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Optimizing multiple selection with a randam number of objects - full information case

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We consider the generalization of the no-information secretary problem to the full-information case, allowing also multiple choices and a random number of objects. The goal is to maximize the probability of choosing the overall best. Previously, different authors studied no-information cases with multiple choices and fixed number of objects; or, as Porosinski(1987) did, extended Presman and Sonin's secretary problem with random number of objects to the full-information case with a single choice. He showed that if (P) $d_j(x) \geq 0$ implies $d_{j+k}(y) \geq 0$ for $k \geq 1, y \geq x$, then the problem is monotone, where $d_j(x) \equiv P(N=j) - \int_x^1 \sum_{k>j} P(N=k) y^{k-j-1} dy$. It is reasonable to expect that if single-choice problem is monotone, then two-choices, three-choices, $\cdots$, m-choices problems are also monotone. We investigate this monotonicity related to the condition (P) through a recursive function on m constructed from the optimality equation. As an example, the case of uniform number of objects is studied. This case satisfies (P). The optimal stopping rules is shown to be a threshold rule with multiple threshold values, which can be described as follows: The optimal stopping time, when we can make m more choices, is $\tau_m = \min\{j \geq 1 : x \geq s_j^{(m)}\}$, where the threshold value $s_j^{(m)}$ is a unique solution in $[0,1]$ of the equation

$$h_j^{(m)}(x) = h_j^{(1)}(x) + \sum_{n>j} \int_{x \vee s_n^{(m-1)}}^1 x^{n-j-1} h_n^{(m-1)}(y) dy$$

with $h_j^{(1)}(x) = \sum_{n \geq j} x^{n-j} d_n(x)$. $s_j^{(m)}$ is nonincreasing in j and in m.