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Pressure distribution under a two-dimensional sandpile

二次元空間内の砂山底面での圧力分布に関する数値計算

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ABSTRACT: In the experiments about a pressure distribution under a sandpile, an interesting phenomenon was previously found. Just under the apex of a pile, a vertical pressure minimum, called a dip, may be seen. Whether a dip appears or not depends on the way grains are poured. Dip is reproduced in our numerical experiment with a distinct element method (DEM) that is a kind of molecular dynamics simulation. We choose two different methods of supplying grains and compare the results (e.g., the distributions of the resulting contact force direction, Coulomb angle distributions) as well as the pressures in order to recognize the difference between the piles with two different histories. We made sure that the Coulomb angle distribution is thought to be connected with a dip.

1 Introduction

Concerning granular materials, there is no clear boundary between the states (gas, liquid, and solid). Energy is always dissipated by the collisions of grains. By the energy dissipation, clusters are formed locally, and a homogeneous state is hardly kept dynamically. Energy supply is

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perpetually needed to keep a steady state. Once the supply is halted, kinetic energy is completely lost, and a static state is obtained.

In a dynamical state, the interactions between grains and the media around them, liquid or gas, are too complex to analyze directly. In this paper, we focus on a static state, and reproduce a dip and make it clear how the stresses propagate through granular media with DEM, a kind of molecular dynamics simulation.

2 Stress propagation inside a pile

Among various static states of granular matter, the simplest setting is a pile constructed by just pouring grains on a floor. You can see a sandpile at a beach, or in a desert. Such a pile exhibits a certain characteristic angle called an angle of repose. It depends on a boundary condition, humidity, a particle shape, etc. The mechanism for the angle of repose still remains unknown. In contrast, in the case of liquid drop on a floor, it takes a shape like a dome due to surface tension. On the other hand, the overall shape of a lot of water just extend flatly without making a pile.

There are some previous research (e.g., [3],[10],[17],[19]) on reporting an interesting phenomenon called a "dip" under piles made of such granular materials, as rape seeds, sands, frosted grass beads, lead shots, etc. For instance, Vanel *et al.*[19] poured sands from a point source and raised the source gradually as a pile grew up. The pressure distribution along the bottom non-intuitively showed a local minimum just under the apex of a pile. Their experiment also showed the dependency of a dip on the history of how sands were poured. By constructing piles with a point source but at a fixed height, the dip became deeper than the one poured from the rising point source. In contrast, when grains were supplied homogeneously like a rain using a sieve, a dip never appeared despite the similarity in the overall shape of the piles created in both methods.

Many hypothesis (e.g., [2],[5],[20]) and models (e.g., [12],[13],[15]) have been proposed in attempts to explain and reproduce a dip. In this paper, we numerically compare the two types of grain sources in order to investigate the history dependence.

3 Distinct Element Method

DEM is a method commonly used, for instance, in manufacturing engineering. Since the paper by Cundall and Struck [4], it has also been used from the scientific point of view in simulating granular dynamics. DEM is to trace the movements of each grain like the description of Lagrange. The main idea of the method is to solve many-body problems by considering two-body interactions at the moment of contact, based on Newton 's equation of motion ($\vec{F} = m\vec{a}$).

Although DEM has problems left in details of modeling contact forces, the method is very intuitive, and constitutive equations commonly used in a continuous description are never needed. A continuous model would provide a platform easily treated in a theoretical analysis. However, the effect of the grain size is lost by coarse-grained averaging. Such micro-mechanical effects are supposed to have an important role in the macroscopic granular behavior. DEM can take these effects into account naturally. Assumptions have to be made on modeling contact forces and tuning parameters, but many kinds of phenomena (e.g., plugging in a flow[16], dilatancy[9], flows in a chut[11], convection and fluidization in a vibrated bed[18]) have been reproduced by DEM. We can use it as a very powerful tool to understand the mechanism of behavior of granular materials. There is constantly an effort to construct a model with simple contact forces but applicable in wide situations.

As mentioned in Sect.2, in the case of the simplest setting, a pile on a floor, we carry out a numerical calculation with DEM. Two-dimensional discs are piled up on a floor, and the pressure at the bottom is measured. Using DEM with polygonal grains and a smooth bottom, the experimental results have been already reproduced.[13] In this article, we show a dip also even with perfectly round particles (discs) and a rough bottom, decreasing the number of the parameters in the model.

3.1 Model

Contact forces are calculated at each contact point, and a resulting force acting on each grain is the sum of the contact force vectors for that grain. This enables us to solve the equations of motion time by time. In our model, the following three kinds of contact forces are taken into account, in addition to the gravitational force. [8]

1. elastic force, proportional to the relative displacement between grains
2. viscous force, proportional to the relative velocity between grains
3. rolling friction, proportional to each contact area and the relative rotational velocity

For the tangential components, contributions from translational as well as rotational motions are included. The total contact force is the sum of the three. The tangential component is cut off at a threshold known as the Coulomb friction as follows;

$$F_t \leq \mu F_n \tag{1}$$

where F_t and F_n denote tangential and normal components of the total force, and μ is the coefficient of Coulomb friction.

Introduction of the rolling friction follows the modification of DEM by Iwashita *et al.*[9] as a possible cause for propagation of rotational moments. Granular materials are deformed by the translational movement and the rotation of grains. Before, it had been regarded that the slip was the main effect of the deformation, and the rotational movement had not been considered. But Oda *et al.*[14] showed the importance of the rotations in the development of dilatancy in their experiment. Moreover, Bardet *et al.*[1] showed by their numerical investigation that the rotation of grains concentrated near the shear bands, and there was a high gradient of particle rotation along the shear band boundaries Therefore, rotational movement of grains is here regarded important.

In this simulation, it can be expected that the shear forces act more effectively to convey energy of the surface flow into a pile by introducing the rotational friction. Both in the dynamics and statics of granular materials, microscopic friction between grains in contact should in no doubt have great effect. Since details are still unknown, however, the static friction is not introduced explicitly in our model.

3.2 Calculation

We follow the algorithm of A.Shimosaka [21] and H.Hayakawa [7]. The governing equations are non-dimensionalized in order to decrease the number of parameters in the equations. The characteristic length and time scales are d and $\sqrt{d/g}$, respectively, so that the maximum diameter of the grain as well as the gravitational acceleration are both taken to be unity. We adopt the Euler scheme with second order accuracy for time updating of positions, velocities, and angular velocities of grains with the time step $\delta t = 1.0 \times 10^{-4}$ in the dimensionless unit. Actual values of the constants used in this calculation are taken from the note [7]. By this method, we solve the equations of motion of each grain with the sum of the contact forces and the gravitational force.

In this paper, two kinds of methods for supplying grains are carried

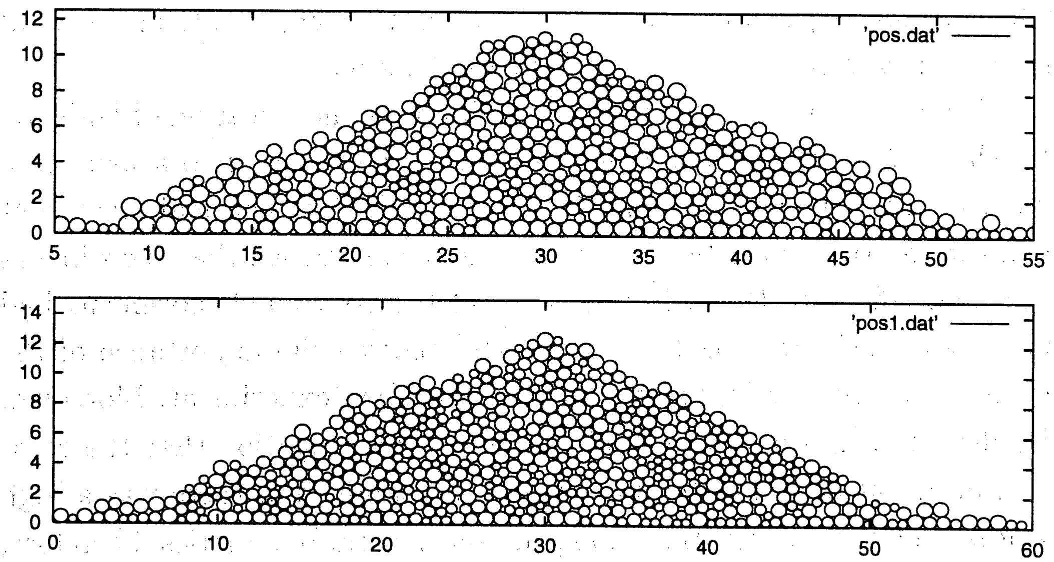

図 1: Configurations of the grains. (Upper: Method 1, with 420 grains. Lower: Method 2, with 529 grains).

out in order to make a comparison with the experiment by Vanel *et al.*[19]. For the "point source", we deposit a pack of grains (6 grains, in this calculation) at a time interval (2.5 time units) in between, from the center of the computational domain, a little above the top of a pile (1 space unit higher). Then we gradually raise the source to keep the same distance from the top all the time. In the following, we call it Method 1. Although we had better carry out a numerical experiment with a fixed height source in order to get a clearer dip, we have to do with a raising source because of the instability caused by the excess energy of grains.

On the other hand, Method 2 uses a homogeneous rain source. Grains are poured layer by layer with the same time interval with Method 1. The width of a layer is gradually decreased as the pile grows. The layers are also released from the adjusted height like in Method 1.

3.3 Results

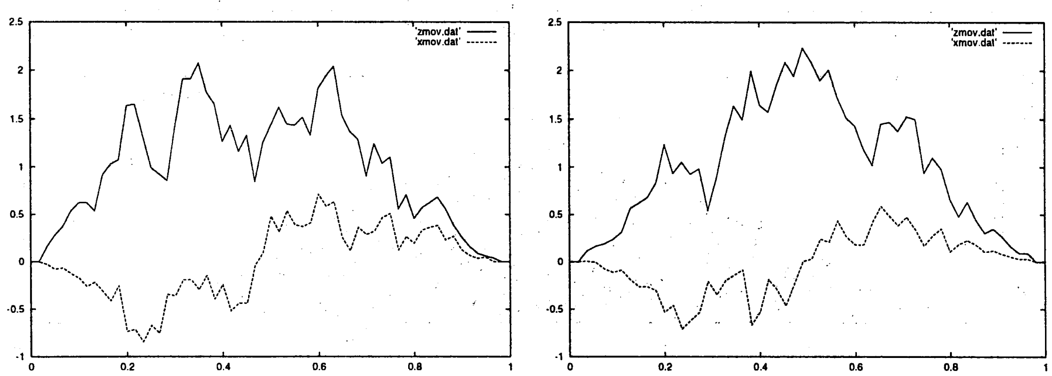

図 2: The pressure distributions along the bottom of the pile. (Left: Method 1, right: Method 2) The pressures are defined to be the amplitude of the force acting on the unit area (width) of the bottom. They are normalized by the total mass of the piles and the width of the bottoms. The solid curves show the downward component of the pressure, showing a local minimum (Left) or a maximum (Right) in the center. The dotted lines show the magnitude of the horizontal pressure component. All curves are smoothed with a moving average of about 5 units.

Figure 1 shows static states obtained from Method 1 and 2 after allowing sufficient relaxation time for avalanches. Each pile has developed nearly straight surfaces inclined at a certain angle, an angle of repose. Both angles of repose are about 25 degrees. The outlines of the piles look almost the same.

In contrast to the similarity in the shapes of the piles, Fig. 2 shows the difference between the pressure distributions by two Methods. A dip can be reproduced by our model with Method 1. In comparison our calculation without a rolling friction, a dip was not obtained. It can be considered that the energy dissipation by rotation has a significant effect on the way of stress propagation through the granular media.

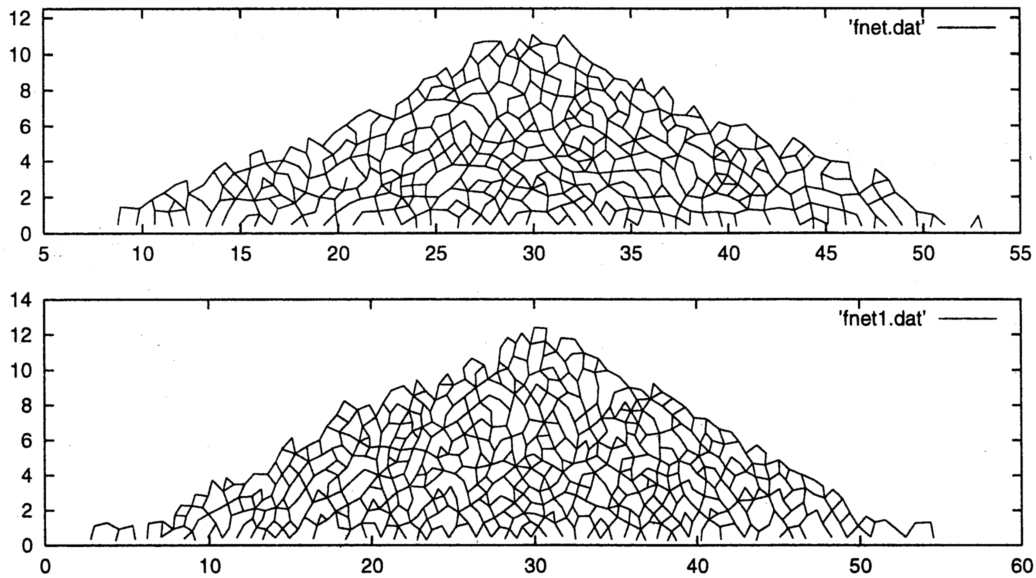

図 3: The centers of the grains are shown connected. (Upper: Method 1, lower: Method 2)

In the following, the internal information that is hardly evaluated in experiments are measured in order to recognize the other differences between these piles. In Fig. 3, the center of the grains in contact are joined. With Method 1, there are more large voids near the surface of a pile than with Method 2. The voids represent formation of arches, poised structures. The non-uniform packing can be seen. Each edge of the force network is inclined, and defines its contact direction.

The contact directions appear to have a favorite angle. The forces will transmit along the edges without the tangential components. The directions along which forces mainly transmit are shown in Fig. 4. The distributions of the resulting contact force directions are plotted. In order to make the effects of strong vectors larger, the each probability of the resulting contact force is weighted by the strength of the force. There are peaks near 40 degrees and 140 degrees. They are symmetric, and a difference between Method 1 and 2 can not be seen clearly.

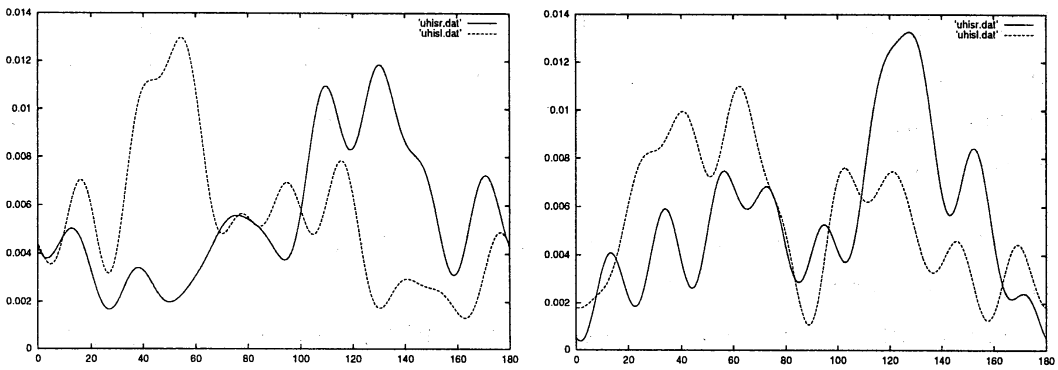

図 4: The distributions of the resulting contact force directions weighted by the strength of forces. (Left: Method 1, right: Method 2) The x axis is a degree against the horizontal line, the y axis is the probability. The solid line shows the direction of the contacts in a right half of a pile. The dotted line corresponds to the left half. They are normalized so that the total probability is 1.

Coulomb angle distributions are measured as shown in Fig. 5, in order to check the extent of distortion of the stresses generated by tangential components. Coulomb angle θ is defined to be the angle the resulting vector of contact forces makes with each contact direction as follows.

$$\theta = \arctan(F_t/F_n) \tag{2}$$

The vector should be inside the Coulomb cone as follows obtained by Eqs.1 and 2.

$$|\theta| \leq \arctan(\mu) \tag{3}$$

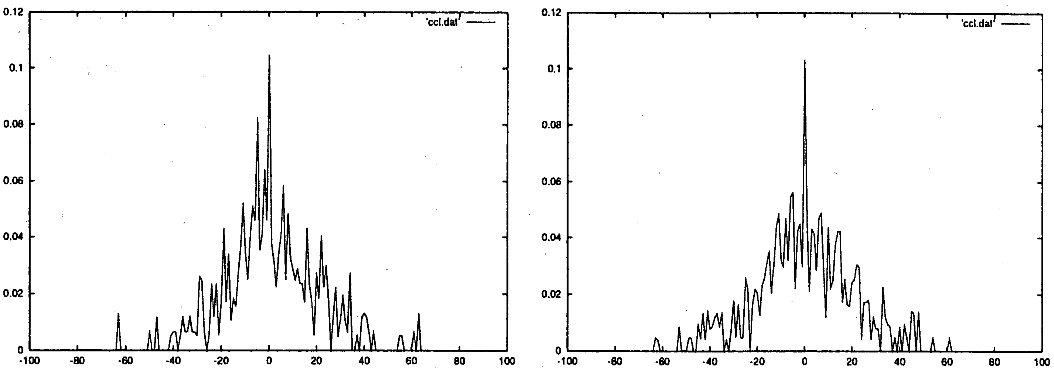

図 5: The horizontal axis denotes the degree of a Coulomb angle, and the vertical axis is the probability. (Left: Method 1, $\bar{\theta} = 1.91$. Right: Method 2, $\bar{\theta} = -0.32$) Only left side of a pile is considered due to symmetry.

Thus, $\theta = 0$ corresponds to contact forces being transmitted along the contact direction.

The Coulomb angles are dispersed around the contact direction inside a Coulomb cone by the effect of each tangential component. The means of the distributions are a little shifted from zero. The shift of Method 1 is larger than the one of Method 2 as shown in the caption of Fig. 5. The positive value means that, in the left side of a pile, the resulting vector is distorted from the contact direction to the outside of a pile. Coulomb angle distribution implies us how the stress propagation is deviated from the contact directions. The local information about the stress distortion is obtained by the Coulomb angle distributions.

The tendency of the Coulomb angle distribution has previously been investigated in an idealized situation by Eloy *et al.*[6]. The constructed a regular lattice piling with the assumption of a bias of Coulomb angle distributions so that the forces were re-directed toward the surface of a pile. They showed the dependency of a pressure distribution on the parameter of the bias as well as reproducing a dip. The averages of the Coulomb angle of the pile Method 1 has a similar bias with the data by Eloy *et al.*[6]. In our simulation, these results coincide with theirs. The bias was obtained without controlling parameters as well as an angle of repose. Therefore, the bias is thought to be important for a dip.

4 Open problems

Although a dip can be reproduced in our model, the modeling of contact forces has mainly two problems left. The first is that although the rolling friction is very intuitive, it has so far no physical evidence. If we cut off the rolling friction, and make the tangential viscous coefficient larger, then it is possible that a dip will also appear. For the appearance of a dip, we need a large shear stress at each contact, and it can be caused not only by the rolling friction. In order to simplification of the model, tangential components of contact forces should be united as much as possible.

As mentioned, another problem is that static friction is not included yet explicitly in the model. But, in contrast with experiments, the behavior of grains and the pressure distributions are well reproduced by our model, and the simpler model should be intended in order to analyze theoretically.

On the other hand, the average pressure distributions of many trial does not show so clear dip. Because the avalanches occurs left and right by turns, and the mass center of a pile is fluctuated in horizontal direction. We have to make sure with a larger pile so that the fluctuation will be small enough against the width of a dip. In addition, there is a dependency of the pressure on the boundary condition. We have to compare the boundary conditions as well as the ways to pour grains.

We intend to describe a stress propagation in a more general style macroscopically treating with the discreteness in space, for example by extending the boundary condition to a vertical container, silo, based on these results.

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