数理解析研究所講究録 1211 巻 2001 年 1-2

Operator Algebras with Hierarchies of Symbols

B.-W. Schulze (Potsdam University)

Abstract for Kyoto:

Manifolds with (geometric) singularities give rise to spaces of typical differential operators that are degenerate in a specific way near the singularities. Examples are conical singularities with Fuchs type operators, edge singularities with edge-degenerate operators, etc. Standard situations such as compact manifolds with smooth boundary also belong to this framework, and pseudo-differential boundary value problems (say, with the transmission property with respect to the boundary) form an operator algebra with a "hierarchy" of (principal) symbols, namely the interior and the boundary symbol (the latter one is operator-valued). Ellipticity requires the bijectivity of all components (that, in general, include additional conditions along the boundary), and the algebra contains the parametrices belonging to the inverse symbols. An analogous program is possible for manifolds with higher, say, polyhedral singularities, for pseudo-differential operators with symbols that are degenerate in a similar way as Laplace-Beltrami operators for singular metrics that model the configuration. This can be understood in an iterative way. From a compact space we can pass to a cone with that base, then to a wedge with that model cone, etc., and in this way reach all "regular" polyhedral singularities.
Parallel to this geometric procedure we generate "pseudo-differential" algebras, starting from given ones on a compact base, construct "conifications" and "edgifications" of those algebras, to reach algebras on the corresponding higher cones and wedges. Then, an invariance consideration and globalisation allows us to start the procedure again. Each conification step produces an additional weight in associated cone Sobolev spaces and an additional operator-valued symbol, the conormal symbol, with values in operators along the cone base. The conormal symbol (in the elliptic case) encodes a global spectral information along the cone base with non-linearly involved "eigenvalues". Each edgification step produces an additional operator-valued edge symbol that is (in the elliptic case) responsible for additional edge conditions

(similarly to boundary conditions in boundary value problems), and we produce a new generation of weighted wedge Sobolev spaces, that are defined in terms of specific (strongly continuous one-parameter) groups of isomorphisms on weighted Sobolev spaces on the model cone, based on homotheties in axial direction. This scenario allows us also to define subspaces with (discrete and continuous) conormal asymptotics that are involved from the very beginning in the understanding of regularity of solutions to elliptic equations near the geometric singularities and in the functional analytic structure of the (operator-valued) symbols of the extra trace and potential conditions on the edge. Our iterative approach to the analysis of pseudo-differential operators on manifolds with higher singularities generates a variety of natural operator ideals that are connected with the system of lower-dimensional skeleta. They contain, for instance, analogues of Green's function of standard boundary value problems, and give a transparent description of reductions of conditions to lower-dimensional skeleta, similarly to the reduction of boundary conditions to the boundary. Our theory has natural applications to problems in applied sciences, e. g. to elasticity or crack theory.

References

B.-W. Schulze: *Pseudo-Differential Boundary Value Problems, Conical Singularities, and Asymptotics.* Akademie Verlag, Berlin, 1994.

B.-W. Schulze: *Boundary Value Problems and Singular Pseudo-Differential Operators.* J. Wiley, Chichester, 1998.

B.-W. Schulze: *Operator algebras with symbol hierarchies on manifolds with singularities.* Preprint 99/30 Institute for Mathematics, Potsdam 1999; Advances in Partial Differential Equations, "Approaches to Singular Analysis", Birkhäuser Verlag, Zürich, 2000 (to appear).