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AN ELEMENTARY APPROACH TO THE MICROSUPPORT-THEORY OF HOLOMORPHIC SOLUTION COMPLEXES FOR $\mathcal{E}_X$- AND $\mathcal{E}_X^{\mathbb{R}}$-MODULES

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Kashiwara-Schapira originated the micro-support theory for complexes of arbitrary sheaves on C^1-differentiable manifolds in 1980. They applied successfully this theory to the microlocal analysis of $\mathcal{D}_X$-modules. For example, let $\mathcal{M}$ be a coherent $\mathcal{D}_X$-module on a complex manifold X, and set the holomorphic solution complex $\mathcal{F}$ by

$$\mathcal{F} = \mathbb{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}; \mathcal{O}_X).$$

Then, the microfunction solution complex for $\mathcal{M}$ is expressed as

$$\mathbb{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}; \mathcal{C}_M) = \mu_M(\mathcal{F}).$$

Here μ_M is the microlocalization functor along M, which operates on complexes of any sheaves on X. Many results on the microfunction solution complex are derived from the estimation of the microsupport of $\mathbb{R}\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}; \mathcal{C}_M)$ from above ; for example, the vanishing or the propagation results on microfunction solutions. The essence of the microsupport theory is the following: Find the microsupport of the most fundamentsal complex like $\mathcal{F}$. Then, give an estimation of the complex derived from $\mathcal{F}$ from above by using some estimation formulas concerning microsupports for several functors. Indeed, Kashiwara proved that the microsupport of $\mathcal{F}$ coincides with the classical characteristic variety of $\mathcal{M}$ (we call this result as the generalization of Cauchy-Kovalevski theorem). Hence many results on PDE reduce to functorial and geometric arguments on the estimation of microsupports; for example, the solvability and propagation results in hyperfunction theory for hyperbolic or microhyperbolic equations.

On the other hand, this beautiful and strong theory has a weak point concerning treating $\mathcal{E}_X$- or $\mathcal{E}_X^{\mathbb{R}}$- equations. This is because the sheaf $\mathcal{O}_X$ is neither $\mathcal{E}_X$- nor $\mathcal{E}_X^{\mathbb{R}}$-modules. Kashiwara-Schapira gave an approach to overcome this difficulty by using more abstract theories; the microlocalization of categories or the theory of Ind-sheaves. Here, we introduce another approach by using more explicit methods. That is, we construct explicitly

$$\mathcal{F} = \mathbb{R}\mathcal{H}om_{\mathcal{E}_X}(\mathcal{M}; \mathcal{O}_X)$$

as a complex of abelian groups, where we abandon the usual sheaf theory. At the same time we define a generalization of microsupports for $\mathcal{F}$ as a conic closed subset of T^*X. Such an approach is not unusual because we often use Martineau's definition of hyperfunctions by using formal sums of defining holomorphic functions in the elementary lecture of microlocal analysis. There, only abelian groups of formal sums of holomorphic functions appear. Owing to a development of FBI-theory, we can prove almost all basic theorems concerning microlocal analysis only by this naive method. We will show here that we can extend this method to the theory of microsupports for $\mathcal{E}_X$- or $\mathcal{E}_X^{\mathbb{R}}$- modules.