
	Component 7	49	85
	Component 8	49	85
	Component 9	85	139
Number of DOFs on the Boundary of Components		1104	1104
Total		1488	1791
Number of Eigenvectors of Superposed Structure		354	704



(a) Node A (Z-direction)

(c) Node C (Z-direction)

(b) Node B (Z-direction)

(d) Node D (Z-direction)

Fig. 3 Frequency responses at several nodes

Table 2 Comparison of CPU time

CPU Time (sec)	Proposed Method	Superelenent (NASTRAN)
Generate Coefficient Matrices with respect to Physical Coordinate	13	8
Reduction of Degree of Freedom of all Structural Components using CMS	141	89
Eigenvalue Analysis of (Reduced) Structure	301	1044
Solving (Reduced) Equation and recovering data	34	52
Total	489	1193

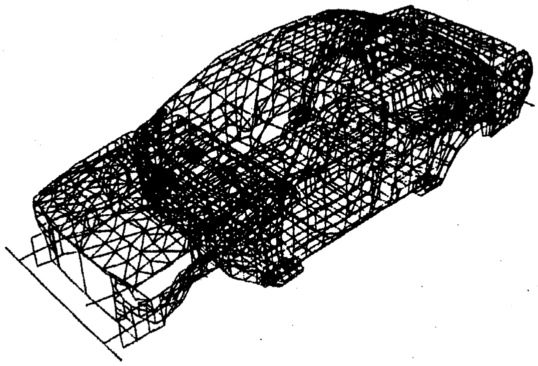
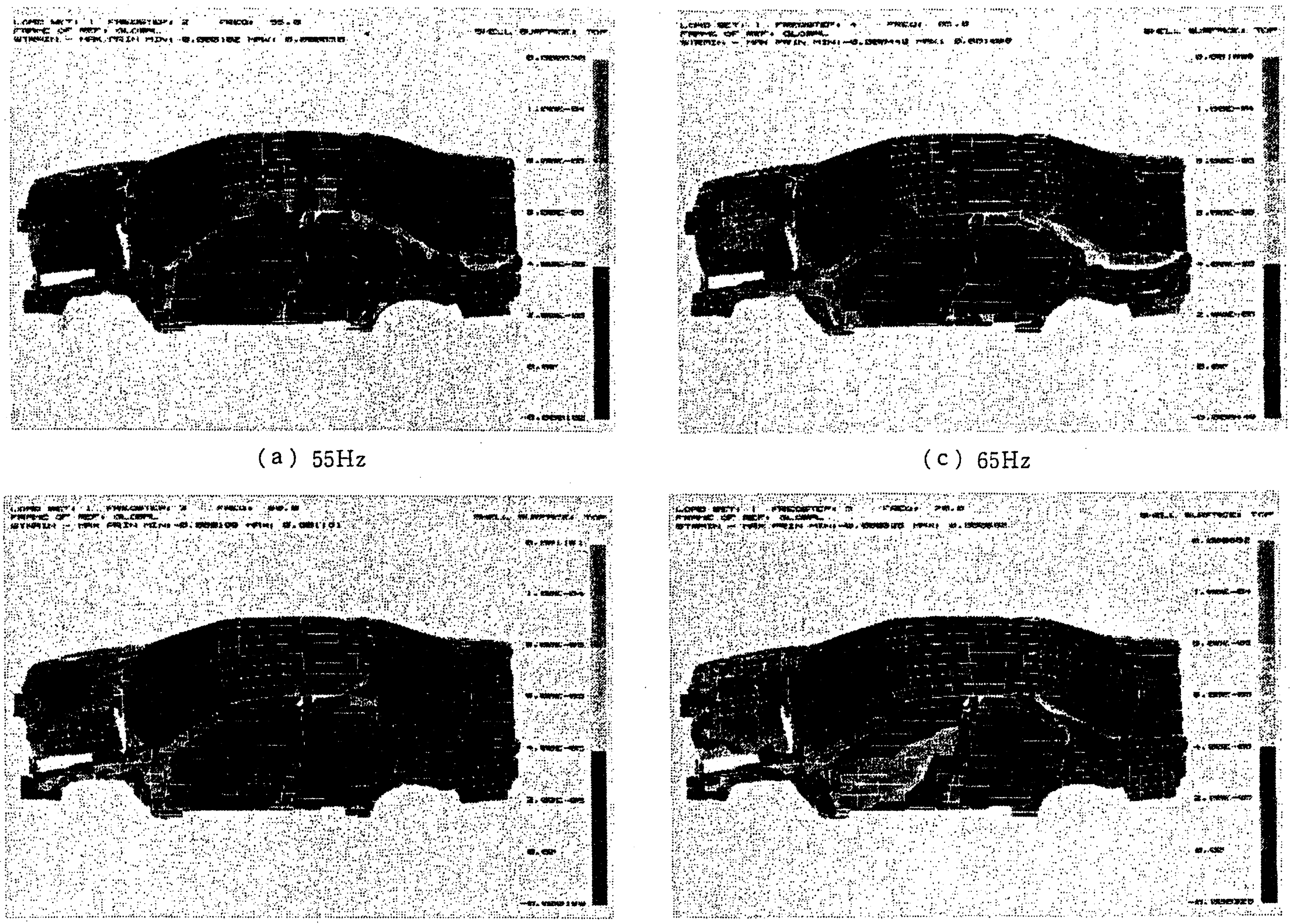


Fig. 4 Finite element mesh of the vehicle body



(a) 55Hz

(c) 65Hz

(b) 60Hz

(d) 70Hz

Fig. 5 Strain distribution on the vehicle body

number of degrees of freedom in the substructures is longer than when using NASTRAN, because the number of computations for generating the transformation matrix is increased even though the total number of component modes is decreased. However, the total number of degrees of freedom of the system after the elimination of degrees in the substructures is reduced due to the truncation of lower component modes, as shown in Table 1, and this leads to the decrease in the CPU time required for the eigenvalue analysis of the system. Therefore, in the present sample, the total CPU time needed for the proposed method is less than half of that required when using NASTRAN.

4.2 Analysis of real vehicle cabin model

The method is applied to the vibration analysis of a 4-door sedan model. The finite element mesh of the model is shown in Fig. 4. The total number of nodes is 3 638. Shell, solid, beam, spring, rigid, and lumped mass elements are used for modeling the vehicle body

The total number of elements is 5 391. The mesh size of this model is rather coarse for analysis in the high-frequency range, but the analysis is carried out in order to verify the versatility of the program implemented by the proposed method. The vehicle body panels were divided into 18 substructures (e.g., roof panel, floor panel, door, engine compartment, etc.). Input vibration is applied to the point of the front right shock absorber of the suspension. The strain distribution of the vehicle body with input frequencies of 55, 60, 65, and 70 Hz are shown in Fig. 5.

5. Conclusion

A component mode synthesis method is presented for analyzing the vibration problems in a high-frequency range. It is based on the mode superposition method that is able to truncate low modes as well as high ones. The results of this method are compared with the multi-purpose code MSC/NASTRAN through numerical study. Several sample solutions show that the analysis in the high-frequency range can be performed effectively using the present method.

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