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# Workers' Learning by Doing and Rigidity in Labor Markets

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**Abstract**

In spite of very weak market conditions for almost ten years, many large firms have been reluctant to change their wage and employment practices, at least until recently. The purpose of this paper is to present a rational explanation of this "rigidity." The argument is based on workers' learning by doing in the long-term employer-employee relationship often found in Japan. A more motivated worker works with higher work intensity, acquires more skills, and becomes more productive. The firm encourages workers' learning by doing by offering long-term wage contracts. The firm keep honoring existing long-term contracts even if economic conditions change unexpectedly, since an adjustment of the wage pledged in the previous period to new conditions in the present period is considered as a breach of the contract.

## 1 Introduction

One of the most distinctive characteristics of the Japanese economy in the 1990s is apparent "rigidity" in the labor market. After the bust of the so-called *Bubble Economy* at the beginning of this decade, the Japanese economy has stagnated for almost ten years, with unprecedented weakness of consumer and investment demand. However, in spite of this very weak market conditions, many large firms have been reluctant to change their wage and employment practices, at least until recently. This lack of adjustment has been a puzzle for economists, since rational economic agents must adjust their decision to changing economic conditions all the time.

The purpose of this paper is to present a rational explanation of this "rigidity" in the wage policy of Japanese large firms. The argument is based on the long-term relationship between employers and employees, which has been emphasized as a hallmark of the labor relationship in most Japanese large firms.

The oft-cited merit of the long-term employer-employee relationship is that it enables workers' learning by doing in production process[1], or equivalently, it takes advantage of the so-called experience curve of workers. As a worker works longer with one firm, he acquires more firm-specific skills in his workplace. These skills increase the worker's productivity in the production process. In addition, these skills may reduce the worker's disutility of labor through better cooperation with his co-workers. This advantage of long-term relationship has been argued to be large in Japan from social and cultural reasons compared with other countries, explaining the prevalence and strength of the Japanese long-term employer-employee relationship. There is ample evidence that such learning by doing, or on-the-job training, is important in Japanese workplaces (see for example, Koike 1988 for an earlier contribution and Chuma 1998 for a recent one).

In this learning-by-doing process, the degree of skill formation is dependent on worker motivation and resulting work intensity. A more motivated worker works with higher work intensity, acquires more skills, and becomes more productive. This dependence of learning by doing on worker motivation and work intensity introduces an intertemporal linkage: workers' high work intensity in the present implies their acquisition of higher skills, which increases their productivity in the future. The firm can motivate the worker by offering a long-term wage contract to him which pledges a higher wage in the future, rewarding high future labor productivity acquired by intensive work done in the present. In fact, large Japanese firms' observed practices can be considered such long-term wage contracts in a implicit form. The firms have a age-related wage profile and the shape of this wage profile is stable over periods, so that the worker can infer its future wage from it.

We argue this intertemporal nature of learning by doing causes rigidity found in the Japanese labor market. Because of the dependence of the present work intensity on future productivity, the firm pledges a high wage in the future to encourage learning by doing in the present. However, once the workers accumulate skills and become old, the firm can cut wages for the old without fearing the wage cut's adverse effect on skill formation of the old. Thus, the firm can increases its profit by reneging the long-term wage contract for the old in the present and cutting their wages.

In the end, however, such an opportunistic behavior may not pay. The renege jeopardizes the firm's credibility and no worker of subsequent generations believes the firm's pledge of future wages.

[1]In the literature, the effect of learning by doing (that is, higher efficiency as experince accumulates) has been recoginized on the side of firms. It is assumed that the firm can accumulate knowlege to improve production efficiency by learning by doing, and such knowledge spilled over to other firms (Arrow 1962). This learning by doing and its spill-over induce endogenous growth. There is a sizable literature on learning by doing and economic growth (see for an example of recent contribution, D'Autume and Michael 1993).

In contrast to this literature, we are concerned with workers' learning by doing in work place in this paper. Although there is a sizable empirical literature on how such learning by doing and accumulation of production skills take place in the Japanese workplace, their theoretical implications in labor market organizations have been largely unexplored. To our knowledge, this paper is the first of this kind.

The level of workers' learning by doing may be reduced substantially, affecting adversely the firm's long-run profit. If this long-run loss of the opportunistic behavior outweighs the short-run gain outlined in the previous paragraph, the firm shuns reneging.

Suppose, then, that market conditions are changed unexpectedly. The firm, a rational economic agent, wants to adjust its wage policy for the young and the old to changed economic conditions. However, since the change in the old's wage constitutes the renege of the long-term wage contract (the wage rate pledged in the previous period), the firm is in a dilemma: to adjust the wage to trigger an adverse reputation effect or to keep the present contract which is now inefficient. The firm rationally chooses between the two. We argue that most Japanese firms rationally choose the latter, that is, rigidity, since the adverse reputation effect is large in Japan.

The organization of the paper is as follows. In Section 2, we present a model of production incorporating workers' learning by doing. We analyze long-term wage contracts in Section 3, and show that there is a pure seniority (age) premium in the optimal long-term contract. In Section 4, we explore the possibility of the firm's opportunistic behavior and the credibility of the firm in offering long-term wage contracts. In Section 5, we examine rigidity in this market when unexpected changes occur in market conditions. We explore conditions for the firm not to adjust the wage to these unexpected changes.

## 2 A Model of Work Intensity and Workers' Learning by Doing

### 2.1 Work Intensity, Learning by Doing and Labor Productivity

To capture characteristics of the labor market of Japanese large firms as simple as possible, we consider a segmented labor market consisting of one firm and a set of workers attached to the firm. The firm is a going concern with infinite horizon, while workers live in two periods. Generation-$t$ workers are born in period $t$, work for the firm in period $t$ as young workers and in period $t+1$ as old workers, and die at the end of period $t+1$. Worker generations overlap one another: generation-$t$ young workers work with generation $t-1$ old workers. Workers are homogeneous within one generation, so that we consider the representative worker in each generation in the following analysis. The firm determines the wages for the young and for the old, while the worker determines his work intensity. To simplify analysis further, we assume that (a) the price of the firm's product is given to the firm, (b) the firm's technology exhibits constant returns to scale, and (c) work hours are fixed and normalized to be unity.

The worker's productivity depends on his work intensity denoted by $X$. In particular, we assume a linear relationship between them in the form of the following production function of the young worker,

$F_{Young}(X_{Young})$:

$$F_{Young}(X_{Young}) = X_{Young}; \quad X_{Young} : \text{Young's work intensity.}$$

Thus, the firm can perfectly infer the level of work intensity from the level of production.

Let us now characterize the effect of learning by doing. The worker acquires skills when young and increases his productivity when old. The level of acquired skills is dependent on the work intensity. To incorporate this learning by doing, we assume the following production function of the old worker

$$F_{Old}\left(\overline{X_{Young}}, X_{Old}\right) = \left(1 + \delta\overline{X_{Young}}\right) X_{Old}; \quad X_{Old} : \text{Old's work intensity,}$$

where $\delta > 0$ and the upper bar over $X_{Young}$ indicates that the variable is determined in the previous period and fixed in this period.

Let $P$ be the price of the firm's product. Under the assumption of price-taking behavior, the total revenue of the firm is

$$\begin{aligned} \text{Revenue} &= P\left\{F_{Young}(X_{Young}) + F_{Old}\left(\overline{X_{Young}}, X_{Old}\right)\right\} \\ &= P(X_{Young} + X_{Old}) + P\left(\delta\overline{X_{Young}}\right) X_{Old}, \end{aligned}$$

where the last term represents the proceeds from learning by doing.

We assume that (a) the firm offers different base (piece) wage rate for the young and the old ($W_{Young}$ and $W_{Old}$) and that (b) the proceeds from learning by doing are shared between the firm and the worker by the ratio of $1-\theta$ and $\theta$. We further assume the sharing ratio $\theta$ is exogenously given. Under the above sharing scheme, the wage payments and the profit are, respectively,

$$\text{Wage payments for the young} = W_{Young}X_{Young} \tag{1}$$

$$\text{Wage payments for the old} = W_{old}X_{Old} + \theta P\left(\delta\overline{X_{Young}}\right) X_{Old} \tag{2}$$

and

$$\begin{aligned} \text{Profit} &= (P - W_{Young})\, X_{Young} \\ &\quad + (P - W_{Old})\, X_{Old} + (1-\theta)\, P\delta\overline{X_{Young}}X_{Old} \end{aligned} \tag{3}$$

## 2.2 Worker's Life-Time Utility and Work-Intensity Determination

The representative worker of generation $t$ enters the labor market in period $t$, works for two periods ($t$ and $t+1$) and exits afterwards. The worker's utility depends on the level of consumption, work hours, and work intensity. We have assumed that labor hours are constant and normalized to be unity. Taking this in mind we assume that the generation-$t$ worker's life-time utility is

$$\begin{aligned} u_t &= Z_{t0} - f_1 X_{t0} - \frac{f_2}{2}(X_{t0})^2 \\ &\quad + \beta\left(Z_{t1} - f_1 X_{t1} - \frac{f_2}{2}(X_{t1})^2\right) - f_3 X_{t0} X_{t1} \end{aligned}$$

where $f_1 > 0$ and $f_2 > 0$. $Z$ denotes the consumption of goods, whose price is assumed to be unity. Here subscript $t$ denotes the generation, subscript 0 indicates the value when he is young, 1 indicates the value when he is old. In particular, $X_{t0}$ and $X_{t1}$ are

$X_{t0}$ : generation-$t$ worker's work intensity when young at period $t$

$X_{t1}$ : generation-$t$ worker's work intensity when old at period $t+1$.

We assume utility function is concave. In addition, we assume

$$P > f_1,$$

and

$$4f_2^2 - \beta\left\{2\left(\theta\delta P - \beta^{-1}f_3\right) + P\delta\left(1-\theta\right)\right\}^2 > 0, \tag{4}$$

the latter of which guarantees the concavity of the firm's generational profit function (see Appendix). The former is necessary for production to be economically profitable.

There are several distinctive features in this formulation. First, work intensity has a negative effect on utility and the marginal disutility of work intensity is increasing. Second, there might be a learning-by-doing effect represented by the last term $-f_3X_{t0}X_{t1}$. The worker may acquire skills to get along with his coworkers, which may decrease disutility of work. This case is represented by the *negative* $f_3$ case in the above formulation. The formulation also includes the case of the burned-out effect, in which high work intensity when young increases disutility of work when old (a positive $f_3$). Third, the life-time utility function is quasi-linear, so that there is no income effect on the determination of work intensity, which reduces complexity of the model greatly.

The budget constraint of the representative generation-$t$ worker is, from (1) and (2)

$$Z_{t0} + \beta Z_{t1} = W_{t0}X_{t0} + \beta\left(W_{t1} + \theta\delta P\overline{X_{t0}}\right)X_{t1}.$$

The worker maximizes his life-long utility under the constraint of the life-long budget, with respect to the consumption $Z_{t0}$ and $Z_{t1}$and the work intensity $X_{t0}$ and $X_{t1}$. Since the model is dynamic, we analyze the worker's work-intensity determination backward to first examine the old worker's problem.

**Work Intensity of the Generation-$t$ Old** Since the utility function is quasi-linear, the generation-$t$ old worker's budget-constrained utility maximization is reduced to the following unconstrained maximization:

$$\max_{X_{t1}}\left[\beta\left(W_{t1}X_{t1} + \theta\delta P\overline{X_{t0}}X_{t1} - f_1X_{t1} - \frac{f_2}{2}\left(X_{t1}\right)^2\right) - f_3\overline{X_{t0}}X_{t1}\right]$$

for given $\overline{X_{t0}}$. The first order condition of this problem is

$$W_{t1} + \theta\delta P\overline{X_{t0}} - f_1 - f_2 X_{t1} - \beta^{-1} f_3 \overline{X_{t0}} = 0. \tag{5}$$

Solving this first order condition, we obtain the old's work intensity such that

$$X_{t1} = \widetilde{X^1}\left[\overline{X_{t0}}, W_{t1}\right] \equiv \frac{-f_1 + \xi\overline{X_{t0}} + W_{t1}}{f_2}, \tag{6}$$

where

$$\xi \equiv \theta\delta P - \beta^{-1} f_3. \tag{7}$$

The work-intensity function (6) shows that a higher wage induces a higher work intensity. Moreover, if there is a positive effect of learning by doing ($\delta > 0$ and/or $f_3 < 0$), then $\xi$ is positive. Thus, a higher work intensity in the previous period induces a higher work intensity in the present period.

**Work Intensity of the Generation-$t$ Young** By the same argument as in the previous subsection, the generation-$t$ young's life-time utility maximization is reduced to

$$\max_{X_{t0}} \left( W_{t0} X_{t0} - f_1 X_{t0} - \frac{f_2}{2}\left(X_{t0}\right)^2 \right)$$
$$+\beta\left\{ \left(W_{t1}^e + \theta\delta P X_{t0}\right)\widetilde{X^1} - f_1\widetilde{X^1} - \frac{f_2}{2}\left(\widetilde{X^1}\right)^2 \right\} - f_3 X_{t0}\widetilde{X^1},$$

where $\widetilde{X^1} = \widetilde{X^1}\left[X_{t0}, W_{t1}^e\right]$ defined in (6). Note that $W_{t1}^e$ is the expectations formed in this period about the next-period's wage rate for the old, $W_{t1}$. Taking account of the envelope relation (5), we have the first order condition such that

$$W_{t0} - f_1 - f_2 X_{t0} + \beta\left(\theta\delta P - \beta^{-1} f_3\right)\widetilde{X^1}\left[X_{t0}, W_{t1}^e\right] = 0.$$

Consequently, the work intensity of the young, $X_{t0}$, is

$$X_{t0} = X^0\left[W_{t0}, W_{t1}^e\right] \equiv \frac{-f_1 f_2 - \beta f_1 \xi + f_2 W_{t0} + \beta\xi W_{t1}^e}{\left(f_2\right)^2 - \beta\xi^2} \tag{8}$$

Since (4) implies

$$\left(f_2\right)^2 - \beta\xi^2 > 0, \tag{9}$$

we know that a higher current wage induces a higher current work intensity. Moreover, if there is a positive effect of learning by doing ($\delta > 0$ and $f_3 < 0$), a higher expected wage in the next period induces a higher work intensity in the present period. A higher wage in the next period implies a higher utility return from learning by doing, and thus the worker increases its work intensity in the present.[2]

Substituting (8) into (6) and substituting $W_{t1}$ with $W_{t1}^e$, we obtain the planned work intensity as a function of ($W_{t0}$,$W_{t1}^e$):

$$X_{t1}^{Planned} = X^1 \left[W_{t0}, W_{t1}^e\right] \equiv \frac{-f_1 (f_2 + \xi) + \xi W_{t0} + f_2 W_{t1}^e}{(f_2)^2 - \beta\xi^2} \tag{10}$$

## 2.3 The Current Profit and the Value of the Firm

Substituting the results obtained in the previous sub-section into (3), the current profit for the firm in the period $t$ is

$$\begin{aligned} & \widetilde{\pi(t)} \left(\overline{X_{t-1,0}}, W_{t-1,1}, W_{t0}, W_{t1}^e\right) \\ = \; & (P - W_{t0}) X^0 \left[W_{t0}, W_{t1}^e\right] \\ & + \left[P \left\{1 + (1-\theta)\,\delta \overline{X_{t-1,0}}\right\} - W_{t-1,1}\right] \widetilde{X^1} \left[\overline{X_{t-1,0}}, W_{t-1,1}\right]. \end{aligned} \tag{11}$$

Similarly, the current profit at in the period $t+1$ is

$$\begin{aligned} & \pi \widetilde{(t+1)} \left(X^0 \left[W_{t0}, W_{t1}^e\right], W_{t1}, W_{t+1,0}, W_{t+1,1}^e\right) \\ = \; & (P - W_{t+1,0}) X^0 \left[W_{t+1,0}, W_{t+1,1}^e\right] \\ & + \left[P \left\{1 + (1-\theta)\,\delta X^0 \left[W_{t0}, W_{t1}^e\right]\right\} - W_{t1}\right] \widetilde{X^1} \left[X^0 \left[W_{t0}, W_{t1}^e\right], W_{t1}\right]. \end{aligned} \tag{12}$$

Consequently, the value of the firm is thus determined as

$$\begin{aligned} V \; = \; & \widetilde{\pi(t)} \left(\overline{X_{t-1,0}}, W_{t-1,1}, W_{t0}, W_{t1}^e\right) \\ & + \beta \pi \widetilde{(t+1)} \left(X^0 \left[W_{t0}, W_{t1}^e\right], W_{t1}, W_{t+1,0}, W_{t+1,1}^e\right) \\ & + \bullet\bullet\bullet \; . \end{aligned} \tag{13}$$

# 3 Overlapping Long-Term Wage Contracts and Incentive to Renege

It is often observed that large Japanese firms have an age-based wage profile, in which old workers receive higher wages than young workers, and that the shape of this age-related wage profile is stable over periods. Thus, young workers can infer their wages when they become old from this age-based wage profile. The age-based wage profile and its long-run stability can be interpreted as the firm's

[2] Marginal utility of the worker from learning by doing is $\beta\theta\delta P X_{t1}$. An increase in the future wage $W_{t1}$ increases $X_{t1}$ and thus increases the marginal utility of learning by doing.

offering long-term wage contracts to young workers specifying the wage when they are young and when they are old, and honoring past long-term wage contracts for old workers. However, these long-term wage contracts are implicit, since future wages of the young when they become old are not specified in actual wage contracts.

In Section 3.1, we examine wage and work-intensity determination in our model of learning by doing under the regime of long-term wage contracts and show that the wage profile is in fact age-based with pure seniority (age) premium. The long-term wage contract for generation-$t$ workers is here defined as a pair of the wage rates when young and when old for each generation, $\{(W_{t0}, W_{t1})\, \textit{for generation } t\}$. Since there are generation-$t$ young workers and generation-$t-1$ old ones in period $t$, these generational wage contracts overlaps one another.

The implicit long-term wage contract, however, has an inherent problem of a renege incentive. We show in Section 3.2 that the firm has a strong incentive to renege the long-term wage contract when workers are old, if young workers still believe the firm's pledge to honor the long-term wage contract for them in the future. Such a temptation to renege is overcome only when the renege of the wage contract for one generation triggers mistrust of the firm's pledge in subsequent generations, which in turn reduces the firm's profits in the future substantially. In Section 3.3, we examine conditions under which this is the case, so that implicit long-term wage contracts offered by the firm is credible.

## 3.1 Wages and Work Intensity under the Long-Term-Contract Regime

Suppose that the implicit long-term wage contract is credible, so that the workers trust the firm's pledge and that the firm honors its pledge. In this case, we have $W_{t1}^e = W_{t1}$ for all $t$. In addition, since the firm honors its past pledge, $\overline{W_{t-1,1}}$ (wage for the period-$t$ old pledged in the previous period) is given. Then, for given $\overline{X_{t-1,0}}$ (work intensity of the previous period), the firm's value is

$$\begin{aligned} V &= \widetilde{\pi(t)}\left(\overline{X_{t-1,0}}, \overline{W_{t-1,1}}, W_{t0}, W_{t1}\right) \\ &\quad + \beta \widetilde{\pi(t+1)}\left(X^0\left[W_{t0}, W_{t1}\right], W_{t1}, W_{t+1,0}, W_{t+1,1}\right) \\ &\quad + \ldots \end{aligned}$$

The firm maximizes its value with respect to the long-term wage contracts $\{(W_{t0}, W_{t1})\, \textit{for generation } t\}$. Thus, the first order condition is for $t \geq 0$

$$\begin{aligned} \left[\widetilde{\pi(t)}\right]_3 + \beta\left[\widetilde{\pi(t+1)}\right]_1 \left[X^0\right]_1 &= 0; \\ \left[\widetilde{\pi(t)}\right]_4 + \beta\left(\left[\widetilde{\pi(t+1)}\right]_1 \left[X^0\right]_2 + \left[\widetilde{\pi(t+1)}\right]_2\right) &= 0 \end{aligned}$$

Here $F_i$ denotes the derivative of the function $F$ with respect to the $i$-th argument. By solving these equations, the long-term wage contract $\left(W_0^O, W_1^O\right)$ is

$$W_0^O = \frac{P+f_1}{2} + \frac{1}{2}\beta P\delta(1-\theta)(P-f_1)\frac{P\delta(1-\theta)+2(f_2+\xi)}{4f_2^2-\beta(2\xi+(1-\theta)P\delta)^2} \text{ (wage for the young)} \tag{14}$$

$$W_1^O = W_0^O + \delta(P-f_1)\frac{f_2 P(1-\theta)(1-\beta)}{4(f_2)^2-\beta(2\xi+(1-\theta)P\delta)^2} \text{ (wage for the old)} \tag{15}$$

Here superscript $O$ indicates that optimal wages under the long-term-contract regime are the optimal Open-Loop policy in the terminology of control theory.

We have assumed that $P > f_1$ and $4f_2^2 - \beta(2\xi + P\delta(1-\theta))^2 > 0$. Under these conditions, (14) and (15) show that $\partial W_0^O/\partial\delta > 0$, and $\partial W_1^O/\partial\delta > 0$ so long as $\delta > 0$ and $\theta > 0$. Thus, if there is a positive learning by doing effect on productivity in the future ($\delta > 0$) and that the worker has a positive share of it ($\theta > 0$), the firm increases wages to motivate the worker to learn more by doing in the present with higher work intensity.

Moreover, we have $W_1^O > W_0^O$ when $\delta > 0$ and $\theta > 0$, since $1 > \beta$: The firm offers a higher base wage for the old. It should be noted that the old worker gets its share of the proceeds from learning by doing, in addition to this higher base wage. Thus we have a pure seniority premium (higher wage rate for the old than for the young).

The reason of this seniority effect is intuitive. Both a higher wage for the young $W_{t0}$ and that for the old $W_{t1}$ can induce higher work intensity in the present in a similar way, when both direct and indirect effects are properly accounted for. However, the wage for the old $W_{t1}$ is paid in the future and is discounted in the consumer's utility calculation, while the wage for the young is not. In order to offset this effect, the firm has to pay a higher wage for the old than that for the young.

Substituting (14) and (15) into (8) and (10), we have the work intensity of the young and the old under the implicit long-term wage contract $[W_0^O, W_1^O]$:

$$X^0[W_0^O, W_1^O] = (P-f_1)\frac{\beta P\delta(1-\theta)+2(f_2+\beta\xi)}{4f_2^2-\beta(P\delta(1-\theta)+2\xi)^2} \tag{16}$$

$$X^1[W_0^O, W_1^O] = (P-f_1)\frac{P\delta(1-\theta)+2(f_2+\xi)}{4f_2^2-\beta(P\delta(1-\theta)+2\xi)^2} \tag{17}$$

### 3.2 Firm's Incentive to Renege

Suppose, *as a thought experiment*, that the firm in period $t$ can renege its implicit long-term wage contract $W_{t-1,1} = W_1^O$ for the generation-$t-1$ old worker and offer a new wage $W_{t-1,1}$ but that *the young worker of the generation-t and subsequent generations still believes the firm's pledge to honor long-term wage contract* $(W_{t0}, W_{t1}) = (W_0^O, W_1^O)$ *for them.* In this sub-section, we show that the firm always has an incentive to renege the long-term contract in this case.

If the young worker still believe $(W_{t0}, W_{t1}) = \left(W_0^O, W_1^O\right)$, the only difference in profits between the renege case and the non-renege case is the profit from the generation-$t-1$ old's work, which is from (3) and (6)

$$\tilde{\pi}_t^{Old} = (P - W_{t-1,1})\,\widetilde{X^1}\left[X_{Young}^O, W_{t-1,1}\right] + (1-\theta)\,P\delta X_{Young}^O \widetilde{X^1}\left[X_{Young}^O, W_{t-1,1}\right].$$

where $X_{Young}^O = X^0\left[W_0^O, W_1^O\right]$ is predetermined and fixed in this period. The maximization of this renege profit determines the optimal renege wage for the old, which is

$$W_{t-1,1}^R\left[X_{Young}^O\right] = W_1^O - \frac{1}{2}\xi\,(P - f_1)\,\frac{2\,(f_2 + \beta\xi) + \beta P\delta\,(1-\theta)}{4f_2^2 - \beta\,(P\delta\,(1-\theta) + 2\xi)^2}. \tag{18}$$

Thus, the optimal renege wage $W_{t-1,1}^R$ is always lower than the optimal wage pledge $W_1^O$. The firm always has an incentive to cut the old's wage if such wage cut has no adverse effect on young workers. This is because the firm does not have to take an adverse effect of such a wage cut on the generation-$t-1$ old's learning by doing, since that learning took place in the previous period. Because the wage is lower, the work intensity in the renege case, $X_{Old}^R$, is also lower than the work intensity of the no-renege case, $X_{Old}^O = X^1\left[W_0^O, W_1^O\right]$.

$$\begin{aligned} X_{Old}^R &= \widetilde{X^1}\left[X_{Young}^O, W_{t-1,1}^R\left[X_{Young}^O\right]\right] \\ &= X_{Old}^O - \frac{1}{2}\xi\frac{(P-f_1)\left[2\,(f_2+\beta\xi) + \beta P\delta\,(1-\theta)\right]}{f_2\left(4f_2^2 - \beta\,(2\xi + P\delta\,(1-\theta))^2\right)} \end{aligned}$$

Let us now measure the gain from this renege. Let $\widetilde{\pi(t)}_{Old}^O$ and $\widetilde{\pi(t)}_{Old}^R$ denote the period-$t$ profit from the generation-$t-1$ old's work in the no-renege case and in the renege case, respectively, such that

$$\begin{aligned} \widetilde{\pi(t)}_{Old}^O &\equiv \left[P - W_1^O\right] X_{Old}^O + (1-\theta)\left(P\delta X_{Young}^O\right) X_{Old}^O \\ \widetilde{\pi(t)}_{Old}^R &= \left[P - W_{t-1,1}^R\left(X_{Young}^O\right)\right] X_{Old}^R + (1-\theta)\left(P\delta X_{Young}^O\right) X_{Old}^R \end{aligned}$$

Substituting the results of (??) and (18), we obtain the gain from the renege

$$\widetilde{\pi(t)}_{Old}^R - \widetilde{\pi(t)}_{Old}^O = \frac{1}{4}\,(P-f_1)^2\,\frac{\xi^2}{f_2}\frac{\left(2\,(f_2+\beta\xi) + \beta P\delta\,(1-\theta)\right)^2}{\left(4f_2^2 - \beta\,(P\delta\,(1-\theta) + 2\xi)^2\right)^2}$$

If there is a positive effect of learning by doing ($\delta > 0$ and/or $f_3 < 0$) and that the worker has a positive share of the proceeds from learning by doing, the term $\xi = \theta\delta P - \beta^{-1} f_3$ is positive. Then, it is evident that the gain from the renege is positive, so that the firm has a strong incentive to renege in this case.

## 3.3 Worker Reaction to the Renege and Conditions for Credible Long-Term Wage Contracts

In the previous section, we have assumed as a thought experiment that the young worker of the generation-$t$ and subsequent generations still believe the firm's pledge to honor long-term wage contract for them in the future even if the firm reneges the contract for the old in the present, and have shown that the firm has a strong incentive to renege the long-term wage contract. However, once the firm reneges the long-term wage contract for generation-$t-1$ old workers, it is very likely that young workers of generation-$t$ and subsequent generations do not trust the firm's pledge of honoring the long-term wage contract for them in the future. If this is the case, the firm can only offer spot wage contracts, specifying only the current wages for the young and the old.

In Section 3.3.1, we first derive the firm value under the regime of spot wage contracts. By using this result, the firm value is compared between the renege case and the no-renege case in Section 3.3.2, and derive the net gain of keeping the long-term wage contract.

Under rational expectations hypothesis, long-term contract offers are credible only if the net gain of keeping the long-term wage contract is positive. In Section 3.3.3, we derive the conditions under which the firm chooses not to renege and keeps to honor the long-term wage contract, that is, conditions for credible long-term wage contracts.

### 3.3.1 Firm Value under the Spot-Contract Regime

Suppose that the firm can only offer spot wage contracts, specifying the current wage for the young and the old. The spot wage contract in period $t$ is a pair of the wage for the old and that for the young, $(W_{t-1,1}, W_{t0})$.

The firm's problem is now to maximize its value (13), which is rewritten here for convenience,

$$\begin{aligned} V &= \widetilde{\pi(t)}\left(\overline{X_{t-1,0}}, W_{t-1,1}, W_{t0}, W_{t1}^{e}\right) \\ &\quad +\beta\pi\,\widetilde{(t+1)}\left(X^{0}\left[W_{t0}, W_{t1}^{e}\right], W_{t1}, W_{t+1,0}, W_{t+1,1}^{e}\right) \\ &\quad +\ldots \end{aligned}$$

with respect to the spot wage contract $(W_{t-1,1}, W_{t0})$ for a given level of $\overline{X_{t-1,0}}$ (the present-period old worker's work intensity when he was young in the previous period), taking account of the expectation formation of the workers. Here $\pi\widetilde{(t)}$ and $\pi\widetilde{(t+1)}$ are defined in (11) and (12). As for expectation formation of the workers, we assume rational expectations.

The firm value in this case can be derived by the method of dynamic programming. Assume, as an educated guess, that the representative worker's expectation formation is characterized by

$$W_{t1}^{e} = h\left(W_{t0}\right) \equiv h_0 + h_1 W_{t0}, \tag{19}$$

where $h_0$ and $h_1$ are undetermined coefficients. Then, the necessary condition for the optimum is the following Bellman equation:

$$V^C(X_{t-1,0}) = \max_{W_{t-1,1}, W_{t0}} \left[ \begin{array}{c} \widetilde{\pi(t)}\left(X_{t-1,0}, W_{t-1,1}, W_{t0}, h\left(W_{t0}\right)\right) \\ \\ +\beta V^C\left(X^0\left[W_{t0}, h\left(W_{t0}\right)\right]\right) \end{array} \right], \tag{20}$$

where $V^C(X_{t-1,0})$ is the firm's value under spot-contract regime.

Assume again as an educated guess that

$$V^C(x) = \eta_0 + \eta_1 x + \eta_2 x^2 \tag{21}$$

where $\eta_0$, $\eta_1$ and $\eta_2$ are undetermined coefficients. Then, by using the undermined coefficient method, we obtain the following solution of the Bellman equation and the expectation function (see Appendix). The parameters in the expectation function $h$ are

$$h_0 = \frac{P\left(f_2^2 - \beta\xi^2 - \delta f_1(1-\theta)(f_2+\beta\xi)\right) + f_2 f_1 (f_2+\xi)}{2(f_2)^2 - \beta\xi^2 - (1-\theta)\delta P\beta\xi}; \tag{22}$$

$$h_1 = \frac{f_2\left((1-\theta)\delta P - \xi\right)}{2(f_2)^2 - \beta\xi^2 - (1-\theta)\delta P\beta\xi}. \tag{23}$$

while the parameters in the value function $V^C$ are

$$\eta_0 = \frac{1}{1-\beta}\frac{(P-f_1)^2}{4f_2}\left(1 + \frac{(2f_2 + 2\beta\xi + \beta P\delta(1-\theta))^2}{4f_2^2 - \beta(3\xi + P\delta(1-\theta))(\xi + P\delta(1-\theta))}\right); \tag{24}$$

$$\eta_1 = (P-f_1)\frac{(P(1-\theta)\delta+\xi)}{2f_2}; \tag{25}$$

$$\eta_2 = \frac{((1-\theta)\delta P+\xi)^2}{4f_2} \tag{26}$$

### 3.3.2 Net Gains from Honoring the Long-Term Wage Contract

Let us now derive the firm value when the firm reneges the long-run wage contract for the old in one generation, *under the assumption that such a renege jeopardizes the confidence of the subsequent worker generations in the firm's pledge.* We compare this with the firm value when the firm keeps to honor the long-term wage contract.

Suppose as in the previous section that the firm has kept to honor the long-term wage contract up until period $t-1$, but that the firm decides to renege the long-term wage contract in period $t$. Then, the old worker's work intensity in the previous period is $X^0\left[W_0^O, W_1^O\right]$, so that the result in the previous period implies that the value of the reneging firm is $V^C\left(X^0\left[W_0^O, W_1^O\right]\right)$.

Next consider the value of the firm honoring its long-term wage contract all the time, which is denoted by $V^O$. Using the results obtained in Section 3.1, we have with some calculation we have

$$\begin{aligned} V^O &= \frac{(P-f_1)^2 (P\delta(1-\theta)+2(f_2+\xi))\left(2\left(f_2^2-\beta\xi^2\right)+P\delta(1-\theta)(f_2-\beta\xi)\right)}{\left(4f_2^2-\beta(P\delta(1-\theta)+2\xi)^2\right)^2} \\ &+\frac{(P-f_1)^2((1+\beta)f_2+\beta P\delta(1-\theta)+2\beta\xi)}{(1-\beta)\left(4f_2^2-\beta(P\delta(1-\theta)+2\xi)^2\right)^2} \end{aligned}$$

By using the above results, the net gain of honoring the long-term contract can be derived, with tedious calculation, which is

$$\begin{aligned} & V^O - V^C\left(X^0\left[W_0^O, W_1^O\right]\right) \\ = & \frac{1}{4f_2}\frac{1}{(1-\beta)}\left[\frac{(P-f_1)^2\xi^2\{2(f_2+\beta\xi)+\beta P\delta(1-\theta)\}^2}{\left[4f_2^2-\beta\{2\xi+P\delta(1-\theta)\}^2\right]^2}\right]\left[\frac{(2\beta-1)\Psi-\beta^2\xi^2}{\Psi}\right] \end{aligned} \tag{27}$$

where

$$\begin{aligned} \Psi &= 4f_2^2-3\beta\xi^2-4\xi\beta P\delta(1-\theta)-\beta\{P\delta(1-\theta)\}^2 \\ &> 4f_2^2-\beta\{2\xi+P\delta(1-\theta)\}^2>0, \end{aligned}$$

since we have assumed the latter inequality in (4) with $\xi \equiv \theta\delta P - \beta^{-1}f_3$. Thus

$$sgn\left[V^O - V^C\left(X^0\left[W_0^O, W_1^O\right]\right)\right] = sgn\left[(2\beta-1)\Psi-\beta^2\xi^2\right] \tag{28}$$

If the net gain is positive, then the firm shuns reneging. Under our rational expectations assumption, workers know this fact and consider credible the firm's pledge to honor the long-term wage contract in the future. Then, the long-term wage contract becomes self-enforcing. However, if the net gain is negative, then the firm reneges the contract in the present and in the future. Rational workers correctly anticipate this behavior and do not trust the pledge of the firm to honor the long-term wage contract. Since no worker believes the pledge, only spot wage contracts are possible.

### 3.3.3 Conditions for Credible Long-Term Wage Contracts

The relation (28) determines the conditions under which the long term wage contract is credible. First, if $\beta < 1/2$, then the net gain is negative. This means that if the discount rate is high, then long term wage contract is not credible. This result implies that the long-term wage contract is not ubiquitous even though there is a substantial learning-by-doing opportunity. The long-term wage contract, and thus a high level of learning by doing, is not likely to be observed in a high-discount-rate economy.

If $\delta > 0$ and $f_3 = 0$, that is, learning by doing matters only in production process, the condition for credible long-term wage contract is substantially simplified. In this case, we have

$$\begin{aligned} (2\beta - 1)\Psi - \beta^2\xi^2 &= 4(2\beta-1) f_2^2 - \beta P^2 \delta^2 \left[(2\beta-1)(2\theta+1) - \beta\theta^2\right] \\ &\Longleftrightarrow \\ \frac{2f_2}{P\sqrt{\beta(2\theta+1) + \frac{\beta^2\theta^2}{2\beta-1}}} &> \delta \end{aligned}$$

Consequently, we have by arranging terms

$$\begin{aligned} sgn\left[V^O - V^C\left(X^0\left[W_0^O, W_1^O\right]\right)\right] &= sgn\left[4(2\beta-1) f_2^2 - \beta P^2\delta^2\left[(2\beta-1)(2\theta+1) - \beta\theta^2\right]\right] \\ &= sgn\left[\frac{2f_2}{P\sqrt{\beta(2\theta+1) + \frac{\beta^2\theta^2}{2\beta-1}}} - \delta\right] \end{aligned}$$

Thus, so long as the learning-by-doing effect $\delta$ is not large, the long-term wage contract is credible. In contrast, if the learning by doing effect is large, then the firm's gain from renege is large, which makes the long-term wage contract unsustainable.

**Numerical Examples** Figures 1 and 2 illustrate the range of the parameters making the long-term wage contract credible. In these figures, we set $f_2 = P = 1$. The area below the thin curve in figure 1 is the domain of $(\theta, \delta)$ that supports credible long-term wage contracts for fixed $\beta$ (=0.6). This figure shows that if the worker's share of the proceeds from learning by doing, the range of the credible long-term wage contract shrinks. However, even if the worker takes all of the proceeds, the firm still has an incentive to honor the long-term contract if $\delta$ is not so large, for example, if $\delta = 0.5 > 0$. (The marginal productivity of the old is $(1 + \delta)$ while that of the young is 1. Therefore, $\delta = 0.5$ means the old's marginal productivity is 1.5 times as much as the young's.)

Figure 2 shows the domain of $(\beta, \delta)$ that supports credible long-term wage contracts for $\theta =$ 0.1 (gray line), 0.5 (black thin line) and 1 (black bold line). The area below each line represents $(\beta, \delta)$ supporting credible long-term contract. As $\beta$ increases, the domain of $\delta$ that supports credible contracts is eventually narrowed.

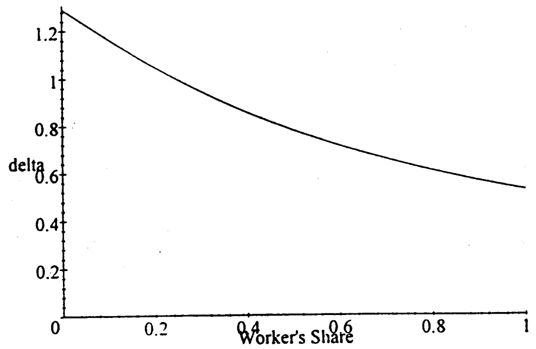


Figure 1: $(\theta, \delta)$ supporting long-term contracts

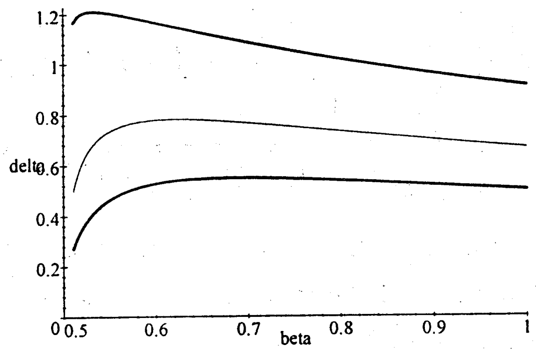


Figure 2: $(\delta, \beta)$ supporting credible long-term wage contracts for $\theta$ =0.1 (Gray), 0.5 (Black, Thin), 1 (Black, Bold)

## 4 Unanticipated Change of $P$ and Wage Rigidity

Let us now take up the issue of wage rigidity. In this section, we consider the case that there is an unexpected change in market conditions, and examine whether the firm adjusts the long-term contract to a new condition or the firm chooses not to adjust it and keeps the long-term contract pledged in the past.

Suppose that the market has been in a stable condition for a long time, and the firm's long-term contracts have been the optimal one for that condition. Suppose then that the market condition has changed unexpectedly. In particular, we assume that the firm's product price has changed from $P$ to $P - 100 * \Delta_P$ permanently. That is, the product price has dropped by $\Delta_P\%$.

As in the previous section, we assume that the firm's renege of the long-term contract for the old worker damages the confidence of the young worker on the firm's long-term wage contract, so that once the firm reneges, the firm is no longer able to use long-term contracts.

**Direct Effect of Price Change on Work Intensity.** In our model, the change in the market price directly affects the proceeds of the learning by doing. Since the worker shares a part of these proceeds (see (2)), the price change directly influences the worker's work-intensity decision. From (6), (8) and (10), new work-intensity function is, for the old,

$$\widetilde{X^1}\left[\overline{X_{t0}}, W_{t1}; P(1-\Delta_P)\right] \equiv \frac{-f_1 + \left(\theta\delta P(1-\Delta_P) - \beta^{-1} f_3\right)\overline{X_{t0}} + W_{t1}}{f_2}.$$

Similarly, the young's work-intensity is

$$\begin{aligned} & X^0\left[W_{t0}, W_{t1}^e; P(1-\Delta_P)\right] \\ \equiv\ & \frac{-f_1 f_2 - \beta(\xi - \theta P\delta\Delta_P) + f_2 W_{t0} + \beta(\xi - \theta P\delta\Delta_P) W_{t1}^e}{(f_2)^2 - \beta(\xi - \theta P\delta\Delta_P)^2}, \end{aligned}$$

and the young's planned work-intensity in the next period is

$$\begin{aligned} & X^1\left[W_{t0}, W_{t1}^e; P(1-\Delta_P)\right] \\ \equiv\ & \frac{-f_1 f_2 - f_1(\xi - \theta P\delta\Delta_P) + (\xi - \theta P\delta\Delta_P) W_{t0} + f_2 W_{t1}^e}{f_2^2 - \beta(\xi - \theta P\delta\Delta_P)^2} \end{aligned}$$

**The Value of Firm Honoring the Long-Term Contract (Wage Rigidity).** Suppose that the firm honors the long-term contract for the old (the case of rigid wages). Thus, the generation-$t-1$ workers get $W_1^O$. Consequently, the generation-$t-1$ old's work intensity is

$$\widetilde{X^1}\left[X^0\left[W_0^O, W_1^O\right], W_1^O; P(1-\Delta_P)\right] = \left[X^1\right]^O - \frac{\theta\delta P\Delta_P}{f_2}\left[X^0\right]^O$$

where $\left[X^0\right]^O = X^0\left[W_0^O, W_1^O\right]$ and $\left[X^1\right]^O = X^1\left[W_0^O, W_1^O\right]$. Thus, the profit from the work of the old when $\Delta_P > 0$, $\widetilde{\pi(t)}_{Old}^{O}(\Delta_P)$, is

$$\begin{aligned} \widetilde{\pi(t)}_{Old}^{O} \mid_{\Delta_P>0} &= \widetilde{\pi(t)}_{Old}^{O} \mid_{\Delta_P=0} \\ & -P\Delta_P \left[\begin{array}{l} \left\{1 + (1-\theta)\delta\left[X^0\right]^O\right\} \times \left(\left[X^1\right]^O + \frac{P\theta\delta}{f_2}(1-\Delta_P)\left[X^0\right]\right) \\ \qquad -\frac{\theta\delta}{f_2}\left[X^0\right]^O \end{array}\right] \end{aligned}$$

where $\widetilde{\pi(t)}_{Old}^{O} \mid_{\Delta_P=0}$ is the profit when the price is $P$.

If the firm honors the long-term contract for the generation-$t-1$ old, the generation-$t$ workers accept the long-term wage contract for them, which is adjusted to new price condition. Therefore, the value of the firm keeping the long-term contract, $V^O\mid_{\Delta_P=0}$ is

$$\begin{aligned}
V^O \mid_{\Delta_P=0} &= \widetilde{\pi(t)}^{O}_{Old}\mid_{\Delta_P=0} \\
&\quad -P\Delta_P\left[\begin{array}{c}\left\{1+(1-\theta)\,\delta\left[X^0\right]^O\right\}\times\left(\left[X^1\right]^O+\frac{P\theta\delta}{f_2}(1-\Delta_P)\left[X^0\right]\right)\\ -\frac{\theta\delta}{f_2}\left[X^0\right]^O\end{array}\right] \\
&\quad +\left[\frac{(P(1-\Delta_P)-f_1)^2}{1-\beta}\right]\left[\frac{\left\{\begin{array}{c}(1+\beta)\,f_2+\beta P(1-\Delta_P)\,\delta(1-\theta)\\ +2\beta\left(\theta\delta P(1-\Delta_P)-\beta^{-1}f_3\right)\end{array}\right\}}{\left[4f_2^2-\beta\left(\begin{array}{c}P(1-\Delta_P)\,\delta(1-\theta)\\ +2\left(\theta\delta P(1-\Delta_P)-\beta^{-1}f_3\right)\end{array}\right)\right]^2}\right].
\end{aligned}$$

**The Value of the Firm Reneging the Contract (Wage Adjustment).** Let us now consider the value of the firm that adjusts the long-term contract for the old to the new economic condition. The value of the firm in this case can be calculated in the same way as in the previous section with $P$ replaced by $P(1-\Delta_P)$. Consequently, we have

$$V^C\left(X^0\left[W_0^O, W_1^O\right]\right)\mid_{\Delta_P>0}=\eta_0'+\eta_1'x+\eta_2'x^2$$

where

$$\begin{aligned}
\eta_0' &= \frac{1}{1-\beta}\frac{(P(1-\Delta_P)-f_1)^2}{4f_2}\times \\
&\quad \left(1+\frac{\left(2f_2+2\beta\left(\theta\delta P(1-\Delta_P)-\beta^{-1}f_3\right)+\beta P(1-\Delta_P)\,\delta(1-\theta)\right)^2}{4f_2^2-\beta\left(\delta P(1-\Delta_P)-\beta^{-1}f_3\right)\left(\delta P(1-\Delta_P)(1+2\theta)-3\beta^{-1}f_3\right)}\right); \\
\eta_1' &= \frac{(P(1-\Delta_P)-f_1)\left((1-\theta)\,\delta P(1-\Delta_P)+\left(\theta\delta P(1-\Delta_P)-\beta^{-1}f_3\right)\right)}{2f_2}; \\
\eta_2' &= \frac{\left((1-\theta)\,\delta P(1-\Delta_P)+\left(\theta\delta P(1-\Delta_P)-\beta^{-1}f_3\right)\right)^2}{4f_2}.
\end{aligned}$$

**Net Gain of Wage Rigidity and Parameter Values Supporting Wage Rigidity.** Using the results obtained so far, we can calculate the net gain of honoring the long-term contract, which is

$$V^O\mid_{\Delta_P=0}-V^C\left(X^0\left[W_0^O, W_1^O\right]\right)\mid_{\Delta_P>0}.$$

Although the result is complicated mathematically, the general characteristics of the net gain of wage rigidity can be easily illustrated by way of numerical examples and corresponding figures. In the following numerical examples, we set $f_3=0$ (no learning buy doing on the disutility of work intensity) and assume $P=f_2=1$, and $f_1=0$.

Figure 3 illustrates the relation between percentage decrease of price, $\Delta_P$ (horizontal axis) and the net gain of wage rigidity (vertical axis) for parameter values of $\beta=0.7$, $\delta=0.5$, and $\theta=0.5$.

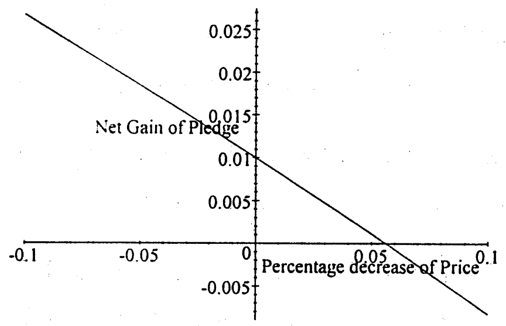


Figure 3: Price Decrease and Net Gain of Wage Rigidity

This figure implies that the permanent 6% decrease in the price may cause the renege and adjustment.

Figure 4 shows the domain of $(\theta, \Delta_P)$ that supports wage rigidity when $\beta = 0.7$ and $\delta = 0.5$ The range below the curve is the long-term-contract supporting domain of $(\theta, \Delta_P)$.

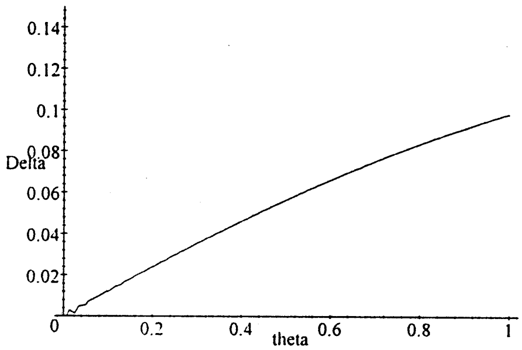


Figure 4: $(\theta, \Delta p)$ supporting wage rigidity

This figure illustrates that as the workers' share increases, the long-term wage contract becomes more robust with respect to changes in the price.

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