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TAMENESS OF PSEUDOVARIETIES OF SEMIGROUPS

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ABSTRACT. Tameness is a property introduced in 1997 by Steinberg and the author in connection with the Krohn-Rhodes complexity problem in order to establish the decidability of semidirect products of pseudovarieties of semigroups. Since then a number of works have been dedicated to proving tameness of pseudovarieties. This paper is a survey of work in this area.

1. THE SEMIDIRECT PRODUCT OF PSEUDOVARIETIES

A *pseudovariety of semigroups* is a class of finite semigroups which is closed under taking homomorphic images, subsemigroups and finite direct products. A pseudovariety is said to be *decidable* if there is an algorithm to test membership in it of a given finite semigroup. Many applications of finite semigroup theory in other areas such as language theory, logic, and complexity theory depend on proving the decidability of specific pseudovarieties.

For a semigroup T, T^1 denotes the least monoid containing T (which is unique up to isomorphism). Given a monoid homomorphism $T^1 \to \operatorname{End} S$ into the monoid of endomorphisms of S, we consider the associated *semidirect product* $S * T$ which is the set $S \times T$ endowed with the operation

$$(s_1, t_1)\,(s_2, t_2) = (s_1\,{}^{t_1}s_2, t_1 t_2)$$

where ${}^{t_1}s_2$ denotes the image of s_2 under the endomorphism associated with t_1. It is easy to see that $S * T$ is again a semigroup.

For pseudovarieties $\mathbf{V}$ and $\mathbf{W}$, their *semidirect product* $\mathbf{V} * \mathbf{W}$ is the pseudovariety generated by all semidirect products $S * T$ with $S \in \mathbf{V}$ and $T \in \mathbf{W}$. A basic question concerning this operation is under what conditions on the factors the semidirect product is decidable. By working with a special type of semidirect product known as the *wreath product*, it can be shown that the semidirect product of pseudovarieties is an associative operation. Thus, a more general question is to find conditions on the factors $\mathbf{V}_1, \ldots, \mathbf{V}_n$ that ensure that the semidirect product $\mathbf{V}_1 * \cdots * \mathbf{V}_n$ is decidable. A potential application of this question appears when one considers such an iterated semidirect product in which the factors are either the pseudovariety $\mathbf{G}$ of all finite groups or the pseudovariety $\mathbf{A}$ of all finite aperiodic (i.e., with only trivial subgroups) semigroups, both of which are idempotents for $*$. Krohn and Rhodes [21, 22] have shown that every finite semigroup belongs to some such semidirect product of $\mathbf{A}$'s and $\mathbf{G}$'s. They defined the least number

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of **G**'s in such a semidirect product containing a given finite semigroup S to be the *complexity* of S and asked whether the complexity function can be effectively computed.

Rhodes [27] has exhibited decidable pseudovarieties $\mathbf{V}$ and $\mathbf{W}$ such that $\mathbf{V} * \mathbf{W}$ is undecidable. One such undecidable example is obtained by taking the semidirect product $\mathbf{Sl} * [\![\Sigma]\!]$ of the pseudovariety $\mathbf{Sl}$ of all finite semilattices by the pseudovariety defined by a certain finite set of identities Σ.

In contrast, Steinberg and the author [8, 9] have established the following result which provides the main motivation for the notion of tameness to be introduced below.

Theorem 1. *If* $\mathbf{V}_1, \dots, \mathbf{V}_n$ *are* tame *pseudovarieties of semigroups then* $\mathbf{V}_1 * \cdots * \mathbf{V}_n$ *is decidable.*

So, in particular, the Krohn-Rhodes complexity problem will be settled affirmatively once it is shown that both **G** and **A** are tame pseudovarieties. For **G** this follows from results of Ash [13]. For **A**, this has been announced by Rhodes [28].

In the next section we introduce formally the notion of tameness while the remainder of the paper discusses particular examples of tame pseudovarieties. While no previous knowledge of finite semigroup theory is assumed to read this paper, many details on the required background are omitted and so a familiarity with some of the literature would certainly help. See [26] for an introduction and motivation, [2] for a more comprehensive treatment, and [4] for an exposition of the profinite approach.

2. Tameness

Throughout this section $\mathbf{V}$ denotes a pseudovariety of semigroups.

A *topological semigroup* is a semigroup S endowed with a topology with respect to which the semigroup operation $S \times S \to S$ is continuous. Finite semigroups are viewed as topological semigroups with the discrete topology. By a *pro-*$\mathbf{V}$ *semigroup* we mean a compact semigroup S which is residually in $\mathbf{V}$ in the sense that, for any two points $s_1, s_2 \in S$, there is a continuous homomorphism $\varphi : S \to T$ into a finite semigroup such that $\varphi s_1 \neq \varphi s_2$.

It can be shown that there is a *free pro-*$\mathbf{V}$ *semigroup on a set* A. This is a (up to homeomorphic isomorphism, unique) pro-$\mathbf{V}$ semigroup $\overline{\Omega}_A\mathbf{V}$ endowed with a *generating* mapping $\iota : A \to \overline{\Omega}_A\mathbf{V}$ such that, for every mapping $\varphi : A \to S$ into a pro-$\mathbf{V}$ semigroup S, there is a unique continuous homomorphism $\hat{\varphi}$ such that the following diagram commutes:

$$\begin{tikzcd} A \arrow[r, "\iota"] \arrow[dr, "\varphi"'] & \overline{\Omega}_A\mathbf{V} \arrow[d, dashed, "\hat{\varphi}"] \\ & S \end{tikzcd}$$

Given $\pi \in \overline{\Omega}_A\mathbf{V}$, this defines an operation

$$\begin{aligned} \pi_S : S^A &\to S \\ \varphi &\mapsto \hat{\varphi}(\pi) \end{aligned}$$

Such an operation is *implicit* in the sense that, for any continuous homomorphism $h : S \to T$ between pro-$\mathbf{V}$ semigroups, the following diagram commutes:

$$\begin{tikzcd} S^A \arrow[r, "\pi_S"] \arrow[d, "h^A"] & S \arrow[d, "h"] \\ T^A \arrow[r, "\pi_T"] & T \end{tikzcd}$$

The mapping

$$\pi \in \overline{\Omega}_A \mathbf{V} \to (\pi_S)_{S \in \mathbf{V}}$$

turns out to be a bijection with the set of A-ary implicit operations on $\mathbf{V}$.

Two very important examples of implicit operations are the following:

- multiplication: $\_ \cdot \_ : (s,t) \mapsto s \cdot t$, interpreted simply as the semigroup operation on each finite semigroup;
- $(\omega - 1)$-power: $\_^{\omega-1} : s \mapsto s^{\omega-1}$ where, for a finite semigroup S and $s \in S$, $t = s^{\omega-1}$ is the unique power of s such that $tst = t$; in particular, $s^\omega = ss^{\omega-1}$ is an idempotent.

In case the arity set A has n elements $x_1, \dots, x_n$, the element x_i may be viewed as the ith component projection, picking the ith component s_i from an n-tuple of elements $(s_1, \dots, s_n) \in S^n$. We then also say that A-ary implicit operations are *n-ary*.

Implicit operations may be composed to obtain again implicit operations: for m implicit operations $\pi_1, \dots, \pi_m$ of arity n and an m-ary implicit operation ρ, the operation $\rho(\pi_1, \dots, \pi_m)$ is defined by the formula

$$(\rho(\pi_1, \dots, \pi_m))_S(s_1, \dots, s_n) = \rho_S(\pi_{1S}(s_1, \dots, s_n), \dots, \pi_{mS}(s_1, \dots, s_n))$$

for $(s_1, \dots, s_n) \in S^n$ and is again an implicit operation.

Denote by $\mathbf{S}$ the pseudovariety of all finite semigroups. A *profinite semigroup* is simply a pro-$\mathbf{S}$ semigroup. By an *implicit signature* we mean a set σ of implicit operations on $\mathbf{S}$ containing the basic semigroup multiplication. For example, $\kappa = \{\_ \cdot \_, \_^{\omega-1}\}$ is called the *canonical signature.* Here the word *canonical* has no technical connotation as it is used just because κ is the most commonly used signature.

Let σ be an implicit signature. Since implicit operations have natural interpretations as operations on profinite semigroups, every profinite semigroup is thus naturally endowed with a structure for the enlarged algebraic signature σ. We call a semigroup with interpretations of all other operations from σ a *σ-semigroup.* The free σ-semigroup on a set A in the variety generated by $\mathbf{V}$ is denoted $\Omega_A^\sigma \mathbf{V}$ and it is easy to show that it may be constructed as the σ-subsemigroup of $\overline{\Omega}_A \mathbf{V}$ generated by the image of the generating mapping $\iota : A \to \overline{\Omega}_A \mathbf{V}$. In other words, $\Omega_A^\sigma \mathbf{V}$ is the set of all A-ary implicit operations which can be obtained from the component projections by composing with the operations from σ. We say that $\mathbf{V}$ is *σ-recursive* if the word problem for $\Omega_A^\sigma \mathbf{V}$ is algorithmically solvable, i.e., if there is an algorithm to test when two such composites define the same implicit operation on $\mathbf{V}$.

We consider here a *graph* to be a set $\Gamma = V \mathbin{\mathring{\cup}} E$ consisting of two types of elements, respectively *vertices* and *edges,* endowed with two mappings $\alpha, \omega : E \to V$ giving respectively the begining and the ending of a vertex; pictorially this is described by $\alpha e \xrightarrow{e} \omega e$. A *labeling* of Γ is a function $\gamma : \Gamma \to S^1$ such that $\gamma E \subseteq S$. We say that the labeling γ is *consistent* if, for all $e \in E$, $(\gamma \alpha e)(\gamma e) = \gamma \omega e$.

A *relational morphism* is a relation $\mu : S \to T$ between two semigroups S and T with domain S which is a subsemigroup of $S \times T$. Note that a homomorphism and the inverse of an onto homomorphism are relational morphisms and that the composite of relational morphisms is again a relational morphism. A labeling $\gamma : \Gamma \to S^1$ is said to be *μ-inevitable* if there is a consistent labeling $\delta : \Gamma \to T^1$ which is μ-related with γ in the sense that $(\gamma z, \delta z) \in \mu \cup \{(1,1)\}$ for every $e \in E$. A labeling $\gamma : \Gamma \to S^1$ of a finite graph by a finite semigroup is said to be $\mathbf{V}$*-inevitable* if it is μ-inevitable for every relational morphism $\mu : S \to T$ into $T \in \mathbf{V}$.

For an A-generated finite semigroup S, the composite

$$\begin{tikzcd} & \Omega_A^\sigma \mathbf{S} \arrow[dl, "\varphi"'] \arrow[dr, "\psi"] & \\ S \arrow[rr, dashed, "\mu_{\mathbf{V}}^\sigma"'] & & \Omega_A^\sigma \mathbf{V} \end{tikzcd}$$

is called the *natural σ-relational morphism* (associated with the choice of generators), where φ is the homomorphism determined by the choice of generators and ψ is defined by restriction. In other words, $\mu_{\mathbf{V}}^\sigma$ is the σ-subsemigroup of $S \times \Omega_A^\sigma \mathbf{V}$ generated by all pairs $(\varphi a, \psi a)$ with $a \in A$. It is easy to check that inevitability of a labeling of a graph by a finite semigroup S with respect to a natural σ-relational morphism does not depend on the choice of generators of S.

The pseudovariety $\mathbf{V}$ is said to be *σ-reducible* if every $\mathbf{V}$-inevitable labeling of a finite graph by a finite semigroup is $\mu_{\mathbf{V}}^\sigma$-inevitable. A compactness argument shows that every pseudovariety is reducible with respect to the signature consisting of all implicit operations. Also say that $\mathbf{V}$ is *weakly σ-reducible* if every $\mathbf{V}$-inevitable labeling of a finite graph by a finite A-generated semigroup S is $\overline{\mu_{\mathbf{V}}^\sigma}$-inevitable with respect to the closure of the natural σ-relational morphism in the product topology of $S \times \Omega_A^\sigma \mathbf{V}$.

A pseudovariety is said to be *σ-tame* if it is recursively enumerable, σ-recursive, and σ-reducible. For this notion to be useful, we require some further computability assumptions on the signature σ. We say that σ is *highly computable* if it is recursively enumerable and consists of computable implicit operations. This is certainly the case of the canonical signature κ. Finally, we say that a pseudovariety is *tame* if it is σ-tame with respect to some highly computable implicit signature σ. This is the notion used in Theorem 1. The proof of that result is based on an earlier syntactic characterization of the semidirect product $\mathbf{V} * \mathbf{W}$ due to Weil and the author [5] which led first to the notion of *hyperdecidability* [3], which failed to deal with semidirect products with more than two factors, and later to tameness. A pseudovariety $\mathbf{V}$ is said to be *hyperdecidable* if it is decidable whether a given labeling of a finite graph by a finite semigroup is $\mathbf{V}$-inevitable. The notion of tameness is basically a refinement of hyperdecidability in a sense giving, in general, a very inefficient but uniform type of algorithm to solve the same decision problem. In particular, tame pseudovarieties are hyperdecidable. However, in handling specific examples, it appears that proving the stronger property is actually easier as one simply ignores the difficulties of finding efficient algorithms. The algorithms coming from proving tameness are very inefficient because they involve in particular examining, up to isomorphism, all semigroups and all graphs up to a given size and there are simply too many of those.

3. Tame pseudovarieties

We start by giving some examples of tame pseudovarieties. The first is a reformulation of a celebrated result due to Ash [13] which proved, in particular, the Rhodes *type II* conjecture and the Henckell and Rhodes *pointlike* conjecture. See [18] for the immediate significance of this result for the theory of finite semigroups and for a history and applications of those two conjectures.

Theorem 2. *The pseudovariety* $\mathbf{G}$ *is* κ*-tame.*

Since $\Omega_A^\kappa \mathbf{G}$ is the free group on A, κ-recursiveness of $\mathbf{G}$ amounts to the elementary solution of the word problem in free groups. In the case of the pseudovariety $\mathbf{J}$ of all finite semigroups in which every principal ideal admits a unique generator, the word problem for $\Omega_A^\kappa \mathbf{J}$ was solved earlier by the author [1] who also showed that $\overline{\Omega}_A \mathbf{J} = \Omega_A^\kappa \mathbf{J}$. Hyperdecidability of $\mathbf{J}$ was proved by Zeitoun and the author [11] and depends heavily on structural knowledge about $\overline{\Omega}_A \mathbf{J}$. This is a concrete example for which proving the stronger notion of κ-tameness (with algorithms which are outrageously inefficient) is much easier. In fact, it becomes a simple exercise to prove the following theorem taking into account [2, Theorem 8.1.11].

Theorem 3. *The pseudovariety* $\mathbf{J}$ *is* κ*-tame.*

A semigroup S is said to be an *orthogroup* if it is the union of its subgroups and the product of two idempotents is again an idempotent. Finite orthogroups form a pseudovariety denoted $\mathbf{OCR}$. The κ-semigroup $\Omega_A^\kappa \mathbf{OCR}$ is the free orthogroup on the set A. The solution of the word problem for this free algebra was obtained by Gerhard and Petrich [16]. Tameness of $\mathbf{OCR}$ was proved by Trotter and the author [10]. More generally, for a pseudovariety $\mathbf{H}$ of groups, let $\overline{\mathbf{H}}$ be the pseudovariety of all finite semigroups whose subgroups lie in $\mathbf{H}$.

Theorem 4. *If the pseudovariety of groups* $\mathbf{H}$ *is* κ*-tame then so is* $\mathbf{OCR} \cap \overline{\mathbf{H}}$.

Removing the assumption that the product of idempotents is again idempotent one obtains the pseudovariety $\mathbf{CR}$ of so-called *completely regular semigroups*. While the solution of the word problem for the free completely regular semigroup $\Omega_A^\kappa \mathbf{CR}$ is also available in this case, due to Kadourek and Polák [20], the proof of tameness appears to be much harder. In ongoing work, Trotter and the author have shown that a conjectured stronger but closely related property than κ-tameness for $\mathbf{G}$ would imply that $\mathbf{CR}$ is κ-tame.

Another important case is that of the pseudovariety $\mathbf{A}$. A solution of the word problem for the free aperiodic κ-semigroup $\Omega_A^\kappa \mathbf{A}$ has been announced independently by McCammond [25] and Zhilt'sov [33] and depends on the solution of the word problem for free Burnside semigroups, respectively in the version of McCammond [24] and Guba [17]. Tameness of $\mathbf{A}$ has been announced by Rhodes [28].

Theorem 5. *The pseudovariety* $\mathbf{A}$ *is* κ*-tame.*

In view of Theorem 1, from Theorems 2 and 5 we deduce the solution of the Krohn-Rhodes complexity problem.

Corollary 6. *The Krohn-Rhodes complexity function may be effectively computed.*

The pseudovariety $\mathbf{D}$ consists of all finite semigroups in which idempotents are right zeros, also known as *definite semigroups*. Elements of $\overline{\Omega}_A\mathbf{D}$ may be identified with finite and infinite words (to the left) on the alphabet A. The word problem for the relatively free κ-semigroup $\Omega_A^\kappa\mathbf{D}$, consisting of finite and ultimately periodic infinite words, is therefore quite easy to solve. A proof of tameness of $\mathbf{D}$ was obtained by Zeitoun and the author [12].

Theorem 7. *The pseudovariety* $\mathbf{D}$ *is* κ*-tame.*

Denote by $\mathbf{G}_p$ the pseudovariety of all finite p-groups. Since the free group is residually a finite p-group [14], the relatively free κ-semigroup $\Omega_A^\kappa\mathbf{G}_p$ is an absolutely free group. Steinberg and the author [8] had shown that this implies that $\mathbf{G}_p$ cannot be κ-tame. On the other hand, extending Ash's arguments, Steinberg [32] has shown that $\mathbf{G}_p$ is weakly κ-reducible. More generally, after Ribes and Zalesskiĭ [30], as in [15, 32], call a pseudovariety $\mathbf{H}$ of groups an $\mathcal{RZ}$*-pseudovariety* (respectively an $\overline{\mathcal{RZ}}$*-pseudovariety*) if the product of finitely generated subgroups of $\Omega_A^\kappa\mathbf{H}$ is closed (respectively, if the product of closed finitely generated subgroups of $\Omega_A^\kappa\mathbf{H}$ is closed and the closure of a finitely generated subgroup of $\Omega_A^\kappa\mathbf{H}$ is itself finitely generated). Here the topology is the induced topology from that of $\overline{\Omega}_A\mathbf{H}$ and is called the *pro-*$\mathbf{H}$ *topology*. Call $\mathbf{H}$ *arborescent* if $(\mathbf{H}\cap\mathbf{Ab})*\mathbf{H} \subseteq \mathbf{H}$ where $\mathbf{Ab}$ denotes the pseudovariety of all finite Abelian groups and where the name arborescent comes from the fact that this property characterizes those pseudovarieties of groups whose profinite Cayley graphs are profinite trees (see [4]). Steinberg [32] proved more generally the following result.

Theorem 8. *Every arborescent* $\overline{\mathcal{RZ}}$*-pseudovariety (in particular, every extension closed pseudovariety) of groups is weakly* κ*-reducible.*

Relying on this result as well on the methods developed by Ribes and Zalesskiĭ [31] and further explored by Margolis, Sapir and Weil [23], the author has shown that $\mathbf{G}_p$ is tame as a consequence of the following more general result.

Theorem 9. *Every recursively enumerable extension closed pseudovariety of groups* $\mathbf{H}$ *for which there is an algorithm to test whether a finitely generated subgroup of a free group is dense with respect to the pro-*$\mathbf{H}$ *topology is tame.*

It remains an open problem whether say the pseudovariety $\mathbf{G}_{\mathrm{sol}}$ of all finite solvable groups possesses the algorithmic property of Theorem 9. On the other hand, removing the assumption that the pseudovariety of groups is extension closed seems to render the problem harder. It remains for instance an open problem whether $\mathbf{Ab}$ and the pseudovariety $\mathbf{G}_{\mathrm{nil}}$ of all finite nilpotent groups are tame although the algorithmic property of Theorem 9 (understood with respect to the relatively free group) holds for both of them (see [15, 23]).

Note that, by a simple counting argument, most pseudovarieties are not tame (in fact, not even decidable). On the other hand every pseudovariety for which relatively free profinite semigroups over finite sets are finite and computable are easily shown to be tame and so, in particular, **Sl** is tame. Hence, in view of Theorem 1, the Rhodes pseudovariety $[\![\Sigma]\!]$ mentioned in Section 1 is decidable but not tame (see also [29]). However, it seems reasonable to expect that "natural" pseudovarieties should be tame and this is what Rhodes would call the *natural conjecture* [27].

Delgado and the author [7] have observed that, in the case of an $\mathcal{RZ}$-pseudovariety of groups $\mathbf{H}$, denoting $F = \Omega_A^\kappa \mathbf{H}$, κ-tameness is equivalent to the following property: for every system of equations of the form

$$x_{\alpha e} x_e = x_{\omega e} \quad (e \in E) \tag{1}$$

which has no solution in F satisfying constraints of the form

$$x_z \in g_z H_{1z} \dots H_{n_z z} \quad (z \in \Gamma) \tag{2}$$

with the $g_z \in F$ and the H_{iz} closed finitely generated subgroups, the H_{iz} may be replaced by subgroups K_{iz} of finite index containing them such that the system (1) remains without solution for the weaker constraints. This property provides a surprising bridge between this area and Model Theory which we proceed to present.

A class $\mathcal{C}$ of relational structures satisfies the *finite extension property for partial automorphisms* (FEPPA) if, for every finite $S \in \mathcal{C}$ and every set $\mathcal{P}$ of partial automorphisms of S, if there is some structure $T \in \mathcal{C}$ of which S is a substructure such that every element of $\mathcal{P}$ extends to a full automorphism of T, then there exists such a structure $T \in \mathcal{C}$ which is finite. A *homomorphic extension* of a structure S is a structure T for which there is a mapping $\varphi : S \to T$ which respects all the relations in the language; we then write $T \leq_h S$. The *exclusion class* of $\mathcal{C}$ is the class

$$\mathrm{Excl}(\mathcal{C}) = \{S : (\neg \exists T \in \mathcal{C})\ T \leq_h S\}.$$

Herwig and Lascar [19] proved the following results by establishing the first by model theoretic methods and showing the formal equivalence with the second.

Theorem 10. *If $\mathcal{C}$ is a finite set of finite structures of a finite relational language, then the class Excl($\mathcal{C}$) has the FEPPA.*

For a subgroup H of a group G and elements $g_1, g_2 \in G$, write $g_1 \equiv_H g_2$ if the left cosets $g_1 H$ and $g_2 H$ are equal.

Theorem 11. *Consider a system of* equations *of the forms*

$$X \equiv_H Y g \qquad \textit{and} \qquad X \equiv_H g,$$

where the H are finitely generated subgroups of $F = \Omega_A^\kappa \mathbf{G}$ and the $g \in F$; if the system has no solution in F in the variables $X, Y, \dots$ then one may replace each subgroup H by a subgroup of F of finite index containing H such that the system remains without solution in F.

In turn, Delgado and the author [6, 7] proved the formal equivalence of these results with tameness of $\mathbf{G}$ by showing that Theorem 11 may be reformulated as stating that, for every system of equations of the form (1) which has no solution in F satisfying constraints of the form

$$x_z \in g_z H_z \quad (z \in \Gamma) \tag{3}$$

with the $g_z \in F$ and the H_z finitely generated subgroups, the H_z may be replaced by subgroups K_z of finite index containing them such that the system (1) remains without solution for the weaker constraints. Moreover, the equivalence between these two results about systems of equations over a free group hold relative to any pseudovariety $\mathbf{H}$ of groups, replacing the absolutely free group by the corresponding relatively free group.

The reduction of the general constraints of the form (2) to the special form (3) also provides a proof that an $\mathcal{RZ}$-pseudovariety of groups which is *κ-tame for labelings of finite graphs by finite inverse semigroups* is in fact κ-tame [7]. The original proof by Ash [13, Sections 8–10] for the case of $\mathbf{G}$, although not explicitly using the fact that $\mathbf{G}$ is an $\mathcal{RZ}$-pseudovariety, was considerably longer.

It would be of interest to find model-theoretic formulations of tameness for other pseudovarieties of groups and to further explore the connection between the two topics. Also of interest would be to determine for which systems of equations instead of (1) the above results hold.

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