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On the Global Stability, Dynamic Trade Patterns and Asset-Debt Positions of the Two Country, Two Good Endogenous Growth Model with Adjustment Costs of Educational Investment

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Abstract

The dynamic trade patterns and asset-debt positions of the two country, two good endogenous growth model are analyzed for two types of adjustment costs of educational investment. The existence and uniqueness of the stationary state and the global stability of the closed economy are shown first. Then the properties of the world competitive equilibrium are derived by observing the equivalence between the competitive equilibrium and the social planner's optimum. The advanced country with greater initial national wealth can be an importer of the good and a creditor throughout the transitional period.

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I. Introduction

The purpose of this paper is to analyze the global stability and the dynamic trade patterns of the two sector endogenous open model with adjustment costs of educational investment. As a preliminary task, the uniqueness of the stationary state, the global stability and the optimal per capita consumption path in a closed economy are derived. We assume utility is maximized over time subject to the laws of motion of physical capital and human capital.

Following the standard two sector endogenous model of a good sector and an educational service sector (see Mino (1996) and Bond and Wong and Yip (BWY) (1996)), both the good and the educational service are produced under constant-returns-to-scale technologies employing physical capital and human capital. The good is used for both consumption and physical investment. Education (educational service) is used to increase human capital, and is subject to adjustment costs. That is, given the amount of human capital only part of educational service is used for educational investment to increase human capital since part of educational service is lost to adjustment costs, Such a characteristic of educational investment adjustment costs is pointed out by Barro and Sala-i-Martin (1995, Chapter 5). They assume that it takes more time to increase human capital than physical capital due to such adjustment costs, suggesting as evidence the long period of economic stagnation after the Black Death in Europe (Hirschleifer 1987, Chapters 1 and 2). To my knowledge, no attempts have been made to incorporate adjustment costs of educational investment into the endogenous growth model. First the uniqueness and the existence of the stationary state of the closed economy are derived (Theorem 1). Then the global stability (Theorem 2) and the characteristic of the optimal per capita consumption path of the closed economy are derived generalizing the results of Mino (1996) and BWY (1996).

Next, based on the results of the closed economy, the dynamic trade patterns and asset-debt positions are discussed assuming two identical countries (the home country and the foreign country) with different amounts of initial gross national wealth. In the open economy without adjustment costs of educational investment capital-labor ratios of both countries become equalized always, which does not seem to be realistic. This is one of the rationalization as to why such adjustment costs should be introduced. First the global stability of such an open economy is shown (Theorem 3). Here the home country, possessing greater initial gross national wealth, can be an importer of goods as well as a creditor throughout entire transitional period. (Theorem 4) In short, trade patterns and asset-debt positions remain stable. This seems consistent with the historical experience of several large economies. If we review the long run trends of the U.K., U.S.A., German and Japanese trade accounts and returns of foreign investment reflecting their asset-debt positions, we can conclude that these countries' trade account patterns and returns on foreign investment have remained stable.

For the U. K. trade accounts have remained negative since the 1820's, while returns on foreign investment became positive by the 1810's and have remained so since.

Similarly for the U.S.A., the trade accounts balance became negative in the 1970's and has remained so since, while the returns on foreign investment have remained positive since the 1910's.

For Germany, the trade accounts balance has remained positive since the 1950's, while the returns on foreign investment have remained positive since the 1980's.

For Japan, the trade accounts balance has been positive since the 1960's, while the returns on foreign investment have been positive since the 1970's. [1/]

In the next section, the model of the closed economy is introduced.

II. Closed Economy

Let X and Y be respectively the amounts of the good and of educational service produced using physical capital and human capital. Let K and H be respectively the physical capital and human capital endowments used in the two sectors. The amount of X depends on Y, K and H, and goods are used either for consumption C or physical investment I.

Hence X is expressed as;

$$X = X(K, H, Y) = C + I \tag{1}$$

where the function X represents the production possibility curve which is concave and homogeneous of degree one in (K, H, Y). The equation of motion of physical capital is expressed as;

$$\dot{K} = I - \delta K \tag{2}$$

where $\dot{K}$ is the time rate of change in K and $\delta > 0$ is the constant depreciation rate of physical capital (Every variable is a function of time. But time dependency is omitted for notational simplicity unless necessary. If it is necessary, it is denoted, e.g., as $K = K(t)$.). As discussed by Barro & Sala-i-Martin (1995), the adjustment costs of educational investment seem much higher than those of physical investment. In fact, presumably it takes much more time for the human capital to recover to the original level once destroyed by say, epidemic (as in the case of the Black Death) than for the physical capital destroyed by say, war. Then as a rough approximation to the reality, it would be appropriate to assume the absence of the adjustment costs of physical capital, while its presence of the human capital. Then, taking into accounts of the adjustment costs of the educational investment, the equation of motion of human capital is expressed as;

$$\dot{H} = G(Y, H) - \eta H \tag{3}$$

where $\eta > 0$ is the constant depreciation rate of human capital. The function G reflects the adjustment costs of educational investment. G is concave and homogeneous of degree one in (Y, H). By letting $g(y) = G(y, 1)$ where $y = Y / H$ being the per capita educational service, we observe $g(0) = 0$, $g'(y) > 0$, $g'(0) = 1$ and $g''(y) < 0$ due to the adjustment costs. Intuitively this implies that given the amount of human capital H and the amount of education Y, only $g \cdot Y(< Y)$ helps to increase physical capital. We call this the g-type adjustment cost. This type of adjustment cost was introduced first by Uzawa (1969) in the context of adjustment cost of physical capital. Here we introduce another type of adjustment cost called the ϕ-type.

(1) is replaced by;

$$X = X(K, H, Y(1+\phi)) = C + I \tag{1'}$$

where $\phi = \phi(y) \geq 0$ with $\phi(0) = 0$, $\phi'(y) > 0$ and $\phi''(y) > 0$. (2) remains valid. (3) is changed into;

$$\dot{H} = Y - \eta H . \tag{3'}$$

Intuitively, the ϕ-type of adjustment cost implies that given the amounts of physical capital H and educational service $Y(1+\phi)\,(> Y)$, only Y units of the educational service contribute to increase human capital. This type of adjustment cost was introduced by Eisner and Strotz (1963), Lucas (1967) and Abel and Blanchard (1983), among others. Henceforth we analyze only the g-type case. The results of the ϕ-type case are shown in Appendix III.

Utility Maximization

Here we consider the following utility maximization problem the social planner faces;

$$\max \int_0^\infty \frac{1}{1-\sigma} C^{1-\sigma} e^{-\rho t} dt$$

subject to (1), (2) and (3) where $\sigma > 0$ is the constant intertemporal rate of substitution of consumption, $\rho > 0$ is the constant time preference rate and $t = 0$ is the initial time. By constructing the following current value Hamiltonian

$$\widetilde{H} = \frac{1}{1-\sigma} C^{1-\sigma} + \mu(X(K, H, Y) - C - \delta K) + \lambda(G(Y, H) - \eta H) \tag{4}$$

we obtain the first order conditions;

$$C^{-\sigma} = \mu \tag{5}$$

$$-\mu X_Y = \lambda G_Y \tag{6}$$

$$\dot{\mu} = \rho\mu - \mu X_K + \delta\mu \tag{7}$$

$$\dot{\lambda} = \rho\lambda - \mu X_H - \lambda G_H + \eta\lambda \tag{8}$$

and the transversality conditions $\lim_{t\to\infty} \mu K e^{-\rho t} = 0$, and $\lim_{t\to\infty} \lambda H e^{-\rho t} = 0$, where μ and λ are interpreted respectively as the shadow prices of physical capital and of human capital. By letting $p = -X_y$ be the relative price of education (the good is numeraire), $q = \lambda / \mu$ be the relative shadow price of human capital, $r = X_K$ be the rental price of physical capital, $w = X_H$ be the wage rate, $k = K / H$ be the capital intensity, and $c = C / H$ be the per capita consumption, we obtain the following system of differential equations;

$$\dot{q} / q = r - \delta - w / q - g(y) + yg'(y) + \eta \tag{9}$$

from (7), (8) and $G_H(Y, H) = g(y) - yg'(y)$,

$$\dot{k} / k = (x - c) / k - \delta - g(y) + \eta \tag{10}$$

from (1), (2) and (3) where $x = x(k, y) = X(k, 1, y)$,

$$\dot{c} / c = (r - \rho - \delta) / \sigma - g(y) + \eta \tag{11}$$

from (3) and (5), and

$$p = qg'(y) \tag{12}$$

from (6). Here we note $r = r(p)$ and $w = w(p)$.

Existence and Uniqueness of the Stationary State

Next we consider the existence and the uniqueness of the stationary state where physical capital, human capital and consumption grow at the same rate, n and the relative shadow price of human capital q remains unchanged. Here we introduce the following assumptions;

A. 1 Both sectors satisfy the Inada condition.

A. 2 For $\sigma < 1$, $-n_0 < n < \rho / (1 - \sigma)$ holds and for $\sigma > 1$, $-n_1 < n$ holds where $n_0 = \min(\eta,\ (\rho + \delta) / \sigma,\ \delta)$ and $n_1 = \min(\eta,\ \rho / (\sigma - 1),\ (\rho + \delta) / \sigma, \delta)$.

A. 1 is assumed throughout the paper and A. 2 is for the g-type cost case. (A. 2 is replaced by A. 2' for the ϕ-type cost case.)

A. 2 is required for all variables to be positive and generalizes BWY's (1996) assumption $\rho - (1 - \sigma)n > 0$ and sets the upper and lower limits for the stationary growth rate n. Then

Theorem 1

Under A. 1 and A.2, there exists a unique stationary state.

Proof

In the stationary state $\dot{k} / k = \dot{c} / c = \dot{q} / q = 0$ holds. Hence $n = g - \eta = (x - c) / k - \delta = (r - \rho - \delta) / \sigma = r - \delta - w / q + yg'$ holds. From this and (12), we obtain;

$$r(p) = \sigma n + \rho + \delta \tag{13}$$

and

$$w(p) / p = (\rho - (1 - \sigma)n) / g'(y) + y. \tag{14}$$

From $g(y) - \eta = n$, we observe $y = y(n)$ with $y'(n) = 1 / g'(y) \geq 1$. Then both $f(n) = \sigma n + \rho + \delta > 0$ and $h(n) = (\rho - (1 - \sigma)n) / g'(y(n)) + y(n) > 0$ are increasing functions of n $(h'(n) = \sigma / g' - g'' y'(\rho - (1 - \sigma)n) / g'^2 > 0)$, noting $\rho - (1 - \sigma)n > 0$ from A. 2.

(1) First we consider capital intensive good case. Then A. 1 implies that there exists a unique stationary state (n_∞, p_∞) such that $r(p_\infty) = f(n_\infty)$ (see Fig. 1.) and $w(p_\infty)/p_\infty = h(n_\infty)$, observing $r(p) \to 0$ and $w/p \to \infty$ as $p \to \infty$, and $r(p) \to \infty$ and $w/p \to 0$ as $p \to 0$ from A. 1. (The ∞ subscript denotes the values of variables at the stationary state.) Under A. 2, at (n_∞, p_∞), g, r, w/p and i are positive.

■

(2) Next we consider labor intensive good case. Observing $r(p) \to \infty$ and $w/p \to 0$ as $p \to \infty$, and $r(p) \to 0$ and $w/p \to \infty$ as $p \to 0$, and $r(p) = f(n)$ is positively sloped, and $w(p)/p = h(n)$ is negatively sloped, we obtain that there exists a unique stationary state (n_∞, p_∞) (In Fig. 1, by interchanging the role of f and h we obtain the similar figure for Case (2).) such that $r(p_\infty) = f(n_\infty)$ and $w(p_\infty)/p_\infty = h(n_\infty)$, and that g, r, w/p and i are positive at (n_∞, p_∞) under A. 2.

Fig. 1

The growth rate n_∞ at the stationary state is seen to depend on σ (the intertemporal rate of substitution of consumption), ρ (the time preference rate) as well as the depreciation rates δ and η and the adjustment cost, characterizing the endogenous growth model.

Next we show the global stability. Here we introduce the value function W;

$$W(K_0, H_0) = \max_{\dot{K}, \dot{H}} \int_0^\infty \frac{1}{1-\sigma} C^{1-\sigma} e^{-\rho t} dt$$

where $K_0 = K(0)$ and $H_0 = H(0)$ are respectively initial values of K and H.[2/] Since the value function is concave and homogeneous of degree $1-\sigma$ in (K, H), $W_K = \mu$ and $W_H = \lambda$ are homogeneous of degree $-\sigma$ in (K, H). From this we obtain $q = \lambda / \mu$ is an increasing function of k, i.e., $q = q(k)$ with $q'(k) > 0$. (See Appendix I.)[3/]

Optimal Consumption Path

Next we show the property of the optimal per capita consumption path. From $C^{-\sigma} = \mu$ (Eq. (5)) and $W_K(k, 1) = H^\sigma \mu$ ((A-1) with $s = 1/H$), we obtain $-\sigma(dc/dk)/c = W_{KK}(k, 1) < 0$ from the concavity of the value function W, showing $dc/dk > 0$. This is a generalization of the results obtained by Mino (1996) and BWY (1996)[4/] for the no-adjustment cost case of educational investment. To show global stability, we assume

A. 3 $\quad G_H(Y, H) \to \infty$ as $H \to 0$.

Intuitively A. 3 implies that the marginal contribution of human capital to increase education becomes infinite as it approaches zero. A. 3 is assumed throughout the paper. Utilizing this result, we obtain;

Theorem 2.

Under A.1 through A. 3 the economy expressed by the system of differential equations (9), (10) and (11) is globally stable.

Proof

(1) First we consider capital intensive good case.
Since $y = \tilde{y}(\underset{+}{p}, \underset{-}{k})$ from the definition of the Rybczynski function and $k = k(\underset{+}{q})$ hold, $p = qg'(\tilde{y}(p,\ k(q)))$ defines p to be a function of q with $p = p(q)$ and $dp/dq = (g' + qg''\tilde{y}_k k'(q))/(1 - qg''\tilde{y}_p) > 0$. The right hand side of (9) is a function of q alone.

Now let $\bar{p} = \lim_{q \to \infty} qg'$.

Then

① if $\bar{p} = +\infty$, and $\overline{\lim_{q \to \infty}} y = +\infty$, then $\lim_{y \to \infty}(g(y) - yg'(y)) = \lim_{H \to 0} G_H(Y,\ H) = +\infty$ and $\dot{q}/q \to -\infty$ as $q \to \infty$.

② If $\bar{p} = +\infty$ and $\overline{\lim_{q \to \infty}} y < +\infty$, then $(w/p)g'(y) \to \infty$, and $\dot{q}/q \to -\infty$ as $q \to \infty$.

③ If $\bar{p} < +\infty$ then $\overline{\lim_{q \to \infty}} y = +\infty$, and hence $\lim_{H \to 0} G_H(Y,\ H) = +\infty$, showing $\dot{q}/q \to -\infty$ as $q \to \infty$.

Fig. 2

Then as drawn in Fig. 2, the $\dot{q}/q$ curve intersects with the horizontal axis q at q_∞ with $\dot{q} < 0 \Leftrightarrow q > q_\infty$, showing the global stability.

(2) Next we consider labor intensive good case. Since $y = \tilde{y}(\underset{+}{p}, \underset{+}{k})$ holds from the definition of the Rubczynski function, observing $q = q(\underset{+}{k})$ and (12) we can see that p, y and k depend only on q. Let $\bar{y} = \overline{\lim_{q \to \infty}} y(q)$, $\bar{k} = \overline{\lim_{q \to \infty}} k(q)$ and $\bar{p} = \overline{\lim_{q \to \infty}} qg'(y(q))$.

Then there exist two subcases;

Case (i) $\bar{p} < +\infty$

and

Case (ii) $\bar{p} = +\infty$.

For Case (i), $\bar{p} < +\infty$ implies $\bar{y} = +\infty$ from (12). Furthermore from $x = \tilde{x}(\underset{-}{p}, \underset{-}{k})$, $\bar{x} = \tilde{x}(\bar{p}, \bar{k}) = 0$[5] and $\bar{c} = c(\bar{k}) \le \bar{x} = 0$ show $\dot{k}/k \to -\infty$ as $k \to \infty$ from (10). Then the $\dot{k}/k$ curve can be drawn as in Fig. 2 replacing q with k, showing global stability.

Now we consider Case (ii), $\bar{p}=+\infty$. In view of $x=\tilde{x}(\underset{-}{p},\underset{-}{k})$ and $y=\tilde{y}(\underset{+}{p},\underset{+}{k})$ for the case of labor intensive good case, we observe $(\bar{x}-\bar{c})/\bar{k}<(x_\infty-c_\infty)/k_\infty$ and $g(\bar{y})>g(y_\infty)$ from $\bar{x}<x_\infty$, $\bar{c}>c_\infty$, $\bar{k}>k_\infty$(derived from $\bar{q}(=q(\bar{k})=+\infty)>q_\infty$ and $q=q(\underset{+}{k})$) and $\bar{p}>p_\infty$. This shows from (10), at $k=\bar{k}$,

$$\dot{k}/k=(\bar{x}-\bar{c})/\bar{k}-g(\bar{y})-\delta+\eta<(x_\infty-c_\infty)/k_\infty-g(y_\infty)-\delta+\eta=0$$

holds, implying again the $\dot{k}/k$ curve is drawn as in Fig. 2 with k in place of q, and the global stability is obtained. ■

III. Open Economy

Now we consider the case of two identical countries, the home country and the foreign country, producing a good for consumption or investment, and education. The two countries are identical except for the amount of initial national wealth. First we consider the case of competitive equilibrium.

Competitive Equilibrium

The home consumers maximize

$$\int_0^\infty \frac{1}{1-\sigma}C^{1-\sigma}e^{-\rho t}dt$$

subject to the flow budget constraint;

$$\dot{b}=Rb+X-I-C \tag{15}$$

where $b(\text{resp.}-b)>0$ is the bond(resp. debt) held by the home consumers, $\dot{b}$ is its time rate of change, R is the international interest rate on bonds, and $EX=X-I-C>0(-EX>0)$ is the amount of the traded good exported (imported) by the home country. That is, the consumer can buy(resp.sell) a bond with interest rate R which is an equity claim on a physical asset, in exchange for the export(resp. import) of the good in the international market. Here education is nontraded. By constructing the current value Hamiltonian, we obtain the following first order conditions;

$$C^{-\sigma}=\mu, \tag{16}$$

and

$$\dot{\mu}=\rho\mu-R\mu, \tag{17}$$

and the transversality condition $\lim_{t\to\infty}\mu b e^{-\rho t}=0$.

The home firms maximize the present value of the net cash flow $\pi=X+PY-(I+PY)$ $=X-I$, i.e.,

$$\max\int_0^\infty (X-I)\theta(0,\ t)dt$$

subject to (2) and (3) where $\theta(0,\ t)=\exp\left[-\int_0^t R(\tau)d\tau\right]$ is the discount rate for the firm given the stream of interest rates $\{R(t)\}_{t=0}^{\infty}$. By constructing the current value Hamiltonian,

$$\widetilde{H}=X(K,H,Y)-I+\xi(I-\delta K)+q(G(Y,H)-\eta H)$$

we obtain the first order conditions;

$$1=\xi \tag{18}$$

$$-X_Y=p=qG_Y, \tag{19}$$

$$\dot{\xi}=R\xi-X_K+\delta\xi, \tag{20}$$

and

$$\dot{q}=Rq-X_H-qG_H+\eta q, \tag{21}$$

and the transversality conditions $\lim_{t\to\infty}\xi K\theta(0,t)=0$ and $\lim_{t\to\infty}qK\theta(0,t)=0$.

From (18) and (20), we obtain $R=r-\delta$ with $X_K=r$. Hence from (21) we observe;

$$\dot{q}/q=r-\delta-w/q-g+yg'+\eta \tag{9}$$

with $w=X_H$ and $G_H=g-yg'$.

Furthermore, from (2) and (3), we obtain;

$$\dot{k}/k=i-\delta-g+\eta \tag{22}$$

where $i=I/K$. Then from (3), (16) and (17), we obtain;

$$\dot{c}/c=(r-\rho-\delta)/\sigma-g+\eta. \tag{11}$$

From (16), (17) and the transversality condition $\lim_{t\to\infty}\mu be^{-\rho t}=0$, we obtain the demand function for consumption;

$$C(t)=h(t)m(0)^{6/} \tag{23}$$

where $m(0)=b(0)+V(0)+W(0)$ is the initial national wealth of the home country, $b(0)$ is the initial bond amount held by the home country's consumers, $V(0)=\int_0^{\infty}(X-I)\theta(0,t)d\tau$ is the initial firm value of the home country and $W(0)=\int_0^{\infty}w\theta(0,t)d\tau$ is the initial value of human wealth capital. We obtain similar equations for the foreign country. Then from the foreign counterparts of (18) and (20) we observe $r=r^*$ where r^* is the foreign rental price of physical capital. (The super script asterisk * denotes the variables, parameters and equations of the foreign country.) Then from $r=r(p)$ and $r^*=r(p^*)$ we also observe that $p=p^*$ follows. This further implies $w=w^*$ holds.

Then for the foreign country;

$$\dot{q}^*/q^*=r-\delta-w/q^*-g(y^*)+y^*g'(y^*)+\eta, \tag{9}^*$$

$$\dot{c}^*/c^*=(r-\rho-\delta)/\sigma-g(y^*)+\eta, \tag{11}^*$$

and

$$\dot{k}^*/k^*=i^*-\delta-g(y^*)+\eta \tag{22}^*$$

with

$$p = q^* g'(y^*) \tag{19}^*$$

hold. Of course in competitive equilibrium the amount of a good demanded must equal the amount supplied;

$$X(K,H,Y) + X(K^*,H^*,Y^*) = C + C^* + I + I^* \tag{24}$$

where C^* is consumption, I^* is investment, K^* is the amount of physical capital, H^* is the amount of human capital, Y^* is the amount of education, and $X^* = X(K^*, H^*, Y^*)$ is the amount of goods of the foreign country. We assume that free trade prevails in the world economy. Henceforth $m(0) > m^*(0)$ is assumed, where $m^*(0)$ is the initial national wealth of the foreign country.

Social Planner's Optimum

Here we introduce the social planner's optimization problem;

$$\max \int_0^\infty \frac{1}{1-\sigma}(C^{1-\sigma} + \gamma C^{*1-\sigma}) e^{-\rho t} dt$$

subject to (2), (3), and their foreign counterparts, and (24) where $\gamma = (m^*(0)/m(0))^{1/\sigma} < 1$ is constant. By constructing the current value Hamiltonian

$$\tilde{H} = \frac{1}{1-\sigma}(C^{1-\sigma} + \gamma C^{*1-\sigma}) + \xi(X(K,H,Y) + X(K^*, H^*, Y^*) - C - C^* - I - I^*)$$
$$+\mu(I - \delta K) + \mu^*(I^* - \delta K^*) + \lambda(G(Y,H) - \eta H) + \lambda^*(G(Y^*, H^*) - \eta H^*),$$

we obtain the first order conditions;

$$C^{-\sigma} = \gamma C^{*-\sigma} = \xi, \tag{25}$$
$$\xi = \mu = \mu^*, \tag{26}$$
$$-\xi X_Y = \lambda G_Y, \tag{27}$$
$$-\xi X^*_{Y^*} = \lambda^* G^*_{Y^*}, \tag{28}$$
$$\dot{\mu} = \rho\mu - \xi X_K + \mu\delta, \tag{29}$$
$$\dot{\mu}^* = \rho\mu^* - \xi X^*_{K^*} + \mu^*\delta, \tag{30}$$
$$\dot{\lambda} = \rho\lambda - \xi X_H - \lambda G_H + \lambda\eta, \tag{31}$$
$$\dot{\lambda}^* = \rho\lambda^* - \xi X^*_{H^*} - \lambda^* G^*_{H^*} + \lambda^*\eta \tag{32}$$

and the transversality conditions $\lim_{t\to\infty} \mu K e^{-\rho t} = 0$, $\lim_{t\to\infty} \mu^* K^* e^{-\rho t} = 0$, $\lim_{t\to\infty} \lambda H e^{-\rho t} = 0$, and $\lim_{t\to\infty} \lambda^* H^* e^{-\rho t} = 0$. (26), (29) and (30) imply $X_K = X^*_{K^*}$, and hence $r = r^*$. Furthermore since $r = r(p)$ and $r = r(p^*)$ hold, $p = p^*$ follows. Then since $X_H = w = w(p)$ and $X^*_{H^*} = w^* = w(p^*)$ hold, $w = w^*$ also follows. By letting $q = \lambda/\mu$ and $q^* = \lambda^*/\mu^*$ we can again obtain (9), (9)*, (11), (11)*, (22), (22)*, (19) and (19)*. In short, the equivalence between competitive equilibrium and the social planner's optimum is derived.

Existence and Uniqueness of the Stationary State

Now we consider the existence and uniqueness of the stationary state. From (9), (11), (22) and their foreign counterparts, with $\dot{k}/k = \dot{k}^*/k^* = \dot{c}/c = \dot{c}^*/c^* = \dot{q}/q = \dot{q}^*/q^* = 0$, we obtain the existence and uniqueness of the stationary growth rate n_∞ and the stationary relative price of education p_∞ using the same method as in Theorem 1. Here the stationary growth rate n_∞ is the same for both countries. Furthermore $y_\infty = y^*_\infty$, i.e., the stationary values of y and y^* are equal, as are $q_\infty = q^*_\infty$ and $i_\infty = i^*_\infty$. From $p = -x_2(k, y) = -x_2(k^*, y^*)$, we observe $k_\infty = k^*_\infty$, and hence $x_\infty = x^*_\infty$. Here these stationary values are all unique since $y_\infty = y(n_\infty)$ and p_∞ are unique.

Value Function $W = W(K_0, K^*_0, H_0, H^*_0)$

As in the closed economy, we introduce the value function W;

$$W(K_0, K^*_0, H_0, H^*_0) = \max_{K, K^*, H, H^*} \int_0^\infty \frac{1}{1-\sigma}(C^{1-\sigma} + \gamma C^{*1-\sigma})e^{-\rho t} dt .$$

This is again concave and homogeneous of degree $1-\sigma$ in (K, K^*, H, H^*). Then $W_K = \mu = \mu^* = W_{K^*}$ implies that W is expressed as;

$$W = \widetilde{W}(K_W, H, H^*)$$

where $K_W = K + K^*$, with $W_{KW} = \mu$, $W_H = \lambda$ and $W_{H^*} = \lambda^*$ being homogeneous of degree $-\sigma$. Then we obtain q and q^* to be functions of h and h^* where $h = H / K_W$ and $h^* = H^* / K_W$, which are expressed as;

$$q = q(h, h^*) \tag{33}$$

and

$$q^* = q^*(h, h^*) \tag{34}$$

from

$$w_H(h, h^*) / w_{KW}(h, h^*) = q$$

and

$$w_{H^*}(h, h^*) / w_{KW}(h, h^*) = q^*$$

where $w_{KW}(h, h^*) = \widetilde{W}_{KW}(1, h, h^*) = K^\sigma_W \mu$, $w_H(h, h^*) = \widetilde{W}_H(1, h, h^*) = K^\sigma_W \lambda$, and $w_{H^*}(h, h^*) = \widetilde{W}_{H^*}(1, h, h^*) = K^\sigma_W \lambda^*$.

By definition;

$$h + h^* = 1 / k_W \tag{35}$$

and

$$y_W = k_W(hy + h^* y^*) \tag{36}$$

hold where $k_W = K_W / H_W$, $y_W = Y_W / H_W$, $Y_W = Y + Y^*$ and $H_W = H + H^*$.

$$p = -x_2(k_W, y_W) = -x_2(k, y) = -x_2(k^*, y^*) \tag{37}$$

holds from $p = -X_Y(K, H, Y) = -X_{Y^*}(K^*, H^*, Y^*)$, $r = X_K(K, H, Y) = X_{K^*}(K^*, H^*, Y^*)$ and $w = X_H(K, H, Y) = X_{H^*}(K^*, H^*, Y^*)$.[7] Then (19), (19)*, (33), (34), (35), (36) and

(37) define p, y, y^*, q, q^*, k_W, y_W, h, h^* to be functions of k and k^*. Then (9) and (9)* constitute a system of two differential equations of k and k^*.

To show the global stability, the following assumption A. 4 and Lemma 1 are used;

A.4 $\delta > \eta$, i.e., the depreciation rate of phiscal capital δ is higher than that of human capital η.

Lemma 1. Poincare-Bendixon Theorem (Hsu and Meyer (1968) Section 5.8)
For a two dimensional autonomous differential equation system, the path (trajectory) must become unbounded or converge to a limit cycle or to a point.

To employ Lemma 1 for (22) and (22)*, we observe first k and k^* are bounded. (See Appendix II.) Then from Lemma 1, the optimal path of (k,k^*) converges either to the stationary point E or to a limit cycle as shown in Fig. 3.

Fig. 3

To show that the optimal path of (k,k^*) converges monotomically to the stationary point E, let $k_0 = k_0^*$, i.e., at the initial point, the capital labor ratios of both countries be equal. Then from (12), (12)*, $y = \tilde{y}(p,k)$ and $y^* = \tilde{y}(p,k^*)$, $y_0 = y_0^*$ and $q_0 = q_0^*$ follow. Hence from (9) and (9)* $\dot{q} = \dot{q}^*$ holds at $t = 0$, implying $q = q^*$ for $t \geq 0$, and hence from (12) and (12)* $k = k^*$ for $t \geq 0$. In short $k_0 = k_0^*$ implies $k = k^*$ for $t \geq 0$. In Fig. 3 this is shown by the movement of optimal path of (k,k^*) along 45° degree line toward the stationary point $E(k_\infty, k_\infty^*)$ which starts either point A or B. Furthermore from the uniqueness of the optimal path given initial point (k_0,k_0^*), the optimal path starting off 45° degree line never crosses this line, implying

$$k_0 > k_0^* \Rightarrow k > k^* \text{ for } t \geq 0.$$

This shows the monotonic convergence not to a limit cycle but to the point.

Hence we obtain

Theorem 3.

The social planner's optimum expressed by (9), (11), (19), (22), their foreign counterparts and (24) are globally stable and converge to a unique stationary state. Furthermore if $k_0 > k_0^*$ holds initially, $k > k^*$ holds always. (i.e., (k,k^*) never crosses 45° line in $k - k^*$ plane.)[8/]

Let $\ell = H^*/H$ be the ratio of the foreign human capital on the home human capital. Then from the assumption of the capital labor ratios, $k_0 > k_0^*$, we have obtained that $k > k^*$ holds always and so does $H^*/H > K^*/K$. At the stationary state where K, K^*, H and H^* glow at the same rate, ℓ becomes constant, i.e., $\ell = \ell_\infty$. From (25), we obtain

$$\gamma^{1/\sigma} c = \ell c^*. \tag{38}$$

Furthermore from (23), we observe

$$\gamma^{1/\sigma} = C^*/C = m^*(0)/m(0) = (b^*(0)+V^*(0)+W^*(0))/(b(0)+V(0)+W(0)) \tag{39}$$

where $V(0)=\widetilde{V}(k_0)$ with $\widetilde{V}(k_0)=\xi_0 k_0$ and $\widetilde{V}(k_0^*)=\xi_0^* k_0^{*\mathfrak{U}}$ and $W^*(0)=W(0)$.

Henceforth we assume

$$b(0) = -b^*(0) > 0,$$

i.e., the home country is initially a creditor. Then we always obtain

$$\gamma^{1/\sigma} < K^*(0)/K(0).$$

First

(1) we consider the case of capital intensive good sector. We obtain $y < y^*$. This implies that $\dot{H}^*/H^* > \dot{H}/H$ from (3) and (3)* recalling $g(y) = G(Y/H, 1) < g(y^*) = G(Y^*/H^*, 1)$. Hence $\ell = H^*/H$ increases to ℓ_∞ showing

$$H^*(0)/H(0) = \ell(0) < \ell < \ell_\infty.$$

Then from (39), we observe

$$\gamma^{1/\sigma} < K^*(0)/K(0) < H^*(0)/H(0) < \ell_\infty. \tag{40}$$

Finally we observe from (24),

$$x - c - ik + \ell(x^* - c^* - i^* k^*) = 0 \tag{41}$$

holds. Especially at the stationary state, the above is expressed as

$$x_\infty - c_\infty - i_\infty k_\infty + \ell_\infty (x_\infty - c_\infty^* - i_\infty k_\infty) = 0. \tag{42}$$

Since $\gamma^{1/\sigma} c_\infty = \ell_\infty c_\infty^*$ holds from (38), we obtain from (40) and (42),

$$ex_\infty = x_\infty - c_\infty - i_\infty k_\infty < 0,$$

i.e., the home country becomes an importer eventually. Furthermore from (15), we obtain

$$b(t) = -\int_t^\infty (X - C - I)\theta(0,t)dt. \tag{43}$$

This shows that the home country becomes eventually a creditor when the good sector is capital intensive.

Next

(2) we consider the case of labor intensive good sector. By the similar arguments as above, we obtain

$$H^*(0)/H(0) = \ell(0) > \ell > \ell_\infty,$$

but at the same time

$$\gamma^{1/\sigma} < K^*(0)/K(0) < H^*(0)/H(0).$$

Hence there exist two subcases for this case.

(1) $\gamma^{1/\sigma} < \ell_\infty$, and hence $ex_\infty < 0$ and hence $b(t) > 0$ eventually, and

(2) $\gamma^{1/\sigma} > \ell_\infty$, and hence $ex_\infty > 0$ and hence $b(t) < 0$ eventually. Summarizing the

above arguments, we obtain

Theorem 4.

Let the home country be initially a creditor.

(1) If the good sector is capital intensive, then the home country eventually becomes an importer of good as well as a creditor. Especially if the home country remains an importer of good always, it also remains a creditor.

(2-i) If the good sector is labor intensive, and $\gamma^{1/\sigma} < \ell_\infty$ (reflecting the initial debt of the foreign country to be rather large), then the conclusion of (1) still hold.

(2-ii) If the good sector is labor intensive and $\gamma^{1/\sigma} > \ell_\infty$ (reflecting the initial debt of the foreign country to be rather small), then the home country eventually becomes an exporter of good as well as a debtor. The asset-debt position of the home country changes during transitional period.

Theorem 4 (1) and (2-i) seem realistic and interesting. Then, the home country (i.e., the developed country), being better endowed with initial national wealth may keep suffering from a current account deficit ($ex < 0$) while remaining a creditor ($b > 0$). This seems to reflect the historical experiences of England and the U. S. A. mentioned earlier. Theorem 4 also imply the possibility of the different trade patterns and asset-debt positions according to the relative capital intensities of good and education sectors.

Concluding Remarks

Here we note all per capita variables are measured not in actual but in an efficient labor unit. That is, if $H = eL$ and $H^* = e^* L^*$ where L and L^* are respectively the numbers in the labor force in the home country and the foreign country, $e, e^* > 1$ reflects the accumulation of human capital in both countries. Then even if $K / L > K^* / L^*$ holds, it is not certain which of $K / H < K^* / H^*$ or $K / H > K^* / H^*$ holds in reality.

To investigate the trade patterns and asset-debt position of specific countries, it would be more appropriate to treat three country model which Ikeda and Ono (1992) analyzed rather than two country model discussed in this paper, although the analysis of global stability would be more difficult. One extension of the present model is to incorporate government expenditure and taxation and analyze there long run as well as short run effects, which would be our next step.

Appendix I

By the definition of homogeneity, we observe

$$W_K(sK, sH) = s^{-\sigma}\mu \tag{A-1}$$

and

$$W_H(sK, sH) = s^{-\sigma}\lambda . \tag{A-2}$$

for $s > 0$. By substituting $s = 1/H$, we obtain

$$q = \lambda / \mu = W_H(k,1) / W_K(k,1).$$

Hence,

$$dq/dk = \{W_{HK}(k,1)W_K(k,1) - W_{KK}(k,1)W_H(k,1)\} / W_K^2(k,1)$$

By differentiating (A-1) and (A-2) with respect to s, and then letting $s = 1$ (For the second equality below $s = H^{-1}$ is substituted into (A-1) and (A-2).) we observe

$$\begin{bmatrix} W_{KK} & W_{KH} \\ W_{HK} & W_{HH} \end{bmatrix} \begin{bmatrix} K \\ H \end{bmatrix} = -\sigma \begin{bmatrix} \mu \\ \lambda \end{bmatrix} = -\sigma H^{-\sigma} \begin{bmatrix} W_K(k,1) \\ W_H(k,1) \end{bmatrix}.$$

Hence

$$H = \sigma H^{-\sigma}\{W_{HK}(k,1)W_K(k,1) - W_{KK}(k,1)W_H(k,1)\} / \det Wij > 0,$$

and the $dq/dk > 0$ follows where $\det Wij$ is the determinant of the Jacobian matrix and positive from the strong concavity of W.

Appendix II

We observe first k_W to be bounded. In fact, from (2), (2)*, (3), (3)* and (24)

$$\dot{k}_W / k_W = \dot{K}_W / K_W - \dot{H}_W / H_W = (I + I^*)/K_W - (\dot{H} + \dot{H})/H_W - \delta$$

$$\le (F_X(K,H) + F_X(K^*,H^*))/K_W - \delta - (g(y)H + g(y^*)H^* - \eta H_W)/H_W$$

where F_X is the production function of good sector. Then

$$\dot{k}_W / k_W \le F_X(K_W, H_W)/K_W - (\delta - \eta) = f_X(k_W)/k_W - (\delta - \eta)$$

where $f_X(k_W) = F_X(k_W, 1)$ being the labor productivity function of good sector. Then we observe $k_W \to +\infty$ implies $f_X(k_W)/k_W \to 0$ from Inada Condition and hence from A. 4. $\dot{k}_W / k_W < 0$ as $k_W \to +\infty$ implying the boundedness of k_W. Next we consider the relationship between capital intensities of good sector and educational sector, k_x and k_y, and wage rental ratio $\omega = w/r$.

Fig. A. 1

Fig. A. 1 illustrates this relationship. Both k_x and k_y are increasing functions of ω. (We consider capital intensive good case. But the other case can be treated similarly.) Then boundedness of k_W implies that of ω. Let $\overline{\omega}$ be the upper bound of ω and $\overline{k}_x$ be that of k_x. Then recalling k and k^* to line between the k_x and k_y, we immediately observe k and $k^* \leq \overline{k}_x$, showing the boundedness of k and k^*.

Appendix III

In this appendix, we discuss the ϕ-type case.

I. Closed Model

Utility maximization over time is expressed as;

$$\max \int_0^{\infty} \frac{1}{1-\sigma} C^{1-\sigma} e^{-\rho t} dt$$

subject to (1)', (2) and (3)'. Then the current value Hamiltonian $\widetilde{H}$ is expressed as;

$$\widetilde{H} = \frac{1}{1-\delta} C^{1-\sigma} + \mu(X[K, H, Y(1+\phi(Y/H))] - C - \delta K) + \lambda(Y - \eta H) \qquad (4)'$$

and the first order conditions are;

$$C^{-\sigma} = \mu \qquad (5)'$$

$$-\mu X_Y (1 + \phi + \phi' \cdot y) = \lambda \qquad (6)'$$

$$\dot{\mu} = \rho\mu - \mu X_K + \delta\mu \qquad (7)'$$

$$\dot{\lambda} = \rho\lambda - \mu X_H + \mu X_Y \cdot y^2 \phi' + \lambda\eta . \qquad (8)'$$

where $y = Y/H$, and the transversality conditions are $\lim_{t\to\infty} \mu K e^{-\rho t} = 0$ and $\lim_{t\to\infty} \lambda H e^{-\rho t} = 0$.

By letting $-X_Y = p,\ \ X_K = r,\ \ X_H = w$ and $q = \lambda / \mu$, we obtain;

$$\dot{q}/q = r - \delta + \eta - w/q - py^2\phi'/q \qquad (9)'$$

$$\dot{k}/k = (x-c)/k - \delta - y + \eta \qquad (10)'$$

$$\dot{c}/c = (r - \rho - \delta)/\sigma - y + \eta \qquad (11)'$$

$$p(1 + \phi + \phi' \cdot y) = q . \qquad (12)'$$

Here again $r = r(p)$ and $w = w(p)$.

Existence and Uniqueness of the Stationary State

By letting $\dot{q}/q = \dot{k}/k = \dot{c}/c = 0$ we obtain;

$$r - \delta = w/q + py^2\phi'/q - \eta = \sigma n + \rho,$$

$$n = (r - \rho - \delta)/\sigma = y - \eta = (x-c)/k - \delta .$$

A. 1 remains valid and A. 2 is replaced by A. 2', i.e., the rate of growth, n, satisfies for $\sigma < 1$, $-n_0' < n < \rho/(1-\sigma)$

and for $\sigma > 1$, $-n_1' < n$

where $n_0' = \min(\eta, (\rho+\delta)/\sigma, (\rho+\eta)/\sigma, \delta)$ and $n_1' = \min(\eta, \rho/(\sigma-1), \delta)$.

Then we obtain;

Theorem 1'

Under A. 1 and A. 2' there exists a unique stationary state.

Proof

By letting;

$$f(n) = \sigma n + \rho + \delta > 0 \qquad (13)'$$

and

$$h(n) = -y^2\phi' + (1+\phi+\phi' y)(\eta + \sigma n + \rho)$$
$$= (\sigma n + \rho - n) y\phi' + (1+\phi)(\eta + \sigma n + \rho) > 0 \qquad (14)'$$

from $y = n + \eta$ and A. 3',

we obtain

$$h'(n) = (\sigma - 1)y\phi' + (\sigma n + \rho - n)(\phi' + y\phi'') + \phi'(\eta + \sigma n + \rho) + \sigma(1+\phi)$$
$$= [2\{(\sigma-1)n + \rho\} + \sigma(n+\eta)]\phi' + \{(\sigma-1)n + \rho\} y\phi'' + \sigma(1+\phi) > 0$$

from A. 2'. Here $\min h(n) = \eta + \sigma n + \rho > 0$ when $y = 0$. Since $f'(n) = \sigma > 0$, we obtain the desired results under A. 2' employing the same arguments as *g*-type; at (n_∞, p_∞), y, r, w/p and i are all positive. ■

Global Stability

From $x = x(k, y(1+\phi))$ and $x = \tilde{x}(p,k)$ (the Rybczynski function), we obtain $y = \tilde{y}(\underset{+}{p}, \underset{-}{k})$.

As in the case of the *g*-type cost, we obtain;

$$q = q(k) \text{ with } q'(k) > 0$$

from the concavity of the value function.

(1) First we consider capital intensive good case. From (12)' and $y = \tilde{y}(p,k)$ we observe

$$p = p(q) \text{ with } p'(q) > 0.$$

Let $\bar{p} = \overline{\lim_{q\to\infty}} q/(1+\phi+\phi'(y))$. In (9)',

① if $\bar{p} = +\infty$, and $\overline{\lim_{q\to\infty}} y = +\infty$, then from $py^2\phi'/q = y^2\phi'/(1+\phi+\phi' y)$ and l'Hopital Theorem $\lim_{y\to\infty} y^2\phi'/(1+\phi+\phi' y) = \lim_{y\to\infty}(2y\phi' + y^2\phi'')/(2\phi' + \phi'' y) = +\infty$, and hence $\dot{q}/q \to -\infty$ as $q \to \infty$.

② If $\bar{p} = +\infty$ and $\overline{\lim_{q\to\infty}} y = \bar{y} < +\infty$, then $w/q = w/p(1+\phi+\phi'(y)) \to \infty$ as $q \to \infty$, implying $\dot{q}/q \to -\infty$ as $q \to \infty$.

③ If $\bar{p} < +\infty$, then $\overline{\lim_{q\to\infty}} y = +\infty$ must follow, which implies $py^2\phi'/q \to +\infty$ as $q \to \infty$, and hence $\dot{q}/q \to -\infty$ as $q \to \infty$.

(2) The proof the labor intensive good case can be done employing the same arguments as g-type case. ■

Open Model

First we consider the **competitive equilibrium**.

The home consumers

$$\text{maximize} \int_0^\infty \frac{1}{1-\sigma} C^{1-\sigma} e^{-\rho t} dt$$

subject to (9), and hence we obtain the first order conditions (16) and (17), and the transversality condition $\lim_{t\to\infty} \mu K e^{-\rho t} = 0$. For net cash flow maximization over time the home firm faces the problem;

$$\max \int_0^\infty (X - I)\theta(0, \tau) dt \text{ subject to (2) and (3)'} .$$

From this we construct the current value Hamiltonian

$$\widetilde{H} = X[K, H, Y(1 + \phi(Y/H))] - I + \xi(I - \delta K) + q(Y - \eta H),$$

and obtain the first order conditions (18);

$$-X_Y(1 + \phi + \phi' y) = p(1 + \phi + \phi' y) = q, \qquad (19)'$$

(20) and

$$\dot{q} = Rq - X_H + X_Y \phi' \cdot y^2 + \eta q \qquad (21)'$$

and the transversality conditions $\lim_{t\to\infty} \xi K\theta(0, t) = 0$ and $\lim_{t\to\infty} qH\theta(0, t) = 0$. From (18) and (20), we obtain $R = r - \delta$. Then we observe (9)', (10)', (11)' and (12)'.

Social Planner's Optimum

Here we consider the social planner's optimum for the open economy. The social planner's maximization problem is

$$\max \int_0^\infty \frac{1}{1-\sigma} (C^{1-\sigma} + \gamma C^{*1-\sigma}) e^{-\rho t} dt$$

subject to;

$$X[K, H, Y(1 + \phi(Y/H))] + X[K^*, H^*, Y^*(1 + \phi(Y^*/H^*))] = C + C^* + I + I^*, \qquad (24)'$$

(2) and (3)', and their foreign counterparts. The current value Hamiltonian $\widetilde{H}$ is;

$$\widetilde{H} = \frac{1}{1-\sigma}(C^{1-\sigma} + \gamma C^{*1-\sigma})$$
$$+\xi\{X[K, H, Y(1 + \phi(Y/H))] + X[K^*, H^*, Y^*(1 + \phi(Y^*/H^*))] - C - C^* - I - I^*\}$$

$$+\mu(I-\delta K)+\mu^*(I^*-\delta K^*)+\lambda(Y-\eta H)+\lambda^*(Y^*-\eta H^*),$$

and the first order conditions are;

$$C^{-\sigma}=\xi=\gamma C^{*-\sigma}, \quad (25)'$$

$$\xi=\mu=\mu^*, \quad (26)'$$

$$-\xi X_Y(1+\phi+\phi'\cdot y)=\lambda, \quad (27)'$$

$$-\xi X^*_{Y*}(1+\phi^*+\phi^{*\prime}\cdot y^*)=\lambda^*, \quad (28)'$$

$$\dot\mu=\rho\mu-\xi X_K+\mu\delta, \quad (29)'$$

$$\dot\mu^*=\rho\mu^*-\xi X^*_{K*}+\mu^*\delta, \quad (30)'$$

$$\dot\lambda=\rho\lambda-\xi X_H+\lambda\eta+\xi X_Y\phi'\cdot y^2, \quad (31)'$$

$$\dot\lambda^*=\rho\lambda^*-\xi X^*_{H*}+\lambda^*\eta+\xi X^*_{Y*}\phi^{*\prime} y^{*2} \quad (32)'$$

and the transversality conditions $\lim_{t\to\infty}\mu Ke^{-\rho t}=0$, $\lim_{t\to\infty}\mu^*K^*e^{-\rho t}=0$, $\lim_{t\to\infty}\lambda He^{-\rho t}=0$, and $\lim_{t\to\infty}\lambda^*H^*e^{-\rho t}=0$. From (26)', (29)' and (30)', we observe $X_K=X_{K*}$, i.e., $r=r^*$, which implies $-X_{\tilde Y}=p=p^*=-X_{\tilde Y*}$ where $\tilde Y=Y(1+\phi(Y/H))$ and $\tilde Y^*=Y^*(1+\phi(Y^*/H^*))$ and hence $X_H=w=w^*=X_{H*}$. From (25)', (26)', (29)' and (30)', we obtain (11)' and its foreign counterpart. By letting $q=\lambda/\mu$ and $q^*=\lambda^*/\mu$, we observe (21)' and its foreign counterpart from (29)', (30)', (31)' and (32)'. (27)' corresponds to (19)', and (28)' to the foreign counterpart of (19)'. Hence we see once again the equivalence of the competitive equilibrium and the social planner's optimum.

Existence and Uniqueness of Equilibrium

By letting $\dot q/q=\dot k/k=\dot c/c=0$, we obtain (13)' and (14)' for the home country, and (13)' and the foreign counterpart of (14)' from $\dot q^*/q^*=\dot k^*/k^*=\dot c^*/c^*=0$. Then under A. 1 and A. 2' the existence and uniqueness of the world equilibrium are obtained. (The equilibrium growth rates of both countries are equal.)

Global Stability of the Social Planner's Optimum

By forming the value function W,

$$W(K_0,K_0^*,H_0,H_0^*)=\max_{\dot K,\dot K^*,\dot H,\dot H^*}\int_0^\infty\frac{1}{1-\sigma}(C^{1-\sigma}+\gamma C^{*1-\sigma})e^{-\rho t}dt$$

we obtain (33) and (34). Then

$$p(1+\phi+\phi' y)=q, \quad (19)'$$

its foreign counterpart, (33)', (34)', (35), (36) and

$$p=-x_2[k_W,k_W\{h(1+\phi(y))+h^*(1+\phi(y^*))\}]=-x_2(k,y(1+\phi(y))$$
$$=-x_2(k^*,y^*(1+\phi(y^*)))^{11/} \quad (37)'$$

define k_W, y_W, y, y^* and p to be functions of k and k^* where $x=x(k,y(1+\phi(y))$

$= X(k, 1, y(1+\phi(y)))$.

Boundedness of *k* and *k**

can be obtained by the same methods as for *g*-type case. For this, see Appendix II.

Patterns of Trade and Asset-Debt Positions

By the same reasoning as in the case of the *g*-type case, we obtain Theorem 4 .

Figures

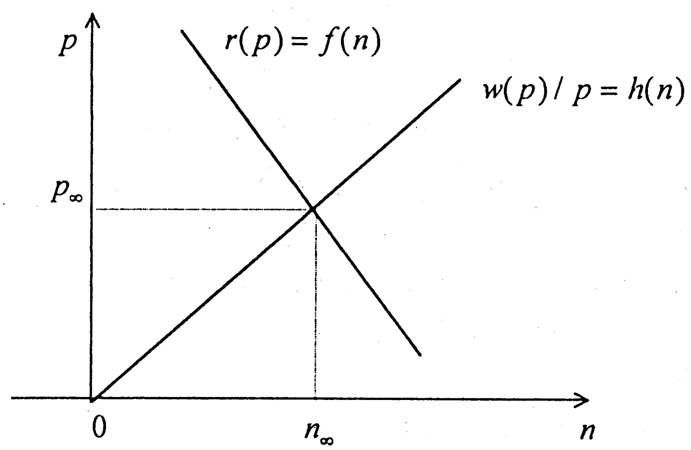

Fig. 1

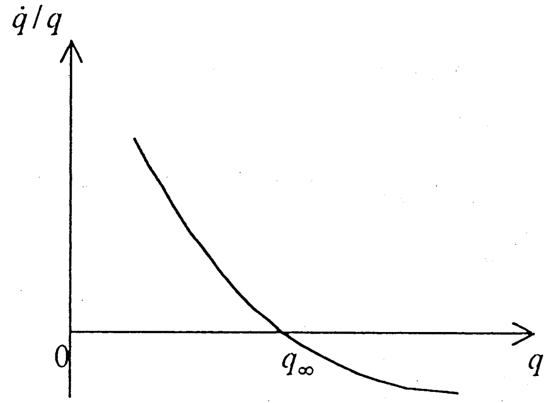

Fig. 2

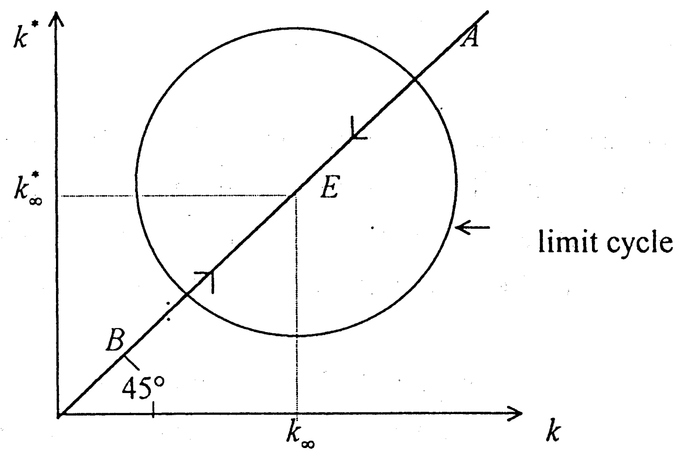

Fig. 3

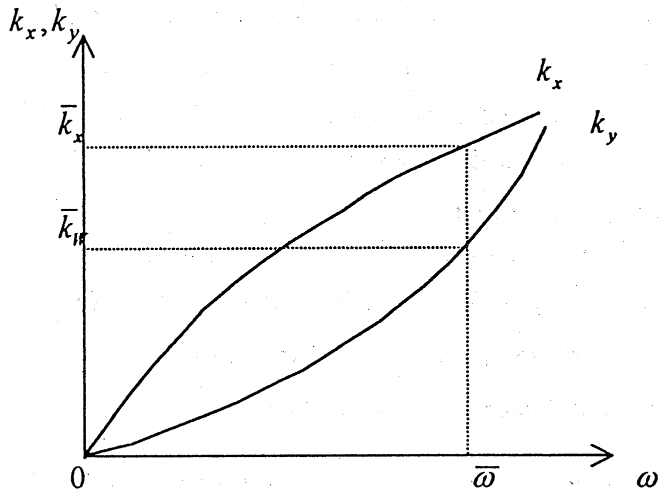

Fig. A. 1

Footnotes

1. For the U. K. data see Mitchell (1962) and the U. K. Central Statistical Office(1943~1997). For the U. S. data, see Mitchell (1993) and the U. S. Department of Commerce (1943~1997). For the Japanese data, see the Japan Economic Planning Agency (1950~1997). For the German data, see the Report of the Deutsche Bundes Bank (1950~1997).
2. Also Cabellé, J. and M. S. Santos (1993), and Ladrón-de-Guevara, A., Oritigueira, S. and M. S. Santos (1997) employed the following property of the value function that its partial derivative to be equal to its co-state variable to show global stability. We follow this line of arguments.
3. As is discussed by BWY (1996), in case of labor intensive good without educational investment adjustment costs, q (which is equal to p without such costs) is independent of k and must be constant. However with educational investment adjustment costs, q depends on k, and hence we cannot assume q to be constant in our case.
4. Although we can use the phase diagram for (10) and (11) (differential equations of k and c) with $q = q(k)$, we obtain both cases of $c'(k) > 0$ and $c'(k) < 0$. This ambiguity disappears by using the value function.
5. The Rybczynski function $\tilde{x}(p,k)$ and $\tilde{y}(p,k)$ are expressed as $\tilde{x}(p,k) = a_x(p) + b_x(p)k$ and $\tilde{y}(p,k) = a_y(p) + b_y(p)k$ where $b_x(p) < 0 < b_y(p)$ and $a_x(p) > 0 > a_y(p)$ in case of labor intensive good. $\bar{y} = +\infty$ implies $\bar{k} = +\infty$. $b_x(\bar{p}) < 0$ implies $a_x(\bar{p}) + b_x(\bar{p})\bar{k} = -\infty$, and hence $\bar{x} = 0$ must hold.
6. From (17), we obtain $\mu(t) = \mu(0)e^{-\int_0^t (R-\rho)d\tau}$. Hence from the transversality condition, NPG (No-Ponzi-Game) condition, $\lim_{t\to\infty} b(t)\theta(0,t) = 0$ is derived. The budget condition (15) is rewritten as $\dot{b} = Rb + \pi - C$, from which we obtain

$$b(t) = b(t_1)\theta(t,t_1) + \int_{t_1}^{t} (\pi - C)\theta(t,\tau)d\tau .$$

By letting $t_1 \to \infty$, and from the NPG condition, we obtain $b(t) = -\int_t^{\infty} (\pi - C)\theta(t,\tau)d\tau$, which implies

$$\int_t^{\infty} C(\tau)\theta(t,\tau)d\tau = b(t) + V(t) + W(t) = m(t)$$

where $V(t) = \int_t^{\infty} \pi\theta(t,\tau)d\tau$ is the firm value at t and $W(t) = \int_t^{\infty} W\theta(t,\tau)d\tau$ the value of human capital wealth at t. By substituting $C(\tau) = C(t)\exp[\int_t^{\tau} (R-\rho)\sigma^{-1}ds]$ obtained from (16) and (17) into the above with $t = 0$ i.e., $\int_0^{\infty} C(t)\theta(0,\tau)d\tau = m(0)$, we obtain

$C(t) = h(t)m(0)$

where $h(t)^{-1} = \int_0^\infty \theta(0,\tau)\exp\int_t^\tau (\rho - R)\sigma^{-1} ds d\tau$. In the case of $\sigma = 1$ (logarithmic utility function) $h(t)^{-1} = \theta(0,\tau)e^{\rho t}\rho^{-1}$ and further in case of $R = \rho$, $h(t) = \rho$.

7. In short, the world efficient production of the good is realized, i.e., max $X(K, H, Y) + X(K^*, H^*, Y^*)$ subject to $K + K^* = K_W$, $H + H^* = H_W$ and $Y + Y^* = Y_W$ for given amount of K_W, H_W and Y_W is obtained. Then $p = -X_2(K_W, H_W, Y_W) = -X_2(K, H, Y) = -X_2(K^*, H^*, Y^*)$ follows. Furthermore in view of homogeneity of degree 0 of X_2 in (K, H, Y), (37) follows.
8. Ladrón-de-Guevara, Ortigueira and Santos (1997) showed the global stability of (closed) two sector endogenous growth model without adjustment costs of educational investment employing the value function.
9. To see that the value of firm $\widetilde{V}(k_0)$ to be equal to $\xi_0 k_0$, see Hayashi (1982).
10. (37)' follows from $p = -X_2(K_W, H_W, \widetilde{Y}_W) = -X_2(K, H, \widetilde{Y}) = -X_2(K^*, H^*, \widetilde{Y}^*)$ and homogeneity of degree zero of X_2 in K, H, $\widetilde{Y}$ where $\widetilde{Y}_W = \widetilde{Y} + \widetilde{Y}^*$, $\widetilde{Y} = Y(1 + \phi(Y/H))$ and $\widetilde{Y}^* = Y^*(1 + \phi(Y^*/H^*))$. For the detailed discussion see footnote 6.

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