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Prediction Problem of Discrete Multiple Time Series

by

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Abstract

Here we present the way to make a prediction of discrete multiple time series. Our prediction method is easy to implement on Personal Computers and has a flexibility in applying to the real world data. It admit us to take into consideration some circumstances which can't be incorporated in ordinary discrete multiple time series prediction.

Subject areas: Prediction, Planning and Forecasting

1 Introduction

In Odanaka and Iwamura[9], we developed Prediction Theory of Discrete Multiple Time Series. . We rewrite here definitions. Let $a_k^{(i)}(1 \leq i \leq n)$ be the given n messages with a discrete time index k, let $b_k^{(i)}(1 \leq i \leq n)$ be the signals and $(b_k^{(i)} - a_k^{(i)})(1 \leq i \leq n)$ be noises. Define the auto-correlation functions

$$\varphi_{ij}^{dd}(v) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} b_{n+v}^{(i)} b_n^{(j)}, \tag{1}$$

$$\varphi_{ij}^{md}(v) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} a_{n+v}^{(i)} b_n^{(j)}, \tag{2}$$

$$\varphi_{ij}^{mm}(v) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} a_{n+v}^{(i)} a_n^{(j)}, \tag{3}$$

We assumed here that all these auto-correlation functions exist and that

$$\varphi_{ij}^{dd}(v) = \varphi_{ji}^{dd}(v) \tag{4}$$

holds for any v, i, j to derive a DP type algorithm to solve
the minimization problem

$$\min \lim_{N\to\infty} \frac{1}{2N+1} \sum_{k=-N}^{N} \left(a_{k+s}^{(1)} - \sum_{i=1}^{n} \sum_{\tau=0}^{M} A_i(\tau) b_{k-\tau}^{(i)} \right)^2 \tag{5}$$

, where $A_i(\tau)(1 \le i \le n, 0 \le \tau \le M)$ are variables to be determined.We easily see that for a stationary multiple time series assumptions (1),(2), (3) hold, whereas assumption (4) doesn't always.

Although there are some algorithms as for the prediction of a single time series in the books such as J.D.Hamilton[5], H.Tong[11], we don't have so much ones as for a multiple time series. Yet, we think that the existence of the functions in (1),(2),(3) and the symmetric condition (4) are not so trivial. If we use Carayannis, Kalouptsidis and Manolakis Algorithm[3] then we can drop condition(4). Still we are restricted to treat just only discrete multiple time series satisfying assumptions (1),(2),(3). We think that the DP algorithm in [9] will be useful for predicting in a long range for almost stationary multiple time series with symmetric condition.

Here we consider a multiple time series which is neither stationary nor symmetric. Hence, generally speaking, we can't make a prediction in a long range, because such a long range predicton will be proved incorrect in the future by a real data. For a bad natured multiple time series, we think that our Prediction Method is more useful in practice. So, we are dealing with a multiple time series for which AIC[1] is not applicable.We can find that there are some other results which were obtained in a similar spirit like ours. The reader can consult Kitagawa and Akaike[6], Chang and Lee[4], Zhao and Iwamura and Liu[13] for this purpose.

2 Prediction Problem of Discrete Multiple Time Series

On the contrary to Odanaka and Iwamura[9], we don't assume the existence of the autocorrelation functions. So, we deal with signals $b_k^{(i)}(1 \le i \le n)$ only.We don't care for $a_k^{(i)}(1 \le i \le n)$. Let's predict signal j at time $t+1$ and so let's write the predicted value $\hat{b}_{t+1}^{(j)}$.Consider a linear combination of $b_{k-\tau}^{(i)}(1 \le i \le n, s \le k \le t, 1 \le \tau \le M)$, i.e.,

$$\sum_{i=1}^{n}\sum_{\tau=1}^{M} a_{i\tau} b_{t-\tau}^{(i)} \tag{6}$$

for a positive integer M.Then let

$$I(M) = \sum_{k=s}^{t}\left(b_k^{(j)} - \sum_{i=1}^{n}\sum_{\tau=1}^{M} a_{i\tau} b_{k-\tau}^{(i)}\right)^2 \tag{7}$$

, where $s < t$ and we have signal observations $b_k^{(i)}(1 \le i \le n, s-M \le k \le t)$. Let M_0 be a given positive integer. For each $M = 1, 2, \cdots, M_0$, calculate $a_{i\tau} = \hat{a}_{i\tau}(1 \le i \le n, 1 \le \tau \le M)$ at which $I(M)$ takes its minimum. Then plot the predicted value $\hat{b}_{t+1}^{(j)}(M)$ for each $M, 1 \le M \le M_0$. We call this prediction model *Short Range Fuzzy Prediction Model*-Combined Model of Autoregressive type Prediction Model and Bellman Model, because we get M_0 predicted values for $\hat{b}_{t+1}^{(j)}$ using full information of $b_k^{(i)}(1 \le i \le n, s-M_0 \le k \le t)$. We use the term Fuzzy because we get M_0 predicted values with different resudials $I(M), M = 1, 2, \cdots, M_0$.

If we take a linear combination of $b_{k-\tau}^{(i)}(1 \le i \le n, i \ne j, s \le k \le t, 1 \le \tau \le M)$, then we get a *Bellman Model* in Bellman[2]. Lastly, if we take a linear combination of $b_{k-\tau}^{(j)}(1 \le \tau \le M)$, then we get a traditional *Autoregression type Model*, which uses only information of itself. Let's consider the situation where two persons are disputing each other what will be the most likely value for $b_{t+1}^{(1)}$ at time t. One predicts $\hat{b}_{t+1}^{(1)}$ based on the observation $b_k^{(1)}$, while the other predicts $\hat{b}_{t+1}^{(1)}$ based on the observation $b_k^{(2)}$ because he thinks that $b_k^{(2)}$ dominates $b_k^{(1)}$ and so insists that his predicted value be better with different pedicted value. In case their values differ largely nobody can determine the predicted value. Futhermore, as is the case for time series data which come from ecomomy, it is sometimes very difficult to determine from which time point to which time point should we take information in multiple time series data. In such situation, the two are advised to make compromize first on time interval from which the two should take information. Then go into the *Short Range Fuzzy Prediction*-Combined Model of Autoregressive type Prediction Model and Bellman Model.

3 An Example

We take two Japanese land price indexes $b_k^{(1)}$ and $b_k^{(2)}$ for 20 years $(1 \leq k \leq 20)$. $b_k^{(1)}$ is a land price index of down town Utsunomiya(merchant use), where $b_k^{(2)}$ is a land price indexof Suginami ward Tokyo(residential use). Hence $n = 2$. Set $1 < s < t \leq 20$ and M_0 so that $1 \leq s - M_0$ holds.In this case, Mayor of Utsunomiya city and its resident living in down town Utsnomiya are disputing on the predicted land price index. Then what model would be the best and what is its predicted value? Writing

$$\chi_{i\tau} = \sum_{k=s}^{t} b_k^{(1)} b_{k-\tau}^{(i)}, \tag{8}$$

$$\varphi_{ij}(\tau,\sigma) = \sum_{k=s}^{t} b_{k-\tau}^{(i)} b_{k-\sigma}^{(j)} (1 \leq i, j \leq 2; 1 \leq \tau, \sigma \leq M) \tag{9}$$

we have to maximize

$$\hat{E}_M = 2\sum_{i=1}^{2}\sum_{\tau=1}^{M} a_{i\tau}\chi_{i\tau} - \sum_{i,j=1}^{2}\sum_{\tau,\sigma=1}^{M} a_{i\tau}a_{j\sigma}\varphi_{ij}(\tau,\sigma) \tag{10}$$

, where $a_{i\tau}(1 \leq i \leq 2; 1 \leq \tau \leq M)$ are variables whose values should be determined. Differetiating (10) by $a_{i_0\tau_0}$, we get

$$\sum_{\tau=1}^{M}((\varphi_{11}(\tau,\tau_0) + \varphi_{11}(\tau_0,\tau))a_{1\tau} + (\varphi_{21}(\tau,\tau_0) + \varphi_{12}(\tau_0,\tau))a_{2\tau}) = 2\chi_{1\tau_0} \tag{11}$$

$$\sum_{\tau=1}^{M}((\varphi_{12}(\tau,\tau_0) + \varphi_{21}(\tau_0,\tau))a_{1\tau} + (\varphi_{22}(\tau,\tau_0) + \varphi_{22}(\tau_0,\tau))a_{2\tau}) = 2\chi_{2\tau_0} \tag{12}$$

Here, we have $2M$ equations with $2M$ unkowns $(a_{1\tau}, a_{2\tau})^t, 1 \leq \tau \leq M$. In order to successively predict $\hat{b}_{t+1}^{(1)}$ for $M = 1, 2, \cdots, M_0$,i.e., *in a fuzzy sense*, we use Carayannis,Kalouptsidis and Manolakis algorithm[3]. Computational results would be shown at the end of the paper.

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Autoregression type Model

Input data s =   16 t =   19 m0 =    4
b( 1,  1 ) =   1.3     b( 2,  1 ) =     1
b( 1,  2 ) =   1.3     b( 2,  2 ) =   1.1
b( 1,  3 ) =   1.3     b( 2,  3 ) =   1.3
b( 1,  4 ) =   1.3     b( 2,  4 ) =   1.5
b( 1,  5 ) =   1.4     b( 2,  5 ) =   1.8
b( 1,  6 ) =   1.5     b( 2,  6 ) =   2.5
b( 1,  7 ) =   1.6     b( 2,  7 ) =   2.7
b( 1,  8 ) =   1.7     b( 2,  8 ) =   2.8
b( 1,  9 ) =  1.75     b( 2,  9 ) =   2.9
b( 1, 10 ) =  1.75     b( 2, 10 ) =   3.2
b( 1, 11 ) =   1.8     b( 2, 11 ) =   7.7
b( 1, 12 ) =   2.4     b( 2, 12 ) =   9.5
b( 1, 13 ) =  2.45     b( 2, 13 ) =   8.6
b( 1, 14 ) =   3.4     b( 2, 14 ) =   8.3
b( 1, 15 ) =   4.4     b( 2, 15 ) =   8.2
b( 1, 16 ) =   4.4     b( 2, 16 ) =   7.2
b( 1, 17 ) =     4     b( 2, 17 ) =   5.4
b( 1, 18 ) =   3.7     b( 2, 18 ) =   4.6
b( 1, 19 ) =   3.4     b( 2, 19 ) =   4.2

BHAT[M], i[M], for m =  1:     3.19772  0.0978585
   a[1] : 1.19118
   a[2] : -0.257021
BHAT[M], i[M], for m =  2:     3.09903   0.018042
   a[1] : 1.30016
   a[2] : -0.486685
   a[3] : 0.13196
BHAT[M], i[M], for m =  3:     3.14766  0.00649255
   a[1] : 1.49012
   a[2] : -0.843548
   a[3] : 0.476912
   a[4] : -0.190382
BHAT[M], i[M], for m =  4:     3.01526 2.73295e-26
```

FIGURE 1

Bellman Model
Input data S = 16 T = 19 M0 = 4 τ1 = 1

B(1, 1) =	0.3	B(2, 1) =	1
B(1, 2) =	1.3	B(2, 2) =	1.1
B(1, 3) =	1.3	B(2, 3) =	1.3
B(1, 4) =	1.3	B(2, 4) =	1.5
B(1, 5) =	1.4	B(2, 5) =	1.8
B(1, 6) =	1.5	B(2, 6) =	2.5
B(1, 7) =	1.6	B(2, 7) =	2.7
B(1, 8) =	1.7	B(2, 8) =	2.8
B(1, 9) =	1.75	B(2, 9) =	2.9
B(1, 10) =	1.75	B(2, 10) =	3.2
B(1, 11) =	1.8	B(2, 11) =	7.7
B(1, 12) =	2.4	B(2, 12) =	9.5
B(1, 13) =	2.45	B(2, 13) =	8.6
B(1, 14) =	3.4	B(2, 14) =	8.3
B(1, 15) =	4.4	B(2, 15) =	8.2
B(1, 16) =	4.4	B(2, 16) =	7.2
B(1, 17) =	4	B(2, 17) =	5.4
B(1, 18) =	3.7	B(2, 18) =	4.6
B(1, 19) =	3.4	B(2, 19) =	4.2
B(1, 20) =	0	B(2, 20) =	0

R=

169.4	190.82	207.68	222.36
190.82	217.13	237.36	253.41
207.68	237.36	261.93	280.18
222.36	253.41	280.18	300.34

D=

-100.5	-114.32	-125.86	-134.79

BHAT[M], i[M] for M = 1	2.49174	0.986328
BHAT[M], i[M] for M = 2	2.42491	0.41935
BHAT[M], i[M] for M = 3	2.63601	0.000155738
BHAT[M], i[M] for M = 4	2.39801	8.21925e-22

FIGURE 2

```
 Proposed Method
 Input data  S=  16 T=  19 MO=    4
 B(1,  1)=  1.3 B(2,  1)=  1.0
 B(1,  2)=  1.3 B(2,  2)=  1.1
 B(1,  3)=  1.3 B(2,  3)=  1.3
 B(1,  4)=  1.3 B(2,  4)=  1.5
 B(1,  5)=  1.4 B(2,  5)=  1.8
 B(1,  6)=  1.5 B(2,  6)=  2.5
 B(1,  7)=  1.6 B(2,  7)=  2.7
 B(1,  8)=  1.7 B(2,  8)=  2.8
 B(1,  9)=  1.8 B(2,  9)=  2.9
 B(1, 10)=  1.8 B(2, 10)=  3.2
 B(1, 11)=  1.8 B(2, 11)=  7.7
 B(1, 12)=  2.4 B(2, 12)=  9.5
 B(1, 13)=  2.5 B(2, 13)=  8.6
 B(1, 14)=  3.4 B(2, 14)=  8.3
 B(1, 15)=  4.4 B(2, 15)=  8.2
 B(1, 16)=  4.4 B(2, 16)=  7.2
 B(1, 17)=  4.0 B(2, 17)=  5.4
 B(1, 18)=  3.7 B(2, 18)=  4.6
 B(1, 19)=  3.4 B(2, 19)=  4.2
R=  0.1368200D+03  0.2127600D+03  0.1334400D+03  0.2427600D+03
R=  0.1192400D+03  0.2676000D+03  0.1024400D+03  0.2863600D+03
R=  0.2127600D+03  0.3388000D+03  0.2034400D+03  0.3816400D+03
R=  0.1771400D+03  0.4153600D+03  0.1518400D+03  0.4447200D+03
R=  0.1334400D+03  0.2034400D+03  0.1325600D+03  0.2351600D+03
R=  0.1205000D+03  0.2612800D+03  0.1030000D+03  0.2789200D+03
R=  0.2427600D+03  0.3816400D+03  0.2351600D+03  0.4342600D+03
R=  0.2073100D+03  0.4747200D+03  0.1765000D+03  0.5068200D+03
R=  0.1192400D+03  0.1771400D+03  0.1205000D+03  0.2073100D+03
R=  0.1125650D+03  0.2341000D+03  0.9706000D+02  0.2502300D+03
R=  0.2676000D+03  0.4153600D+03  0.2612800D+03  0.4747200D+03
R=  0.2341000D+03  0.5238600D+03  0.2010700D+03  0.5603600D+03
R=  0.1024400D+03  0.1518400D+03  0.1030000D+03  0.1765000D+03
R=  0.9706000D+02  0.2010700D+03  0.8536500D+02  0.2163400D+03
R=  0.2863600D+03  0.4447200D+03  0.2789200D+03  0.5068200D+03
R=  0.2502300D+03  0.5603600D+03  0.2163400D+03  0.6006800D+03
 d=-0.129D+03-0.201D+03-0.125D+03-0.229D+03
 d=-0.111D+03-0.252D+03-0.958D+02-0.270D+03
 BHAT(M), I(M)  FOR M= 1          0.30749D+01          0.47146D-01
 BHAT(M), I(M)  FOR M= 2          0.30552D+01          0.42970D-23
 BHAT(M), I(M)  FOR M= 3          0.30552D+01          0.43409D-23
 BHAT(M), I(M)  FOR M= 4          0.30552D+01          0.43370D-23
```

FIGURE 3