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多重線形 Littlewood-Paley 作用素と多重線形 Fourier Multiplier

関西学院大学理学部 藪田 公三 (YABUTA Kôzô)
School of Science, Kwansei Gakuin University

表題の多重線形 Littlewood-Paley 作用素は次の形のもの

$$T(f_1,f_2,\ldots,f_m)(x)=\int_0^\infty((\varphi_1)_t*f_1)(x)((\varphi_2)_t*f_2)(x)\cdots((\varphi_m)_t*f_m)(x)\,b(t)\frac{dt}{t},$$

但し，$\varphi_j(x)\in L^1(\mathbb{R}^n)$ で適当な条件を満たし，少なくとも一つの j に対し $\int_{\mathbb{R}^n}\varphi_j(x)\,dx=0$ であり，$b(t)\in L^\infty(0,\infty)$. 又，ここでも以下でも，$\mathbb{R}^n$ 上の函数 $f(x)$ と $t>0$ に対して，$f_t(x)=t^{-n}f(x/t)$ とする．$\varphi*f$ は φ と f の合成積を表す．Poisson 核 $P(x)=c_n(1+|x|^2)^{-\frac{n+1}{2}}$ を用いて $\psi(x)=\frac{\partial P_t(x)}{\partial t}\big|_{t=1}$ として，

$$\Big(\int_0^\infty\big|(\psi_t*f)(x)\big|^2\frac{dt}{t}\Big)^{1/2}$$

が，$\mathbb{R}^n$ での Littlewood-Paley の g 函数である．

また，多重線形 Fourier Multiplier は次の形のもの

$$\begin{aligned}&M_\sigma(f_1,f_2,\ldots,f_m)(x)\\&=\frac{1}{(2\pi)^{nm}}\int_{(\mathbb{R}^n)^m}e^{ix\cdot(\xi_1+\xi_2+\cdots+\xi_m)}\sigma(\xi_1,\xi_2,\ldots,\xi_m)\hat{f}_1(\xi_1)\hat{f}_2(\xi_2)\cdots\hat{f}_m(\xi_m)\,d\xi_1d\xi_2\cdots d\xi_m.\end{aligned}$$

ここで，表象 $\sigma(\xi_1,\xi_2,\ldots,\xi_m)$ は，例えば，次を満たす．

$$|\partial^\alpha\sigma(\xi_1,\xi_2,\ldots,\xi_m)|\le C_\alpha(|\xi_1|+|\xi_2|+\cdots+|\xi_m|)^{-|\alpha|},\quad |\alpha|\le nm+1$$
$$\text{on } (R^n)^m\setminus\{(0,0,\ldots,0)\}.$$

もう一つ，関連したもので，次の多重線形 Calderón-Zygmund 特異積分がある．

$$\begin{aligned}&T_K(f_1,f_2,\ldots,f_m)(x)\\&=\text{p.v.}\int_{\mathbb{R}^{nm}}K(x-y_1,x-y_2,\ldots,x-y_m)f_1(y_1)f_2(y_2)\cdots f_m(y_m)\,dy_1dy_2\cdots dy_m.\end{aligned}$$

ここで，$K(x)$ は，例えば，次の条件を満たす．

(i) $\displaystyle\int_{S^{nm-1}}K(y_1,y_2\ldots,y_m)\,dy_1dy_2\cdots dy_m=0,$

(ii) $|K(y_1,y_2\ldots,y_m)|\le C/(|y_1|+|y_2|+\cdots+|y_m|)^{nm},$

(iii) $|\nabla K(y_1,y_2\ldots,y_m)|\le C/(|y_1|+|y_2|+\cdots+|y_m|)^{nm+1}.$

これら3つは，Coifman-Meyer [3, 4, 5] の研究以来，特にいろいろな人が関心を寄せている．Coifman-Meyer [3, 4, 5] では，適当な条件の下で $1 < p_j < \infty$ $(j = 1, 2, \ldots, m)$ と $p_0 \geq 1 : 1/p_0 = 1/p_1 + \cdots 1/p_m$ に対して上記の作用素が $L^{p_1} \times L^{p_2} \times \cdots \times L^{p_m} \to L^{p_0}$ 有界になるということである．最近の Grafakos-Torres [7] によれば，積分核，あるいは表象の適当な滑らかさの下に，例えば最初に挙げた条件下で，多重線形 Calderón-Zygmund 特異積分と多重線形 Fourier multiplier の場合には，$p_0 \geq 1$ の制限は取れる，つまり，$p_0 > 1/m$ としてよいということである．(表象に対する条件に現れる滑らかさの指数 $nm + 1$ については，例えば，Yabuta [16] 参照) また，多重線形 Littlewood-Paley 作用素については，Sato-Yabuta [13] で，$b(t) \equiv 1$ の場合に，同様のことを示している．

ここでは，Grafakos-Torres の枠外になる必ずしも滑らかでない表象の多重線形 Fourier multiplier を扱ってみる．具体的には，$n = 1, m = 2$ の場合を扱う．滑らかでない表象の2重線形 Fourier multiplier として，よく知られているものに Calderón の交換子 $C(a, f)$ と2重線形 Hilbert 変換 $H_s(a, f)$ がある．

$$C(a, f)(x) = \text{p.v.} \int_{-\infty}^{\infty} \frac{\int_y^x a(u)\, du}{(x-y)^2} f(y)\, dy = .$$

$$H_s(a, f)(x) = \text{p.v.} \int_{-\infty}^{\infty} \frac{a(x - s(x-y))}{x-y} f(y)\, dy.$$

これらを2重線形 Fourier multiplier として表現したときの表象 σ_C, σ_{H_s} は次のようになる

$$\sigma_C(\xi, \alpha)/(-\pi i) = \left\{1 - \left(1 - \left|\frac{\xi}{\alpha}\right|\right)^+\right\} \operatorname{sgn} \xi + \left(1 - \left|\frac{\xi}{\alpha}\right|\right)^+ \operatorname{sgn} \alpha$$

$$\sigma_{H_s}(\xi, \alpha)/(-\pi i) = \operatorname{sgn}(\xi + s\alpha)$$

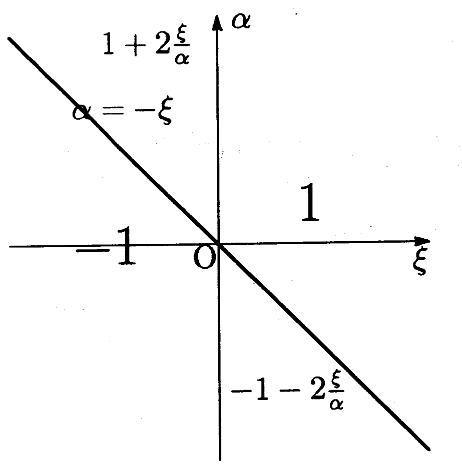

Fig 1. $\sigma_C(\xi, \alpha)/(-\pi i)$

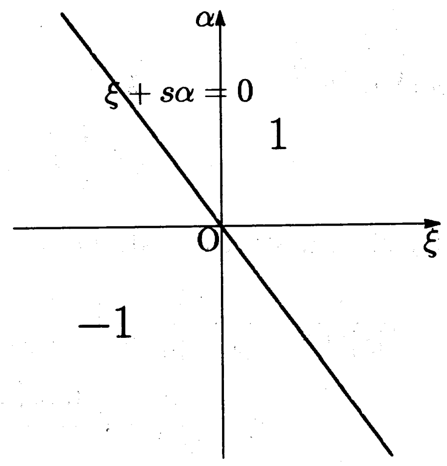

Fig 2. $\sigma_{H_s}(\xi, \alpha)/(-\pi i)$

Calderón の交換子 $C(a, f)$ については C. P. Calderón [2] により，$1 < p_1, p_2 < \infty$, $p_0 > 1/2 : 1/p_0 = 1/p_1 + 1/p_2$ に対して $L^{p_1} \times L^{p_2} \to L^{p_0}$ 有界性が成り立つことが示されている．また，2重線形 Hilbert 変換 $H_s(a, f)$ については，ごく最近 Lacey-Thiele [10, 11] により，上のことが $p_0 > 2/3$ の時，成り立つことが示されている（ただし，$s \neq 1$）．2/3 が最良かどうかは，まだ未解決である．

表象の特徴としては，$\sigma_C(\xi,\alpha)$ は０次斉次で単位円周上で連続，区分的に C^2 であり，$H_s(a,f)$ の方は０次斉次で単位円周上区分的に C^2 だが，不連続点があることである．

以下で，Calderón の交換子と同じような表象の２重線形 Fourier multiplier については，同じ結果が成り立つことを検証してみる．目標は次の定理である（Yabuta [15] では $r \geq 1$ であった）．以下は，英文で記すこととする．

Theorem 1. *Let $\sigma(\xi,\alpha)$ be a continuous and homogeneous function of degree zero in $\mathbb{R}^2\setminus\{(0,0)\}$, such that $\omega(\theta)=\sigma(\cos\theta,\sin\theta)$ is differentiable except atmost coutably many points and $\omega'(\theta)$ is of bounded variation on $[0,2\pi]$. Let T be the bilinear Fourier multiplier defined by*

$$T(f,g)(x)=\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{ix(\xi+\alpha)}\sigma(\xi,\alpha)\hat{f}(\xi)\hat{g}(\alpha)\,d\xi d\alpha.$$

Then, for $1<p,q<\infty$, and $r:\frac{1}{r}=\frac{1}{p}+\frac{1}{q}$, there exists $C>0$ such that

$$\|T(f,g)\|_r\leq C\|f\|_p\|g\|_q.$$

To prove this we prepare some lemmas.

Let $(M_\gamma f)\hat{}(\xi)=|\xi|^{i\gamma}\hat{f}(\xi)$. Then, the distributional kernel $K_\gamma(x)$ of M_γ is given by

$$K_\gamma(x)=c_\gamma|x|^{-1-i\gamma},\qquad c_\gamma=\frac{2^{i\gamma}\Gamma(\frac{1}{2}+\frac{i\gamma}{2})}{\pi^{\frac{1}{2}}\Gamma(-\frac{i\gamma}{2})},\quad |c_\gamma|\sim\sqrt{\frac{|\gamma|}{2\pi}}\text{ as }|\gamma|\to\infty.$$

Lemma 1. *Let $v(\gamma)$ be a nonnegative measurable function on $\mathbb{R}$, and $A\subset\mathbb{R}$ be a measurable set. Then,*

$$\int_{|x|\geq 2|y|}\Big(\int_A\Big|\frac{1}{|x|^{1+i\gamma}}-\frac{1}{|x-y|^{1+i\gamma}}\Big|^2 v(\gamma)\,d\gamma\Big)^{\frac{1}{2}}dx$$
$$\leq C\Big(\int_A v(\gamma)\,d\gamma\Big)^{\frac{1}{2}}\Big(1+\log_2\Big(\int_A\gamma^2 v(\gamma)\,d\gamma\Big/\int_A v(\gamma)\,d\gamma\Big)\Big).$$

Proof. By elementary calculations, we have for $|x|>2|y|$

$$\begin{aligned}\Big|\frac{1}{|x|^{1+i\gamma}}-\frac{1}{|x-y|^{1+i\gamma}}\Big| &\leq\Big|\frac{1}{|x|}-\frac{1}{|x-y|}\Big|+\frac{1}{|x-y|}\Big|\frac{1}{|x|^{i\gamma}}-\frac{1}{|x-y|^{i\gamma}}\Big|\\ &\leq C\frac{|y|}{|x|^2}+C\frac{\min(1,\frac{|\gamma y|}{|x|})}{|x|}.\end{aligned}$$

Hence

$$\begin{aligned}I&=\int_{|x|\geq 2|y|}\Big(\int_A\Big|\frac{1}{|x|^{1+i\gamma}}-\frac{1}{|x-y|^{1+i\gamma}}\Big|^2 v(\gamma)\,d\gamma\Big)^{\frac{1}{2}}dx\\ &\leq\int_{|x|\geq 2|y|}\Big(\int_A\Big(C\frac{|y|}{|x|^2}\Big)^2 v(\gamma)\,d\gamma\Big)^{\frac{1}{2}}dx+\int_{|x|\geq 2|y|}\Big(\int_A\Big(C\frac{\min(1,\frac{|\gamma y|}{|x|})}{|x|}\Big)^2 v(\gamma)\,d\gamma\Big)^{\frac{1}{2}}dx\\ &=:I_1+I_2.\end{aligned}$$

As for I_1,

$$I_1 \le C\Big(\int_A v(\gamma)\,d\gamma\Big)^{\frac12}|y|\int_{2|y|}^{\infty}\frac{1}{r^2}dr \le C\Big(\int_A v(\gamma)\,d\gamma\Big)^{\frac12}.$$

As for I_2,

$$\begin{aligned}
I_2 &= C\sum_{l=1}^{\infty}\int_{2^l|y|\le|x|\le 2^{l+1}|y|}\Big(\int_A\Big(\frac{\min(1,\frac{|\gamma y|}{|x|})}{|x|}\Big)^2 v(\gamma)\,d\gamma\Big)^{\frac12}dx\\
&\le C\sum_{l=1}^{\infty}\Big(\int_{2^l|y|\le|x|\le 2^{l+1}|y|}\int_A\Big(\frac{\min(1,\frac{|\gamma y|}{|x|})}{|x|}\Big)^2 v(\gamma)\,d\gamma\,dx\Big)^{\frac12}\Big(\int_{2^l|y|\le|x|\le 2^{l+1}|y|}dx\Big)^{\frac12}\\
&\le C\sum_{l=1}^{\infty}(2^{l+1}|y|)^{1/2}\Big(\int_A\Big[\int_{2^l|y|\le|x|\le 2^{l+1}|y|}\Big(\frac{\min(1,\frac{|\gamma y|}{|x|})}{|x|}\Big)^2dx\Big]v(\gamma)\,d\gamma\Big)^{\frac12}\\
&\le C\sum_{l=1}^{\infty}(2^{l+1}|y|)^{1/2}\Big(\int_A\Big[\Big(\frac{\min(1,\frac{|\gamma y|}{2^l|y|})}{2^l|y|}\Big)^2 2^l|y|\Big]v(\gamma)\,d\gamma\Big)^{\frac12}\\
&\le C\sum_{l=1}^{\infty}\Big(\int_A\min(1,\gamma^2 2^{-2l})v(\gamma)\,d\gamma\Big)^{1/2}\\
&\le C\sum_{l=1}^{\infty}\min\Big(\Big(\int_A v(\gamma)\,d\gamma\Big)^{\frac12},\Big(\int_A\gamma^2 v(\gamma)\,d\gamma\Big)^{\frac12}2^{-l}\Big)\\
&\le C\sum_{1\le l\le\log_2(\int_A\gamma^2v(\gamma)\,d\gamma/\int_A v(\gamma)\,d\gamma)/2}\Big(\int_A v(\gamma)\,d\gamma\Big)^{\frac12}\\
&\quad+\sum_{\log_2(\int_A\gamma^2v(\gamma)\,d\gamma/\int_A v(\gamma)\,d\gamma)/2<l}\Big(\int_A\gamma^2 v(\gamma)\,d\gamma\Big)^{\frac12}2^{-l}\\
&\le C\Big(\int_A v(\gamma)\,d\gamma\Big)^{\frac12}\Big(1+\log_2\Big(\int_A\gamma^2v(\gamma)\,d\gamma\Big/\int_A v(\gamma)\,d\gamma\Big)\Big).
\end{aligned}$$

□

Taking $v(\gamma) = (1+\sqrt{|\gamma|})^2/(1+\gamma^2)$ in Lemma 1, we have

Lemma 2. *Let $b(\gamma)\in L^\infty(\mathbb{R})$, $A_0=\{|\gamma|<1\}$ and $A_j=\{2^j\le|\gamma|<2^{j+1}\}$ $(j=1,2,\dots)$. Let $K_\gamma(x)$ be the distributional kernel of M_γ, where $(M_\gamma f)\hat{}(\xi)=|\xi|^{i\gamma}\hat f(\xi)$. Then, there exists $C>0$ such that*

$$\int_{|x|\ge 2|y|}\Big(\int_{A_j}|K_\gamma(x)-K_\gamma(x-y)|^2\frac{|b(\gamma)|}{1+\gamma^2}\,d\gamma\Big)^{\frac12}dx\le Cj^{3/2},\qquad j=0,1,2,\dots.$$

Lemma 3. *Let $(M_\gamma f)\hat{}(\xi)=|\xi|^{i\gamma}\hat f(\xi)$, $b(\gamma)\in L^\infty(\mathbb{R})$, and*

$$T(f,g)(x)=\int_{-\infty}^{\infty}M_\gamma f(x)M_{-\gamma}g(x)b(\gamma)\frac{d\gamma}{1+\gamma^2}.$$

Then, for $1 < p, q < \infty$, *and* $r : \frac{1}{r} = \frac{1}{p} + \frac{1}{q}$, *there exists* $C > 0$ *such that*

$$\|T(f,g)\|_r \le C\|f\|_p\|g\|_q.$$

Proof. In the case $1 \le r < \infty$, one can easily show the above by using Minkowski's inequality. We treat the case $1/2 < r < 1$. We treat first the case $1 < p, q \le 2$. Let $A_0 = \{|\gamma| < 1\}$ and $A_j = \{2^j \le |\gamma| < 2^{j+1}\}$ $(j = 1, 2, \dots)$. Put $u(\gamma) = b(\gamma)/(1 + |\gamma|^2)$. Then,

$$\begin{aligned}
\int_{\mathbb{R}} |T(f,g)(x)|^r\,dx &= \int_{\mathbb{R}} \left| \sum_{j=0}^{\infty} \int_{A_j} M_\gamma f(x) M_{-\gamma} g(x) u(\gamma)\,d\gamma \right|^r dx \\
&\le \sum_{j=0}^{\infty} \int_{\mathbb{R}} \left| \int_{A_j} M_\gamma f(x) M_{-\gamma} g(x) u(\gamma)\,d\gamma \right|^r dx \\
&\le \sum_{j=0}^{\infty} \int_{\mathbb{R}} \left[\left(\int_{A_j} |M_\gamma f(x)|^2 |u(\gamma)|\,d\gamma \right)^{1/2} \left(\int_{A_j} |M_{-\gamma} g(x)|^2 |u(\gamma)|\,d\gamma \right)^{1/2} \right]^r dx \\
&\le \sum_{j=0}^{\infty} \left(\int_{\mathbb{R}} \left(\int_{A_j} |M_\gamma f(x)|^2 |u(\gamma)|\,d\gamma \right)^{\frac{p}{2}} dx \right)^{\frac{r}{p}} \left(\int_{\mathbb{R}} \left(\int_{A_j} |M_{-\gamma} g(x)|^2 |u(\gamma)|\,d\gamma \right)^{\frac{q}{2}} dx \right)^{\frac{r}{q}} \\
&\qquad\qquad (\because 1 = \frac{r}{p} + \frac{r}{q}).
\end{aligned}$$

Now,

$$\left\| \left(\int_{A_j} |M_\gamma f(x)|^2 |u(\gamma)|\,d\gamma \right)^{1/2} \right\|_2 = c_n \left(\int_{A_j} |u(\gamma)|\,d\gamma \right)^{1/2} \|f\|_2 = C2^{-j/2}\|f\|_2.$$

So, since $\frac{1}{p} = (1 - (2 - \frac{2}{p})) + \frac{2-\frac{2}{p}}{2}$, by Lemma 2 and a result of Hörmander (M_γ is an $L^2(A_j, |u(\gamma)|\,d\gamma)$-valued singular integral),

$$\left\| \left(\int_{A_j} |M_\gamma f(x)|^2 |u(\gamma)|\,d\gamma \right)^{1/2} \right\|_p \le C(j^{3/2} + 2^{-2j/2})^{\frac{2}{p}-1}(2^{-j/2})^{2-\frac{2}{p}} \le Cj^{\frac{3}{p}-\frac{3}{2}}2^{-j(1-\frac{1}{p})}\|f\|_p.$$

Similarly we have

$$\left\| \left(\int_{A_j} |M_{-\gamma} g(x)|^2 |u(\gamma)|\,d\gamma \right)^{1/2} \right\|_q \le Cj^{\frac{3}{q}-\frac{3}{2}}2^{-j(1-\frac{1}{q})}\|g\|_q.$$

Hence, using $\frac{1}{r} = \frac{1}{p} + \frac{1}{q}$ and $r > 1/2$ we have

$$\begin{aligned}
\int_{\mathbb{R}} |T(f,g)(x)|^r\,dx &\le C \sum_{j=0}^{\infty} (j^{\frac{3}{p}-\frac{3}{2}}2^{-j(1-\frac{1}{p})} j^{\frac{3}{q}-\frac{3}{2}}2^{-j(1-\frac{1}{q})})^r \|f\|_p^r \|g\|_q^r \\
&\le C \sum_{j=0}^{\infty} j^{3-3r}2^{-j(2r-1)}\|f\|_p^r\|g\|_q^r \le C\|f\|_p^r\|g\|_q^r.
\end{aligned}$$

Next, we treat the case $1 < p \le 2$, $2 \le q$ or $2 \le p$, $1 < q \le 2$. We may assume $1 < p \le 2$, $2 \le q$. For $1 < p \le 2$, we can use

$$\left\| \left(\int_{A_j} |M_\gamma f(x)|^2 |u(\gamma)|\, d\gamma \right)^{1/2} \right\|_p \le C j^{\frac{3}{p}-\frac{3}{2}} 2^{-j(1-\frac{1}{p})} \|f\|_p.$$

For $q \ge 2$, we have by duality

$$\left\| \left(\int_{A_j} |M_{-\gamma} g(x)|^2 |u(\gamma)|\, d\gamma \right)^{1/2} \right\|_q \le C j^{3(1-\frac{1}{q})-\frac{3}{2}} 2^{-\frac{j}{q}} \|g\|_q \le C j^{\frac{3}{2}-\frac{3}{q}} 2^{-\frac{j}{q}} \|g\|_q.$$

Since $1 - \frac{1}{p} > 0$, we have

$$\begin{aligned} \int_{\mathbb{R}} |T(f,g)(x)|^r\, dx &\le C \sum_{j=0}^{\infty} (j^{\frac{3}{p}-\frac{3}{2}} 2^{-j(1-\frac{1}{p})} j^{\frac{3}{2}-\frac{3}{q}} 2^{-\frac{j}{q}})^r \|f\|_p^r \|g\|_q^r \\ &\le C \sum_{j=0}^{\infty} j^{3r(\frac{1}{p}-\frac{1}{q})} 2^{-jr(1-\frac{1}{p}+\frac{1}{q})} \|f\|_p^r \|g\|_q^r \le C \|f\|_p^r \|g\|_q^r. \end{aligned}$$

□

Proof of Theorem 1. Let $\sigma_1(\xi,\alpha)$ be a C^∞ homogeneous function of degree zero in $\mathbb{R}^2 \setminus \{(0,0)\}$ such that $\sigma_1(\pm1,0) = \sigma(\pm1,0)$ and $\sigma_1(0,\pm1) = \sigma(0,\pm1)$. Let T_1 be the bilinear Fourier multiplier defined by

$$T_1(f,g)(x) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{ix(\xi+\alpha)} \sigma_1(\xi,\alpha) \hat{f}(\xi) \hat{g}(\alpha)\, d\xi d\alpha.$$

Then, by a theorem of Grafakos and Torres, the conclusion of Theorem 1 holds for this bilinear operator T_1. Hence, to prove Theorem 1, we may assume $\sigma(\pm1,0) = \sigma(0,\pm1) = 0$. Thus, as in the proof of Theorem 4.1 in Yabuta [15, pp. 552-553], there exist four bounded function $b_j(\gamma)$ $(j = 1,2,3,4)$ such that

$$\begin{aligned} T(f,g)(x) = &\int_{-\infty}^{\infty} M_\gamma f(x) M_{-\gamma} g(x) b_1(\gamma) \frac{d\gamma}{1+\gamma^2} + \int_{-\infty}^{\infty} M_\gamma Hf(x) M_{-\gamma} g(x) b_2(\gamma) \frac{d\gamma}{1+\gamma^2} \\ &+ \int_{-\infty}^{\infty} M_\gamma f(x) M_{-\gamma} Hg(x) b_3(\gamma) \frac{d\gamma}{1+\gamma^2} + \int_{-\infty}^{\infty} M_\gamma Hf(x) M_{-\gamma} Hg(x) b_4(\gamma) \frac{d\gamma}{1+\gamma^2}, \end{aligned}$$

where H is the Hilbert transform, defined by $(Hf)\hat{}(\xi) = \frac{\xi}{|\xi|} \hat{f}(\xi)$. Using the L^p-boundedness of the Hilbert transform and Lemma 3, we get the desired conclusion. □

Remark 1. It was my misunderstanding that I could prove the assertion in Remark 1 in Yabuta [15, p. 553]. It is still an open problem whether Theorem 1 holds for $0 < p, q < \infty$ (L^p replaced by H^p), even in the case $\sigma(\pm1,0) = \sigma(0,\pm1)$.

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