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On the partial cooperative games

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Resume

1 Introduction.

Considering the cooperative games in extensive form, we suppose that the coalition membership of players does not change during the game. In this paper we propose a model of dynamic conflict process where conflict participants (players) alter their coalition membership during the process evolution.

Consider a finite non–cooperative game in extensive form with perfect information $\Gamma = \langle K(x_0), P, h\rangle$, where $K(x_0)$ is the game tree with the initial point x_0, P is the player partition $P_1, P_2, \ldots, P_n, P_{n+1}$ (P_{n+1} is the set of endpoints), and $h : P_{n+1} \rightarrow R_+^n$ is the terminal payoff function. Basing on Γ we introduce a class of partial coopertive games where players use both cooperative and non–cooperative behavior.

Take a decision point $x \in K(x_0)$. Assume that x be the decision point of player i. We shall say that player i does not cooperate at x if player i's decision at x is first of all optimal for him. We shall also say that player i cooperates at x if there is a coalition $S \subset N = \{1, \ldots, n\}$, $i \in S$, and the decision of player i at x is first of all optimal for S. Suppose that in pre–play communications each player i defines a set of nodes X_i such that at each decision point in $P_i \cap X_i$ player i is ready to cooperate with anyone. The set X_i is called the cooperative region of player i.

Construct a game where players carry out the following behavior. For a decision point $x \in P_i$ player i is purposed to keep on an individual behavior if $x \notin X_i$. If $x \in X_i$, then at x player i have to be in the coalition $S(x) = \{j \in N | x \in X_j\}$ involving players whose cooperative regions contain x. We call the given behavior the partial cooperative one. The game where players use the partial cooperative behavior called game with partial cooperation.

The classes of partial cooperative games are different with the methods of the player's cooperative region formalization. We propose three approaches.

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2 Model 1.

In [4] the timing interpretation of cooperative regions are considered. It is supposed that the game tree has a special structure. For any game evolution players make decisions in an index order, i.e., at the initial point the decision is made by player 1, at the immediate successors of x_0 the decisions are made by player 2 and so on, after decisions of player n the decisions are again made by player 1 and etc. The set of n sequential moves starting from player 1, is a stage of the game. Also, it assumed that each path of the game has the same length $T+1$. Let a vector $s = (s_1, \dots, s_n)$, $s_i \in \{0, 1, \dots, T+1\}$, $i \in N = \{1, \dots, n\}$, be given in Γ. Note that each stage starts from the decision points of player 1. Denote by $P_1^\tau \subset P_1$ the set of player 1's decision points in the stage τ. The game with partial cooperation, where

$$X_i = \{K(x) | x \in P_1^{t_i}, t_i = T + 1 - s_i\}, \quad i \in N,$$

is denoted by $\Gamma_s(x_0)$. Here, $K(x)$ is the subtree with the initial point x. It is said that s_i is the length of player i's cooperative activity in $\Gamma_s(x_0)$.

3 Model 2.

In [5] an extension of the model 1 is considered. Suppose that the information structure of the game tree and the payoff functions are the same as in the game $\Gamma_s(x_0)$. Assume that in the pre–play communication each player $i \in N$ chooses two stages d_{i1} and d_{i2} satisfying the following conditions. If $d_{i1} \neq T+1$, then $d_{i1} < d_{i2}$. If $d_{i1} = T+1$, then $d_{i1} = d_{i2}$. Here, d_{i1} is the stage of the start of player i's cooperation and $d_{i2} - 1$ is the stage of the end of player i's cooperation. The choices of players are presented by a matrix $D = \{d_{ij}\}$, $i \in N$, $j = 1, 2$. During the game evolution player i have to be ready to cooperate with anyone in the stages $\{d_{i1}, \dots, d_{i2} - 1\}$. In the other stages player i have to play non–cooperatively. In this case the player i's cooperative region is

$$X_i = \{K(x) \setminus (\cup_{y \in P_1^{d_{i2}} \cap K(x))} K(y) | x \in P_1^{d_{i1}}\}.$$

4 Model 3.

In the models 1, 2 the cooperative region has a timing meaning. For any pass of the game evolution player has to cooperate since a given stage. In [6] a weakened definition of the cooperative region is proposed. Suppose that the tree $K(x_0)$ of the game Γ has an arbitrary information structure. We assume that the player's cooperative activity depends on the game evolution. As it is shown in [6], the given approach can be formalized with help of a function called cooperative one. Define functions $f_i : P_i \to \{0, 1\}$, $i \in N$. We shall say that player i cooperates in the decision point $x \in P_i$ if $f_i(x) = 1$. In the case $f_i(x) = 0$, player i plays in x individually. $f = (f_1, \dots, f_n)$ is called a cooperative function if for each player i from the condition $f_i(x) = 1$, $x \in P_i$, it follows that player i cooperates in each of his decision point on the subtree $K(x)$. We denote the given partial cooperative game by $\Gamma_f(x_0)$. The cooperative region of player i in $\Gamma_f(x_0)$ is

$$X_i = \{K(x) | f_i(y) = 1, \forall y \in P_i \cap K(x)\}.$$

5 Coalition structures.

In each proposed partial cooperative game model we can observe the same dynamic of changing of coalition structures. For any decision point x, if x belongs to an intersaction of two and more cooperative regions, at x the player set N is separated into a multi–player coalition $S(x)$ and $|N \setminus S(x)|$ non–cooperating players. Hence, instead of n players in Γ, at x of the partial cooperative game there are $|N \setminus S(x)| + 1$ players, where $S(x)$ is considered as a player–coalition. In the models 1, 3 the number of members of the player–coalition does not decrease during the game evolution. In the model 2 the membership of the player–coalition can both increase and decrease. If the partial cooperative game ends at an endpoint $\bar{x} \in P_{n+1}$, then the payoff of the player–coalition is

$$\sum_{i \in S(\bar{x})} h_i(\bar{x}).$$

The players who do not cooperate in $\bar{x}$ get a payoff in accordance with the terminal payoff function. We propose that the payoffs of players who cooperate in $\bar{x}$ are defined by an optimal in some sense imputation of the player–coalition payoff. In the mentioned models the Shapley value is considered as the optimal imputation. In general case we can take the core, the dominance core and other solutions of cooperative games. The usage of the Shapley value is motivated by its "good" characteristics: the Shapley value is unique and always exists. Also the Shapley value has an appropriate interpretation: in accordance with the Shepley value the shares of players in the payoff of the player–coalition are mathematical expectations.

6 The main results.

1) For all models a path construction procedure is proposed ([4]—[6]). The proposed approach based on the use both Nash equilibrium solution ([3]) and solutions from the cooperative game theory ([7]). We proceed by backward induction, contructing step by step the optimal way of players behavior. The procedure is similar to the one used in the scheme of subgame–perfect Nash equillibrium construction. The difference is in the following. We separate the game tree into the zones such that in each decision point of a zone the coalition structure is the same. Inside a zone with the constant coalition structure we can apply the Nash scheme. Moving from one zone to another, we construct a cooperative subgames to define the share of payoff of players who leave (enter) the multi–player coalition.

2) Denote the constructed path by $\Phi(x_0) = \{x_0, \dots, x^*\}$, $x^* \in P_{n+1}$. In [4]—[6] it is shown that there is a payoff–vector $\bar{r}(x^*)$ related to $\Phi(x_0)$. In general case $\bar{r}(x^*) \neq h(x^*)$. The payoff–vector $\bar{r}(x^*)$ consists form the values of the terminal payoff functions for those players who are not in the multi–player coalition $S(x^*)$, and the components of the Shapley value of the payoff of multi–player coalition $S(x^*)$ for those players who are in $S(x^*)$. The payoff–vector $\bar{r}(x^*)$ is called the value of the partial cooperative game.

3) Denote the value of partial cooperative game in the model 1 by $r(s)$. Let L be the set of all vectors s that can be defined on Γ. Let $\Gamma_L(x_0)$ be a partial cooperative game where players define the vector s in pre–playing communications. After selection of the vector s, the game $\Gamma_L(x_0)$ proceeds as $\Gamma_s(x_0)$. In [4] it is shown that if the function $v : L \to R^1_+$

defined by $v(s) = \sum_{i \in N, s_i > 0} r_i(s)$ is superadditive, then $\Gamma_L(x_0)$ is a multi–choice game (L, v) ([1]).

7 Conclusion.

Consider the case when it is impossible to define a superadditive function v for $\Gamma_L(x_0)$. For such $\Gamma_L(x_0)$ we can propose a scheme of the Nash equilibrium construction. The method is based on the considering of an auxiliary non–cooperative game $\overline{\Gamma}$ with the binary tree $\overline{K}(x_0)$ having the length $T+1$ stages (the same with $K(x_0)$). Let the strategies in a decision points of $\overline{K}(x_0)$ be called "to go left" and "to go right" respectively. We propose that if in accordance with the defined vector s player i have to cooperate in a stage t in $\Gamma_L(x_0)$, then he has to go right in all his decision points on $\overline{K}(x_0)$ in the stage t in $\overline{\Gamma}$. If player i does not cooperate in the stage t in $\Gamma_L(x_0)$, then he has to go left in all his decision points on $\overline{K}(x_0)$ in the stage t in $\overline{\Gamma}$. Hence, we can apply the standard scheme of the Nash equilibrium construction for $\overline{\Gamma}$. It is expected that the obtained solution is the Nash equlibrium of $\Gamma_L(x_0)$.

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