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ON EXISTENCE OF SCATTERING SOLUTIONS FOR DISSIPATIVE SYSTEMS

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In this report we shall give two frameworks (Theorem 1 and 3) for the existence of scattering solutions of dissipative systems and apply these to some dissipative wave equations.

Let $\mathcal{H}$ be a separable Hilbert space with inner product $\langle\cdot,\cdot\rangle_{\mathcal{H}}$. This norm is denoted by $\|\cdot\|_{\mathcal{H}}$. Let $\{V(t)\}_{t\geqq 0}$ and $\{U_0(t)\}_{t\in\mathbf{R}}$ be a contraction semi- group and a unitary group in $\mathcal{H}$, respectively. We denote these generators by A and A_0 ($V(t)=e^{-itA}$ and $U_0(t)=e^{-itA_0}$). We make the following assumptions on A and A_0.

(A1) $\sigma(A_0)=\sigma_{ac}(A_0)=\mathbf{R}$ or $[0,\infty)$.

(A2) $(A-i)^{-1}-(A_0-i)^{-1}$ defined as a form is extended to a compact operator K in $\mathcal{H}$.

(A3) There exist non-zero projection operators P_+ and P_- such that $P_++P_-=I_d$ and

$$\|KU_0(t)\psi(A_0)P_+\|\in L^1(\mathbf{R}_+), \tag{A3.1}$$

$$\|K^*U_0(t)\psi(A_0)P_+\|\in L^1(\mathbf{R}_+), \tag{A3.2}$$

$$\|K^*U_0(-t)\psi(A_0)P_-\|\in L^1(\mathbf{R}_+), \tag{A3.3}$$

$$\mathrm{w}-\lim_{t\to+\infty}U_0(-t)\psi(A_0)P_-f_t=0, \tag{A3.4}$$

for each $\psi\in C_0^\infty(\mathbf{R}\backslash 0)$ and $\{f_t\}_{t\in\mathbf{R}}$ satisfying $\sup_{t\in\mathbf{R}}\|f_t\|_{\mathcal{H}}<\infty$, where $\|\cdot\|$ is the operator norm of bounded operator from $\mathcal{H}$ to $\mathcal{H}$.

Let $\mathcal{H}_b$ be the space generated by the eignvectors of A with real eigenvalues.

Theorem 1. *Assume that (A1) $\sim$ (A3). For any $f\in\mathcal{H}_b^\perp$, the wave opeartor*

$$Wf=\lim_{t\to\infty}U_0(-t)V(t)f$$

exists. Moreover W is not zero as an operator in $\mathcal{H}$.

To prove Theorem 1 we shall use the following facts (see [17] and [14]):

(F1) $\{(A-i)^{-2}Af\in\mathcal{H}:f\in D(A)\cap\mathcal{H}_b^\perp\}$ is dense in $\mathcal{H}_b^\perp$.

(F2) There exists a sequence $\{t_n\}$ such that

$$\lim_{n\to\infty}t_n=\infty$$

and

$$\mathrm{w}-\lim_{n\to\infty} V(t_n)f = 0, \quad \text{for any} \quad f \in \mathcal{H}_b^{\perp}.$$

Theorem 1 implies that there exists scattering states of $\frac{dV(t)f}{dt} = -iAV(t)f, f \in D(A)$ as follows:

Corollary 2. *Assume that (A1) $\sim$ (A3). Then there exist non-trivial initial data $f \in \mathcal{H}$ and $f_+ \in \mathcal{H}$ such that for any $k = 0, 1, 2, \cdots$, and $\zeta_0 \in \mathbf{C}$ satisfying $\Re\zeta_0 > 0$*

$$\lim_{t\to\infty} \|V(t)(A-\zeta_0)^{-k} f - U_0(t)(A_0-\zeta_0)^{-k} f_+\|_{\mathcal{H}} = 0.$$

Theorem 1 is proven by using Enss's approach [3] and [17]. Examples of Theorem 1 contain scattering problem for elastic wave equation with dissipative boundarly condition in a half space of $\mathbf{R}^3$ (cf. [2]). To show (A3) we use the Mellin transformation (cf.[13]). Theorem 1 is not applied to acoustic wave equations with dissipative terms in stratifed media(cf. [19]). Since generalized eigenfunctions of acoustic wave propagation in stratifed media are not smooth at thresholds, the key estimates (A3.1)$\sim$(A3.3) have not been obtained in the neighborhood of each threshold. So we consider the following assumptions to deal with such equations .

Let B_0 be non-negative operator.

(A4) B_0 is A_0-compact.

(A5) Let ζ belong to $\mathbf{C}\backslash\mathbf{R}$. $\sqrt{B_0}(A_0-\zeta)^{-1}\sqrt{B_0}$ can be extended to a bounded operator $Q(\zeta)$ which satisfies that for any $\beta > \alpha > 0$, there exist positive constants $C_{\alpha,\beta}$ and η such that

$$\sup_{\alpha \leqq |Re\zeta| \leqq \beta, 0<|Im\zeta|<\eta} \|Q(\zeta)\| \leqq C_{\alpha,\beta}.$$

We reset $A = A_0 - iB_0$, $D(A) = D(A_0)$. Then [15] (see Theorem X-50) implies that A generates a contraction semi-group ,$\{V(t)\}_{t\geqq 0}(V(t) = e^{-itA})$.

We have the following theorem.

Theorem 3. *Assume that (A1), (A4) and (A5). Then*

(1) *A has no real eigenvalues.*

(2) *The wave operator*

$$W = s - \lim_{t\to\infty} U_0(-t)V(t)$$

exists. Moreover W is not zero as an operator in $\mathcal{H}$.

Corollary 4. *Assume that (A1), (A4) and (A5). Then we have the same conclusion of Corollary 2.*

To prove Theorem 3 we shall used Mochizuki's idea [12] due to Kato's smooth perturbation theory[8].

In §4 we shall apply our frameworks to elastic wave equation with dissipative boundary condition in a half space of $\mathbf{R}^3$ and acoustic wave equation with dissipative term in stratifed media. It seems that there is little literature concerning such dissipative systems (cf. [7]).

2. Proof of Theorem 1 and Corollary 2.

In this section we deal the case $\sigma(A_0) = \sigma_{ac}(A_0) = \mathbf{R}$ only. The another case can be dealt in the same way. We set $F(\lambda) = (\lambda - i)^{-2}\lambda$ and $W(t) = U_0(-t)V(t)$. In this section C is used as positive constants.

Below we shall give the proof of Theorem 1. First we prove the existence of W by refering to [3], [17], [10], [13], [4], [18] and [14]. But we sometimes omit to note the above references.

proof of the existence of W. For any $f \in \mathcal{H}_b^\perp \cap D(A)$ and $t, s > t_n$, note (F1) and

$$\begin{aligned}&\|(W(t) - W(s))F(A)^2 f\|_{\mathcal{H}} \\ \leqq &\|(W(t) - W(t_n))F(A)^2 f\|_{\mathcal{H}} + \|(W(s) - W(t_n))F(A)^2 f\|_{\mathcal{H}}.\end{aligned}$$

Thus in order to prove the existence of W, it is sufficent to show

$$\lim_{n\to\infty} \overline{\lim_{t\to\infty}} \|(W(t) - W(t_n))F(A)^2 f\|_{\mathcal{H}} = 0 \tag{2.1}$$

(cf. [4])

We estimate $\|(W(t) - W(t_n))F(A)^2 f\|_{\mathcal{H}}$ as follows (cf. [17]):

$$\begin{aligned}&\|(W(t) - W(t_n))F(A)^2 f\|_{\mathcal{H}} \\ =&\|U_0(-t)(V(t - t_n) - U_0(t - t_n))F(A)^2 V(t_n) f\|_{\mathcal{H}} \\ \leqq &\sum_{j=1}^{5} \|T_j\|_{\mathcal{H}},\end{aligned}$$

where

$$\begin{aligned}T_1 &= (V(t - t_n) - U_0(t - t_n))(F(A)^2 - F(A_0)^2)V(t_n)f, \\ T_2 &= (V(t - t_n) - U_0(t - t_n))(I_d - \psi_M(A_0))F(A_0)^2 V(t_n)f, \\ T_3 &= (V(t - t_n) - U_0(t - t_n))(\psi_M F)(A_0)P_+ F(A_0)V(t_n)f, \\ T_4 &= (V(t - t_n) - U_0(t - t_n))(\psi_M F)(A_0)P_- F(A_0)(I_d - \psi_M(A_0))V(t_n)f, \\ T_5 &= (V(t - t_n) - U_0(t - t_n))(\psi_M F)(A_0)P_-(\psi_M F)(A_0)V(t_n)f\end{aligned}$$

and $\psi_M(\lambda) \in C_0^\infty(\mathbf{R})$ satisfies $0 \leqq \psi_M(\lambda) \leqq 1, \psi_M(\lambda) = 0(|\lambda| < 1/2M, |\lambda| > 2M)$ and $\psi_M(\lambda) = 1(1/M < |\lambda| < M)$.

First, we note that for any ε, there exists $M > 0$ such that

$$\|T_j\|_{\mathcal{H}} \leqq C\|(1 - \psi_M)F\|_{L^\infty(\mathbf{R})} < \varepsilon \quad (j = 2, 4)$$

Therefore once the limits

$$\lim_{n\to\infty} \overline{\lim_{t\to\infty}} \|T_j\|_{\mathcal{H}} = 0, \quad (j = 1, 3, 5) \tag{2.2}$$

are proved, we obtain (2.1). Below we shall show (2.2). For $j = 1$ (A2) implies that $F(A)^2 - F(A_0)^2$ is a compact operator in $\mathcal{H}$. Using (F2) we have

$$\|T_1\|_{\mathcal{H}} \leqq C\|(F(A)^2 - F(A_0)^2)V(t_n)f\|_{\mathcal{H}} \to 0 \quad (n \to \infty)$$

For $j = 3$, we decompose T_3 as follows

$$T_3 = T_{31} + T_{32} + T_{33},$$

where

$$\begin{aligned} T_{31} &= V(t-t_n)(F(A_0) - F(A))(\psi_M F)(A_0)P_+F(A_0)V(t_n)f \\ T_{32} &= (F(A) - F(A_0))U_0(t-t_n)(\psi_M F)(A_0)P_+F(A_0)V(t_n)f \\ T_{33} &= F(A)(V(t-t_n) - U_0(t-t_n))\psi_M(A_0)P_+F(A_0)V(t_n)f \end{aligned}$$

Same argument as in the proof of T_1 implies

$$\lim_{n\to\infty} \overline{\lim_{t\to\infty}} \|T_{31}\|_{\mathcal{H}} = 0.$$

We have by (A1)

$$\mathrm{w} - \lim_{t\to\infty} U_0(t-t_n)f = 0.$$

Thus (A2) implies

$$\lim_{t\to\infty} \|T_{32}\|_{\mathcal{H}} = 0.$$

To estimate T_{33}, we use Cook-Kuroda method. We have by (A2)

$$\begin{aligned} &\langle T_{33}, g\rangle_{\mathcal{H}} \\ &= -i \int_0^{t-t_n} \langle V(t-t_n-s)A(A-i)^{-1}KU_0(s)\tilde{\psi}_M(A_0)P_+F(A_0)f_n, g\rangle_{\mathcal{H}} ds, \end{aligned}$$

where $g \in \mathcal{H}$, $f_n = V(t_n)f$ and $\tilde{\psi}_M(\lambda) = (\lambda - i)\psi_M(\lambda)$.

Therefore we obtain

$$\|T_{33}\|_{\mathcal{H}} \leqq C \int_0^\infty \|KU_0(s)\tilde{\psi}_M(A_0)P_+F(A_0)f_n\| ds.$$

For each $s \geqq 0$ we have by (F2) and (A2) ,

$$\lim_{n\to\infty} \|KU_0(s)\tilde{\psi}_M(A_0)P_+F(A_0)f_n\|_{\mathcal{H}} = 0.$$

Therefore (A3.1) and Lebesgue's theorem imply

$$\lim_{n\to\infty} \overline{\lim_{t\to\infty}} \|T_{33}\|_{\mathcal{H}} = 0.$$

Now we obtain

$$\lim_{n\to\infty} \overline{\lim_{t\to\infty}} \|T_3\|_{\mathcal{H}} = 0.$$

We estimate T_5 as follows :

$$\begin{aligned} \|T_5\|_{\mathcal{H}}^2 &\leqq C\|P_-(F\psi_M)(A_0)V(t_n)f\|_{\mathcal{H}}^2 \\ &= C\sum_{j=1}^{3} T_{5j}, \end{aligned}$$

where

$$T_{51} = \langle \psi_M(A_0)P_-h_n, (F(A_0) - F(A))V(t_n)f\rangle_{\mathcal{H}}$$
$$T_{52} = \langle \psi_M(A_0)P_-h_n, (V(t_n) - U_0(t_n))F(A)f\rangle_{\mathcal{H}}$$
$$T_{53} = \langle U_0(-t_n)\psi_M(A_0)P_-h_n, F(A)f\rangle_{\mathcal{H}}$$

and $h_n = (F\psi_M)(A_0)V(t_n)f$.

(A2) and (F2) imply

$$\lim_{n\to\infty} T_{51} = 0.$$

(A3.4) implies

$$\lim_{n\to\infty} T_{53} = 0.$$

To estimate T_{52}, again we use Cook-Kuroda method. Note that

$$|T_{52}| \leqq C\|f\|_{\mathcal{H}} \int_0^\infty \|K^* U_0(-s)\overline{\tilde{\psi}}_M(A_0)P_-h_n\|_{\mathcal{H}} ds.$$

Using (A2), (F2) and (A3.2) we have by Lebesgue's theorem

$$\lim_{n\to\infty} T_{52} = 0.$$

Now we obatin

$$\lim_{n\to\infty} \overline{\lim_{t\to\infty}} \|T_5\|_{\mathcal{H}} = 0.$$

Therefore the proof of the existence of W is completed. □

To show $W \not\equiv 0$, we introduce a subspace of $\mathcal{H}$, D, as follows :

$$D = \{f \in \mathcal{H} : \lim_{t\to\infty} V(t)f = 0\}.$$

Since

$$Af = \lambda f, \lambda \in \mathbf{R}, f \in \mathcal{H} \Longrightarrow A^* f = \lambda f$$

(see Lemma 1.1.5 of [14]), we can easily show

$$D \subset \mathcal{H}_b^{\perp}.$$

We prepare the following proposition without the proof.

Proposition 2.1. *Assume that*

$$\mathcal{H}_b^{\perp} \ominus D = \{0\}.$$

Then one has

$$w - \lim_{t\to\infty} U_0(-t)V(t)f = 0 \tag{2.3}$$

for any $f \in \mathcal{H}$.

Below we shall show $W \not\equiv 0$(cf. [12]§3).

proof of $W \not\equiv 0$. For any $f \in \mathcal{H}$ and $g \in \mathcal{H}$, note that

$$(2.6)\qquad \begin{aligned}&\langle U_0(-t)V(t)(A-i)^{-1}f,(A_0+i)^{-1}g\rangle_{\mathcal{H}}\\ &= \langle (A-i)^{-1}f,(A_0+i)^{-1}g\rangle_{\mathcal{H}} + i\int_0^t \langle V(\tau)f, K^*U_0(\tau)g,\rangle_{\mathcal{H}}d\tau.\end{aligned}$$

We assume that $W \equiv 0$, i.e, for any $f \in \mathcal{H}_b^{\perp}$,

$$(2.7)\qquad \|Wf\|_{\mathcal{H}} = \lim_{t\to\infty}\|V(t)f\|_{\mathcal{H}} = 0.$$

(2.7) means

$$\mathcal{H}_b^{\perp} \ominus D = \{0\}.$$

Hence Proposition 2.1 and (2.6) imply

$$\langle (A-i)^{-1}f,(A_0+i)^{-1}g\rangle_{\mathcal{H}} = -i\int_0^\infty \langle V(\tau)f, K^*U_0(\tau)g,\rangle_{\mathcal{H}}d\tau.$$

Putting

$$f = (A_0-i)U_0(s)\psi_M(A_0)P_+h \quad \text{and} \quad g = (A_0+i)U_0(s)\psi_M(A_0)P_+h$$

for any $h \in \mathcal{H}$, we have

$$\begin{aligned}\|\psi_M(A_0)P_+h\|_{\mathcal{H}}^2 \leqq &\|h\|_{\mathcal{H}}(\|((A-i)^{-1}-(A_0-i)^{-1})U_0(s)\tilde{\psi}_M(A_0)P_+h\|_{\mathcal{H}}\\ &+ C_M\int_0^\infty \|K^*U_0(\tau+s)\tilde{\psi}_M(A_0)P_+h\|_{\mathcal{H}}d\tau).\end{aligned}$$

(A1) and (A2) imply

$$\lim_{s\to\infty}\|((A-i)^{-1}-(A_0-i)^{-1})U_0(s)\tilde{\psi}_M(A_0)P_+h\|_{\mathcal{H}} = 0$$

and (A3.2) implies

$$\lim_{s\to\infty}\int_0^\infty \|K^*U_0(\tau+s)\tilde{\psi}_M(A_0)P_+\|d\tau = 0.$$

Therefore we have

$$(2.8)\qquad \|\psi_M(A_0)P_+h\|_{\mathcal{H}} = 0,$$

for any $h \in \mathcal{H}_0$ and any $M > 0$.

(2.8) means $P_+ \equiv 0$. This is a contradiction with (A3). Now we complete the proof of $W \not\equiv 0$. □

We give a brief sketch of the proof of Corollary 2.

proof of Corollary 2. Noting that $U_0(t)$ is unitary in $\mathcal{H}$ we have the case $k = 0$ by Theorem 1. It follows from the case $k = 0$ and (A1) that the case $k = 1$.

We can show the cases $k = 2, 3, 4, \cdots$ by the induction. □

3. Proof of Theorem 3 and Corollary 4.

For the sake of simplicity, we shall also restrict ourselves to the case $\sigma(A_0) = \sigma_{ac}(A_0) = \mathbf{R}$ only.

Let $E(\lambda)$ be the spectral family of A_0. Then we have

$$A_0 = \int_{-\infty}^{\infty} \lambda dE(\lambda).$$

For $\beta > \alpha > 0$, we denote $E((-\beta, -\alpha) \cup (\alpha, \beta))$ by $E_{\alpha,\beta}(A_0)$.

(A3) means that $\sqrt{B}_0 E_{\alpha,\beta}(A_0)$ is A_0-smooth, i.e. for any $g \in \mathcal{H}$

$$\int_{-\infty}^{\infty} \|\sqrt{B}_0 U_0(t) E_{\alpha,\beta}(A_0) g\|_{\mathcal{H}}^2 dt \leqq \tilde{C}_{\alpha,\beta} \|g\|_{\mathcal{H}}^2 \tag{3.1}$$

(cf. [8] or [16]), where $\tilde{C}_{\alpha,\beta}$ is a positive constant which depends on α and β only. Moreover we note the following identity of $V(t)f, f \in D(A)$:

$$\|V(t)f\|_{\mathcal{H}}^2 + 2\int_0^t \|\sqrt{B}_0 V(\tau) f\|_{\mathcal{H}}^2 d\tau = \|f\|_{\mathcal{H}}^2, \tag{3.2}$$

Using (3.1) and (3.2) we prove the following lemma.

Lemma 3.1. *Let $\beta > \alpha > 0$. Then for any $f \in D(A)$ one has*

$$\lim_{t,s\to\infty} \|E_{\alpha,\beta}(A_0)(U_0(-t)V(t) - U_0(-s)V(s))f\|_{\mathcal{H}} = 0.$$

proof. See [12] §3.

By Lemma 3.1 and (A1) we have the following lemma.

Lemma 3.2. *One has*

$$w-\lim_{t\to\infty} V(t) = 0.$$

Using Lemma 3.2 we prove Theorem 3(1) as follows.

proof of Theorem 3(1). Assume that there exists $f \in D(A), \lambda \in \mathbf{R}$ such that $Af = \lambda f$. Then we have

$$\langle V(t)f, f\rangle_{\mathcal{H}} = e^{-it\lambda} \|f\|_{\mathcal{H}}^2$$

This yields a contradiction with Lemma 3.2. □

Theorem 3(1) and (F1) imply that

$$\{(A-i)^{-2} Af \in \mathcal{H} : f \in D(A)\} \quad \text{is dense in} \quad \mathcal{H}. \tag{3.4}$$

Below we prove Theorem 3(2).

proof of Theorem 3(2). First we show the existence of W. Set $F(\lambda) = (\lambda - i)$ By (2.6) it is sufficient to show that $\{U_0(-t)V(t)F(A)f\}_{t \geqq 0}$ is Cauchy in ' $t \to \infty$, where $f \in D(A)$. We estimate as follows (cf. [17]) :

$$\|(U_0(-t)V(t) - U_0(-s)V(s))F(A)f\|_{\mathcal{H}} \leqq \sum_{j=1}^{4} \|T_j\|_{\mathcal{H}},$$

where

$$T_1 = U_0(-t)(F(A) - F(A_0))V(t)f$$
$$T_2 = U_0(-s)(F(A) - F(A_0))V(s)f$$
$$T_3 = F(A_0)(I_d - E_{1/M,M}(A_0))(U_0(-t)V(t) - U_0(-s)V(s))f$$
and
$$T_4 = F(A_0)E_{1/M,M}(A_0)(U_0(-t)V(t) - U_0(-s)V(s))f.$$

We note that for any ε, there exists $M > 1$ such that

$$\|(1 - \chi_{(-M,-1/M)\cup(1/M,M)})F\|_{L^\infty(\mathbf{R})} < \varepsilon.$$

Thus we have

$$\|T_3\|_{\mathcal{H}} < \varepsilon\|f\|_{\mathcal{H}}. \tag{3.5}$$

By (A4), $F(A) - F(A_0)$ is a compact operator. Hence Lemma 3.2 implies

$$\lim_{t\to\infty} \|T_1\|_{\mathcal{H}} = \lim_{s\to\infty} \|T_2\|_{\mathcal{H}} = 0. \tag{3.6}$$

Lemma 3.1 implies

$$\lim_{t,s\to\infty} \|T_4\|_{\mathcal{H}} = 0. \tag{3.7}$$

(3.5), (3.6) and (3.7) imply the existence of W.

Next we prove $W \not\equiv 0$ (cf. [12]§3). Assume that $W \equiv 0$ i.e. for any $f \in \mathcal{H}$

$$\lim_{t\to\infty} \|V(t)f\|_{\mathcal{H}} = 0. \tag{3.8}$$

We set $G(\lambda) = (\lambda - i)^{-1}$. Then noting

$$\langle U_0(-t)V(t)G(A)f, G(A_0)f\rangle_{\mathcal{H}}$$
$$= \langle G(A)f. G(A_0)f\rangle_{\mathcal{H}} - \int_0^t \langle U_0(-\tau)BV(\tau)G(A)f, G(A_0)f\rangle_{\mathcal{H}} d\tau,$$

we have by (3.8) and Schwartz inequality

(3.9)
$$|\langle G(A)f, G(A_0)f\rangle_{\mathcal{H}}|$$
$$\leqq (\int_0^\infty \|\sqrt{B}V(\tau)G(A)f\|_{\mathcal{H}}^2 d\tau)^{\frac{1}{2}} \times (\int_0^\infty \|\sqrt{B}U_0(\tau)G(A_0)f\|_{\mathcal{H}}^2 d\tau)^{\frac{1}{2}}.$$

(3.2) and (3.8) imply

$$2\int_0^\infty \|\sqrt{B}V(\tau)G(A)f\|_{\mathcal{H}}^2 d\tau = \|G(A)f\|_{\mathcal{H}}^2. \tag{3.10}$$

Hence we have by (3.9) and (3.10)

$$\|G(A_0)f\|_{\mathcal{H}}^2 \leqq \|f\|_{\mathcal{H}}\{\|(G(A)-G(A_0))f\|_{\mathcal{H}} + (\frac{1}{2}\int_0^\infty \|\sqrt{B}U_0(\tau)G(A_0)f\|_{\mathcal{H}}^2 d\tau)^{\frac{1}{2}}\}.$$

Let fix $M > 1$. Put $f = U_0(s)g, g$ satisfying $E_{1/M,M}(A_0)g = g$. Then we have

(3.11)

$$\begin{aligned}\|G(A_0)g\|_{\mathcal{H}}^2 \leqq& \|g\|_{\mathcal{H}}\{\|(G(A)-G(A_0))U_0(s)g\|_{\mathcal{H}} \\ &+ (\frac{1}{2}\int_s^\infty \|\sqrt{B}E_{1/M,M}(A_0)U_0(\tau)G(A_0)g\|_{\mathcal{H}}^2 d\tau)^{\frac{1}{2}}\}.\end{aligned}$$

(A1) and (A4) imply

$$\lim_{s\to\infty} \|(G(A)-G(A_0))U_0(s)g\|_{\mathcal{H}} = 0. \tag{3.12}$$

(3.1) implies

$$\lim_{s\to\infty}\int_s^\infty \|\sqrt{B}E_{1/M,M}(A_0)U_0(\tau)G(A_0)g\|_{\mathcal{H}}^2 d\tau = 0. \tag{3.13}$$

Therefore it follows from (3.11), (3.12) and (3.13) that $g \equiv 0$. This is a contradiction. Therefore we have $W \not\equiv 0$. □

To prove Corollary 4 we should repeat the same way as in the proof of Corollary 2. Here we omit to do it.

4. Applications.

Application 1 (Elastic wave equation with dissipative boundary condition in a half space of $\mathbf{R}^3$).

We shall apply Theorem 1. In this section we also use C as positive constants.

Let $x = (x_1, x_2, x_3) = (y, x_3) \in \mathbf{R}^2 \times \mathbf{R}_+$ and $\mu_0 > 0, \rho_0 > 0, \lambda_0 \in \mathbf{R}$ satisfying $3\lambda_0 + 2\mu_0 > 0$. We use $O_{3\times 3}$ and $I_{3\times 3}$ as zero and unit matrix of 3×3 type, respectively.

We set

$$\varepsilon_{hj}(u(x)) = \frac{1}{2}(\frac{\partial u_h}{\partial x_j} + \frac{\partial u_j}{\partial x_h})$$

and

$$\sigma_{hj}(u(x)) = \lambda_0(\nabla_x \cdot u)\delta_{hj} + 2\mu_0\varepsilon_{hj}(u)$$

where $h, j = 1, 2, 3, u(x) ={}^t (u_1(x), u_2(x), u_3(x)) \in \mathbf{C}^3$ and $\nabla_x = (\partial/\partial_1, \partial/\partial_2, \partial/\partial_3)$.

We define operators $\tilde{L}_0$ as

$$(\tilde{L}_0 u)_h = -\sum_{j=1}^{3} \frac{1}{\rho_0} \frac{\partial \sigma_{hj}(u(x))}{\partial x_j} \quad (h = 1, 2, 3).$$

We consider two elastic wave equations as follows:

$$(4.1) \quad \begin{cases} \partial_t^2 u(x,t) + \tilde{L}_0 u(x,t) = 0, (x,t) \in \mathbf{R}^3_+ \times [0,\infty), \\ {}^t(\sigma_{13}(u), \sigma_{23}(u), \sigma_{33}(u)) \mid_{x_3=0} = B(y)\partial_t u \mid_{x_3=0} \end{cases}$$

and

$$(4.2) \quad \begin{cases} \partial_t^2 u(x,t) + \tilde{L}_0 u(x,t) = 0, (x,t) \in \mathbf{R}^3_+ \times \mathbf{R}, \\ \sigma_{i3}(u) \mid_{x_3=0} = 0 (i = 1, 2, 3). \end{cases}$$

To set assumptions for $B(y)$ we introduce a function space $B^k(\Omega)$ as follows :

$$B^k(\Omega) = \{u \in C^k(\Omega); \sum_{|\alpha| \leqq k} \|\partial^\alpha u\|_{L^\infty(\Omega)} < \infty\},$$

where $\Omega \subset \mathbf{R}^n$.

Assume that

(4.3) $B(y)$ belongs to $B^1(\mathbf{R}^2, \mathbf{M}_{3\times3})$ and satisfies

$$O_{3\times3} \leqq B(y) \leqq \varphi(|y|) I_{3\times3},$$

where $\varphi(r)$ is a non-increasing function and belongs to $L^1(\mathbf{R}_+)$. $\mathbf{M}_{3\times3}$ is the class of 3×3 matrix.

The following operator L_0 in $\mathcal{G} = L^2(\mathbf{R}^3_+, \mathbf{C}^3; \rho_0 dx)$:

$$L_0 u = \tilde{L}_0 u$$

and

$$D(L_0) = \{u \in H^1(\mathbf{R}^3_+, \mathbf{C}^3); \tilde{L}_0 u \in \mathcal{G}, \sigma_{h3}(u) \mid_{x_3=0} = 0 (h = 1, 2, 3)\}$$

is a non-negative self-adjoint operator.

Let $\mathcal{H}$ be Hilbert space with inner product :

$$\langle f, g \rangle_{\mathcal{H}} = \int_{\mathbf{R}^3_+} \Big(\sum_{h,j,k,l=1}^{3} a_{hjkl} \varepsilon_{kl}(f_1) \overline{\varepsilon_{hj}(g_1)} + f_2 \overline{g_2} \rho_0 \Big) dx,$$

where $a_{hjkl} = \lambda_0 \delta_{hj} \delta_{kl} + \mu_0 (\delta_{hk} \delta_{jl} + \delta_{hl} \delta_{jk})$ and $f = {}^t(f_1, f_2), g = {}^t(g_1, g_2)$. By Korn's inequality (cf. [5]) we note that $\mathcal{H}$ is equivalent to $\dot{H}^1(\mathbf{R}^3_+, \mathbf{C}^3) \times L^2(\mathbf{R}^3_+, \mathbf{C}^3)$ as Banach space.

We set $f = {}^t(u(x,t), u_t(x,t))$, where $u(x,t)$ is the solution to (4.1) (resp. (4.2)) with a initial data $f_0 = {}^t(u(x,0), u_t(x,0)) \in \mathcal{H}$. Then (4.1) (resp. (4.2)) can be written as

$$\partial_t f = -iAf \qquad (\text{resp.} \quad \partial_t f = -iA_0 f),$$

where

$$A = i\begin{pmatrix} 0 & I_{3\times 3} \\ -\tilde{L}_0 & 0 \end{pmatrix}, \qquad A_0 = i\begin{pmatrix} 0 & I_{3\times 3} \\ -\tilde{L}_0 & 0 \end{pmatrix},$$

$$D(A) = \{f = {}^t(f_1, f_2) \in \mathcal{H}; \tilde{L}_0 f_1 \in L^2(\mathbf{R}^3_+, \mathbf{C}^3), f_2 \in H^1(\mathbf{R}^3_+, \mathbf{C}^3),$$
$${}^t(\sigma_{13}(f_1), \sigma_{23}(f_1), \sigma_{33}(f_1)) \mid_{x_3=0} = B(y) f_2 \mid_{x_3=0}\}$$

and

$$D(A_0) = \{f = {}^t(f_1, f_2) \in \mathcal{H}; \tilde{L}_0 f_1 \in L^2(\mathbf{R}^3_+, \mathbf{C}^3), f_2 \in H^1(\mathbf{R}^3_+, \mathbf{C}^3),$$
$$\sigma_{h3}(f_1) \mid_{x_3=0} = 0 (h = 1, 2, 3)\}$$

According to P210-P211 of [11] or Corollary 1.1.4 of [14] we can show that A generates a contraction semi-group $\{V(t)\}_{t\geqq 0}$ (resp. a unitary group $\{U_0(t)\}_{t\in\mathbf{R}}$) in $\mathcal{H}$. Using $\{V(t)\}_{t\geqq 0}$ (resp. $\{U_0(t)\}_{t\in\mathbf{R}}$) we solve $\partial_t f = -iAf$ (resp. $\partial_t f = -iA_0 f$) as follows

$$f = V(t) f_0 \quad (\text{resp.} f = U_0(t) f_0).$$

Below we make a check on Assumptions (A1),(A2) and (A3)
[2] implies $\sigma(A_0) = \sigma_{ac}(A_0) = \mathbf{R}$. Therefore we have (A1).

Next we show (A2). For $f, g \in \mathcal{H}$, we have by easy caluculation

$$\text{(4.4)} \qquad \langle((A-i)^{-1} - (A_0 - i)^{-1}) f, g\rangle_{\mathcal{H}}$$
$$= i \int_{\mathbf{R}^2} B(y) \Gamma_0((A_0 - i)^{-1} f)_2 \overline{\Gamma_0((A^* + i)^{-1} g)_2} dy,$$

where Γ_0 is a tarce operator which is defined by

$$(\Gamma_0 u)(y) = u(y, 0).$$

Note that $\Gamma_0((A_0-i)^{-1}f)_2$ and $\Gamma_0((A^*+i)^{-1}f)_2$ belong to $H^{1-s}(\mathbf{R}^3_+, \mathbf{C}^3)$ by Korn's inequality for any $s \in (1/2, 1)$. Since $B(y)\Gamma_0\Pi_2(A_0 - i)^{-1}$ is a compact operator from $\mathcal{H}$ to $L^2(\mathbf{R}^2, \mathbf{C}^3)$ by Rellich's theorem, where $\Pi_j\, {}^t(f_1, f_2) = f_j (j = 1, 2)$, the form $(A-i)^{-1} - (A_0 - i)^{-1}$ can be extended to a compact operator,$(\Gamma_0\Pi_2(A^* + i)^{-1})^* B(y)\Gamma_0\Pi_2(A_0 - i)^{-1}$, in $\mathcal{H}$.

To show (A3) we sate a result from [2]. There exist $F_P u, F_S u, F_{SH} u$ and F_R which are partially isometric operators from $\mathcal{G} = L^2(\mathbf{R}^3_+, \mathbf{C}^3; \rho_0 dx)$ onto $L^2(\mathbf{R}^3_+, \mathbf{C}^3)$ and $L^2(\mathbf{R}^2, \mathbf{C}^3)$, respectively. Defining the operator F as follows :

$$Fu = (F_P u, F_S u, F_{SH} u, F_R u) \quad \text{for} \quad u \in \mathcal{G},$$

we have by Theorem 3.6 of [2]

Lemma A. *F is unitary operator from $\mathcal{G}$ to*

$$\hat{\mathcal{H}} = \bigoplus_{j=1}^{3} L^2(\mathbf{R}^3_+, \mathbf{C}^3) \bigoplus L^2(\mathbf{R}^2, \mathbf{C}^3)$$

and for every $u \in D(L_0)$

$$FL_0u = (c_P^2|k|^2F_Pu, c_S^2|k|^2F_Su, c_S^2|k|^2F_{SH}u, c_R^2|p|^2F_Ru),$$

where $k = (p, p_3) \in \mathbf{R}^2 \times \mathbf{R}_+$.

Using $F_j(j = P, S, SH, R)$ as above, we construstrust $P_\pm$ as follows :

$$(4.5) \qquad P_\pm = T^{-1}\{ \sum_{j=P,S,SH} \begin{pmatrix} F_j^*P_\mp^{(3)}I_{3\times3}F_j & O_{3\times3} \\ O_{3\times3} & F_j^*P_\pm^{(3)}I_{3\times3}F_j \end{pmatrix} + \begin{pmatrix} F_R^*P_\mp^{(2)}I_{3\times3}F_R & O_{3\times3} \\ O_{3\times3} & F_R^*P_\pm^{(2)}I_{3\times3}F_R \end{pmatrix} \}T$$

where

$$T = \frac{1}{\sqrt{2}} \begin{pmatrix} L_0^{\frac{1}{2}} & iI_{3\times3} \\ L_0^{\frac{1}{2}} & -iI_{3\times3} \end{pmatrix}$$

and $P_-^{(3)}$(resp. $P_+^{(3)}$) and $P_-^{(2)}$(resp. $P_+^{(2)}$) are negative(resp. positive) spectral projections of

$$D^{(3)} = \frac{1}{2i}(k \cdot \nabla_k + \nabla_k \cdot k) \quad \text{and} \quad D^{(2)} = \frac{1}{2i}(p \cdot \nabla_p + \nabla_p \cdot p), \quad \text{respectively.}$$

Using the representation of the generalized eigenfunction of L_0(see [2]) and the Mellin transformation we show (A3.1)$\sim$(A3.4) (cf. [13] and [6]). The Mellin transformations for $D^{(3)}, D^{(2)}$ are given as

$$(M^{(3)}u)(\lambda, \omega) = (2\pi)^{-1/2} \int_0^{+\infty} r^{1/2-i\lambda}u(r\omega)dr$$

and

$$(M^{(2)}v)(\lambda, \nu) = (2\pi)^{-1/2} \int_0^{+\infty} r^{-i\lambda}v(r\nu)dr,$$

where $u(k) \in C_0^\infty(\mathbf{R}^3_+\backslash\{0\}), v(p) \in C_0^\infty(\mathbf{R}^2\backslash\{0\}), \omega \in \mathbf{S}^2_+ = \{(\omega_1, \omega_2, \omega_3) = (\overline{\omega}, \omega_3) \in \mathbf{S}^2 : \omega_3 > 0\}$ and $\nu \in \mathbf{S}^1$.

Then $M^{(3)}$ (resp. $M^{(2)}$) is extended to a unitary operator from $L^2(\mathbf{R}^3_+)$ (resp. $L^2(\mathbf{R}^2)$) to $L^2(\mathbf{R} \times \mathbf{S}^2_+)$(resp. $L^2(\mathbf{R} \times \mathbf{S}^1)$) (cf.[13] Lemma 2).

Proposition 4.1. *$P_\pm$ as in (4.5) satisfy (A3).*

To show Proposition 4.1 we prepare

Lemma 4.2. *Let $\psi(\lambda)$ be same as in (A3) and $0 < \delta < c_R$ (for c_R, see Appendix). Then for any positive integer N and $t \in \mathbf{R}_\pm$, there exists a positive constant $C_{N,\psi}$ which is independent of t such that*

$$\|\nabla_x(e^{-itA_0}\psi(A_0)P_\pm f)_1\|^{|x|\leqq\delta|t|}_{L^2(\mathbf{R}^3_+,\mathbf{C}^3)} \leqq C_{N,\psi}(1+|t|)^{-N}\|f\|_{\mathcal{H}}, \tag{4.6}$$

$$\|(e^{-itA_0}\psi(A_0)P_\pm f)_2\|^{|x|\leqq\delta|t|}_{L^2(\mathbf{R}^3_+,\mathbf{C}^3)} \leqq C_{N,\psi}(1+|t|)^{-N}\|f\|_{\mathcal{H}} \tag{4.7}$$

and

$$\|\Gamma_0(e^{-itA_0}\psi(A_0)P_\pm f)_2\|^{|y|\leqq\delta|t|}_{L^2(\mathbf{R}^2,\mathbf{C}^3)} \leqq C_{N,\psi}(1+|t|)^{-N}\|f\|_{\mathcal{H}} \tag{4.8}$$

for any $f \in \mathcal{H}_0$, where

$$\|u\|^B_{L^2(\mathbf{R}^3_+,\mathbf{C}^3)} = (\int_B |u|^2dx)^{\frac{1}{2}} \quad \textit{and} \quad \|v\|^B_{L^2(\mathbf{R}^2,\mathbf{C}^3)} = (\int_B |v|^2dy)^{\frac{1}{2}}.$$

This lemma is the key lemma to show (A3). The proof is done by using $M^{(3)}, M^{(2)}$ and Lemma A. But we omit to prove (cf. [13] or [6]).

proof of Proposition 4.1. Lemma A of Appendix implies that P_+ and P_- are projection operators and satisfy $P_+ + P_- = Id$ in $\mathcal{H}$. Below we show (A3.1)~(A3.4).

For any $f, g \in \mathcal{H}$ we have by (4.4)

$$\begin{aligned}&|\langle Ke^{-itA_0}\psi(A_0)P_+f, g\rangle_{\mathcal{H}}|\\ &\leqq CI(t) \times (\|A^*(A^*+i)^{-1}g\|_{\mathcal{H}} + \|(A^*+i)^{-1}g\|_{\mathcal{H}}),\end{aligned}$$

where

$$\begin{aligned}I(t) = (\int_{\mathbf{R}^2} &|B(y)\Gamma_0(e^{-itA_0}(A_0-i)^{-1}\psi(A_0)f)_2|^2dy)^{\frac{1}{2}} \times\\ &\times(\|A^*(A^*+i)^{-1}g\|_{\mathcal{H}} + \|(A^*+i)^{-1}g\|_{\mathcal{H}}).\end{aligned}$$

Decomposing $I(t)$ as follows :

$$\begin{aligned}I(t) \leqq C\{(&\int_{\mathbf{R}^2\cap\{|y|\leqq\delta t\}} |\Gamma_0(e^{-itA_0}(A_0-i)^{-1}\psi(A_0)P_+f)_2|^2dy)^{\frac{1}{2}}\\ &+(\int_{\mathbf{R}^2\cap\{|y|\geqq\delta t\}} |B(y)\Gamma_0(e^{-itA_0}(A_0-i)^{-1}\psi(A_0)P_+f)_2|^2dy)^{\frac{1}{2}}\},\end{aligned}$$

we have by (4.8) of Lemma 4.2 and (4.3)

$$I(t) \leqq C_{N,\psi}\{(1+t)^{-N} + \varphi(\delta t)\}\|f\|_{\mathcal{H}}.$$

Therefore (A3.1) is proven.

To prove (A3.2) and (A3.3) we note

$$\langle f, K^*g\rangle_{\mathcal{H}} = \langle((A-i)^{-1} - (A_0-i)^{-1})f, g\rangle_{\mathcal{H}}$$

for any $f, g \in \mathcal{H}$.

By easy caluculation we have

$$(4.9)\qquad \begin{aligned}&\langle((A-i)^{-1} - (A_0-i)^{-1})f, g\rangle_{\mathcal{H}_0}\\ &= i\int_{\mathbf{R}^2} \Gamma_0((A-i)^{-1}f)_2\overline{B(y)\Gamma_0((A_0+i)^{-1}g)_2}dy.\end{aligned}$$

Then using (4.9) and the same way as in the proof of (A3.1), we obtain (A3.2) and (A3.3). Here we omit the detail.

We show (A3.4). Lemma 4.2 implies

$$\begin{aligned}&|\langle e^{itA_0}\psi(A_0)P_-f_t, g\rangle_{\mathcal{H}}|\\ &\leqq C_{N,\psi}\{(1+t)^{-N}\|g\|_{\mathcal{H}} + \|\nabla_x g_1\|^{|x|\geqq\delta t}_{L^2(\mathbf{R}^3_+,\mathbf{C}^3)} + \|g_2\|^{|x|\geqq\delta t}_{L^2(\mathbf{R}^3_+,\mathbf{C}^3)}\}\|f_t\|_{\mathcal{H}},\end{aligned}$$

for any $g \in \mathcal{H}$ and any positive integer N. Thus, noting $\sup_{t\in\mathbf{R}}\|f_t\|_{\mathcal{H}} < \infty$, we have (A3.4). □

Application 2 (Acoustic wave equations with dissipative terms in stratified media).

We shall apply Theorem 3. First we explain acoustic operator.

Let $n \geqq 1$ and $(x,y) \in \mathbf{R}^n \times \mathbf{R}$. We set

$$c_0(y) = \begin{cases} c_+ & (y \geqq h)\\ c_h & (0 < y < h)\\ c_- & (y \leqq 0),\end{cases}$$

for some positive constants h and c_+, c_-, c_h.

Acoustic operators are

$$L_0 = -c_0(y)^2\triangle,$$

where

$$\triangle = \sum_{j=1}^{n}\frac{\partial^2}{\partial x_j^2} + \frac{\partial^2}{\partial y^2}.$$

Considering the case $c_h < \min(c_+, c_-)$ we find the guided waves (cf. [18] or [19]). But we do not restrict ourselves to such cases.

L_0 is a non-negative self-adjoint operator in $\mathcal{G} = L^2(\mathbf{R}^{n+1}; c_0(y)^{-2}dxdy)$. $D(L_0)$ is given by $H^2(\mathbf{R}^{n+1})$, $H^s(\mathbf{R}^{n+1})$ being Sobolev space of order s over $\mathbf{R}^{n+1}$.

We deal with the following dissipative wave equations :

$$(4.10)\qquad \partial_t^2u(x,y,t) + b(x,y)\partial_tu(x,y,t) + L_0u(x,y,t) = 0$$

and

$$(4.11)\qquad \partial_t^2u(x,y,t) + \langle\partial_tu, \varphi\rangle_{\mathcal{G}}\varphi(x,y) + L_0u(x,y,t) = 0,$$

where $(x, y, t) \in \mathbf{R}^n \times \mathbf{R} \times [0, \infty)$ and $\langle \cdot, \cdot \rangle_{\mathcal{G}}$ is the inner-product of $\mathcal{G}$.

We assume that $b(x, y)$ and $\varphi(x, y)$ are measurable functions which satisfy

$$0 \leqq b(x, y) \leqq C(1 + |x|^2 + y^2)^{-\frac{\theta}{2}}$$

and

$$\varphi(x, y) \in L^2(\mathbf{R}^{n+1}; (1 + |x|^2 + y^2)^{\frac{\theta}{2}} dx dy)$$

for some $\theta > 1$ and $C > 0$.

We shall show the existence of the scattering states for (4.10) and (4.11) which are considered as the pertubed systems of

$$\partial_t^2 u(x, y, t) + L_0 u(x, y, t) = 0, \quad (x, y, t) \in \mathbf{R}^n \times \mathbf{R} \times \mathbf{R} \tag{4.12}$$

In [19], [1], and [21], we can find local resolvent estimates as follows : for any $\beta > \alpha > 0$, there exists positive constants $C_{\alpha,\beta}$ and η such that

$$\sup_{\alpha \leqq |\mathrm{Re}\zeta| \leqq \beta, 0 < |\mathrm{Im}\zeta| < \eta} \|X_{\frac{\theta}{2}}(L_0 - \zeta^2)^{-1} X_{\frac{\theta}{2}}\|_{L^2(\mathbf{R}^{n+1}) \to L^2(\mathbf{R}^{n+1})} \leqq C_{\alpha,\beta}. \tag{4.13}$$

where $\zeta \in \mathbf{C}, X_\gamma = (1 + |x|^2 + y^2)^{-\frac{\gamma}{2}}$ and $\| \cdot \|_{L^2(\mathbf{R}^{n+1}) \to L^2(\mathbf{R}^{n+1})}$ is the norm of the bounded operator in $L^2(\mathbf{R}^{n+1})$.

[12] has already dealt with the case $c_h = c_+ = c_- = 1$ and $n \geqq 2$ of (4.10). His proof has been bacsd on Kato's smooth pertubation theory [10] and global resolvent estimates for L_0(see also [10] Theorem 4.4.1)

We apply Theorem 3(Corollaty 4) to (4.10). We set $f(t) = (u(t, x, y), \partial_t u(t, x, y))$.▮ Then (4.12) and (4.10) can be written as $\partial_t f = -iA_0 f$ and $\partial_t f = -iAf$ respectively, where

$$A_0 = i \begin{pmatrix} 0 & 1 \\ -L_0 & 0 \end{pmatrix}, \qquad A = i \begin{pmatrix} 0 & 1 \\ -L_0 & -b(x, y) \end{pmatrix}.$$

Let $\mathcal{H}$ be Hilbert spaces with inner product

$$\langle f, g \rangle_{\mathcal{H}} = \int_{\mathbf{R}^{n+1}} (\nabla f_1(x, y) \overline{\nabla g_1(x, y)} + f_2(x, y) \overline{g_2(x, y)} c_0^{-2}(y)) dx dy,$$

and $\| \cdot \|_{\mathcal{H}}$ is the corresponding norm, where $f =^t (f_1, f_2), g =^t (g_1, g_2)$.

The domains of A_0 is

$$D(A_0) = \{f \in \mathcal{H}; \Delta f_1 \in L^2(\mathbf{R}^{n+1}), f_2 \in H^1(\mathbf{R}^{n+1})\}.$$

Then A_0 is a self-adjoint operator in $\mathcal{H}$ and generates a unitary group $\{U_0(t)\}_{t \in \mathbf{R}}$ in $\mathcal{H}$. Below we make a cheak on (A1), (A4) and (A5).

Note that

$$T_0 A_0 T_0^{-1} = \begin{pmatrix} \sqrt{L_0} & 0 \\ 0 & -\sqrt{L_0} \end{pmatrix},$$

where

$$T_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{L_0} & i \\ \sqrt{L_0} & -i \end{pmatrix}$$

and T_0 is a unitary operator from $\mathcal{H}$ onto $\mathcal{G} \times \mathcal{G}$. It follows from (4.13) that for any $u \in \mathcal{G}$

$$\sup_{\alpha \leqq |Re\zeta| \leqq \beta, 0<|Im\zeta|<\eta} |\mathrm{Im}\langle (\pm\sqrt{L_0} - \zeta)^{-1} X_{\frac{\theta}{2}} u, X_{\frac{\theta}{2}} u \rangle_{\mathcal{G}}| < \infty.$$

Therefore we have (A1) by [16] Theorem XIII-20.

Since

$$B_0 = \begin{pmatrix} 0 & 0 \\ 0 & b(x,y) \end{pmatrix}$$

is A_0-compact by Rellich's theorem, we have (A2). Therefore A generates a contraction semi-group $\{V(t)\}_{t \geqq 0}$ in $\mathcal{H}$.

In the same argument as in [12]§3 we can show (A5) as follow. Let $g = (g_1, g_2) \in \mathcal{H}$. We set

$$u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = (A_0 - \zeta)^{-1} \sqrt{B_0} \begin{pmatrix} g_1 \\ g_2 \end{pmatrix}.$$

Then we have

$$(L_0 - \zeta^2) u_2 = \zeta \sqrt{b(x,y)} g_2$$

and

$$\sqrt{B_0} (A_0 - \zeta)^{-1} \sqrt{B_0} g = \sqrt{B_0} u =^t (0, \sqrt{b(x,y)} u_2).$$

Therefore we can calculate as follows :

$$\|\sqrt{B_0}(A_0 - \zeta)^{-1}\sqrt{B_0} g\|_{\mathcal{H}} = |\zeta| \|\sqrt{b(x,y)}(L_0 - \zeta^2)^{-1}\sqrt{b(x,y)} g_2\|_{\mathcal{G}_0}. \tag{4.14}$$

(4.13) and (4.14) imply (A5). Thus we have the conclusion of Theorem 3(Corollary 4) for (4.10) and (4.12).

Next we apply Theorem 3(Corollary 4) to (4.11). we set

$$B_0 = \begin{pmatrix} 0 & 0 \\ 0 & \langle \cdot, \varphi \rangle_{\mathcal{G}} \varphi \end{pmatrix}$$

Then B is a compact operator in $\mathcal{H}$. We shall show (A5). Note that

$$|\mathrm{Im}\zeta| \|\sqrt{B}(A_0 - \zeta)^{-1} f\|^2_{\mathcal{H}} \leqq |\mathrm{Im}\zeta| \|X_{\frac{\theta}{2}}((A_0 - \zeta)^{-1} f)_2\|^2_{\mathcal{G}} \times \|X_{-\frac{\theta}{2}} \varphi\|^2_{\mathcal{G}} \tag{4.15}$$

for any $f \in \mathcal{H}$. We set

$$B_1 = \begin{pmatrix} 0 & 0 \\ 0 & X_\theta \end{pmatrix}.$$

Then we have

$$\begin{aligned} |\mathrm{Im}\zeta| \|X_{\frac{\theta}{2}}((A_0 - \zeta)^{-1} f)_2\|^2_{\mathcal{G}} &= |\mathrm{Im}\zeta| \|\sqrt{B_1}(A_0 - \zeta)^{-1} f\|^2_{\mathcal{H}} \\ &\leqq \|\sqrt{B_1}\{(A_0 - \zeta)^{-1} - (A_0 - \bar{\zeta})^{-1}\}\sqrt{B_1}\| \|f\|^2_{\mathcal{H}}. \end{aligned}$$

Noting (4.14) which is changed B_0 and $b(x,y)$ to B_1 and X_θ, respectively we get (A5). Therefore we have the conclusion of Theorem 3(Corollary 4) for (4.11) and

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