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On a Synchronization Problem in Asynchronous DS/CDMA System

多元通信における同期確立の難易度について

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Abstract— Recently studies on spreading sequences (SS) codes generated by a Markov chain have been extensively discussed since it was reported that, for an asynchronous DS/CDMA system, SS codes can achieve smaller bit error rate (BER) than linear feedback shift register (LFSR) sequences. However these results follow from the assumption that the receiver is completely synchronized. In this paper we treat the case where the transmitted signal and its corresponding correlation receiver is incompletely synchronized within a fraction of a chip and give the distribution of autocorrelation functions of SS codes generated by a Markov chain. We also give the BER when a nearly synchonized correlator is employed.

Keywords— asynchronous DS/CDMA, spreading sequences, Markov chain, autocorrelation function

1 Introduction

In direct sequence code division multiple access (DS/CDMA) systems, one of the most important problems is to design "good" spreading sequences, each of which is assigned to each user. In most cases, the linear feedback shift registor (LFSR) sequences have been used.

Recently Mazzini, Rovatti, and Setti [1]–[3] have extensively discussed that some class of Markov sequences is better than independent and identically distributed (i.i.d.) random sequences as well as LFSR sequences in terms of

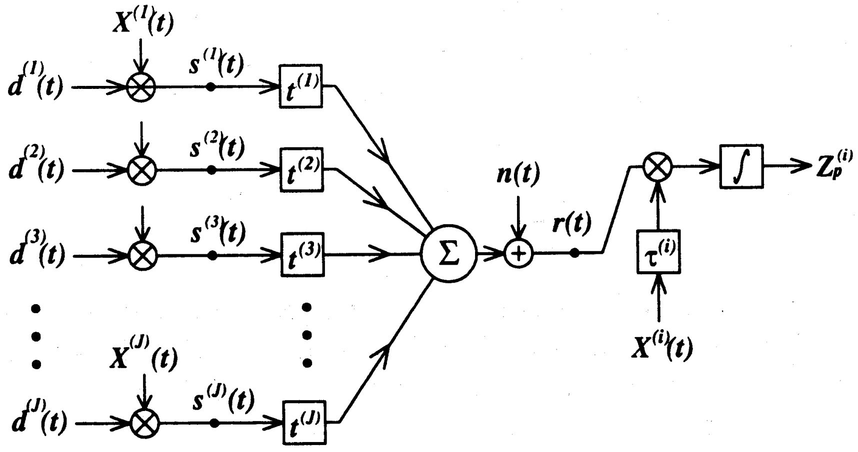

Figure 1: Asynchronous DS/CDMA communication system

bit error rate (BER), which temporarily astonished researchers in communication engineering and applied mathematics who believed unconsciously that sequences of i.i.d. random variables are the best at spreading sequences in terms of BER. This, however, has been supported by several following papers [4]–[6].

Corresponding with such results including almost all of the previous ones, is the fact that the receiver is assumed to be completely synchronized. It is natural to ask: to what extent do the sequence designers sacrifice acquisition and tracking performances of the correlation receiver in order to achieve a smaller BER by using Markov sequences? In this paper we will give some possible answers to this question.

2 Asynchronous DS/CDMA Systems

We consider baseband direct-sequence spread-spectrum (DS/SS) communications of J users (See Figure 1). We define the data signal of the j-th user ($j = 1, 2, \cdots, J$) with duration T and its assigned spread-spectrum code signal with duration T_c, respectively, by

$$d^{(j)}(t) = \sum_{p=-\infty}^{\infty} d_p^{(j)} u_T(t - pT) \tag{1}$$

$$X^{(j)}(t) = \sum_{q=-\infty}^{\infty} X_q^{(j)} u_{T_c}(t - qT_c), \tag{2}$$

where

$$u_D(t) = \begin{cases} 1 & \text{for } 0 \leq t < D \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

The j-th user's spread-spectrum code sequence $\boldsymbol{X}^{(j)} = \{X_q^{(j)}\}_{q=-\infty}^{\infty}$ has period $N = T/T_c$. Without loss of generality, we assume $T_c = 1$ through this paper.

We assume both data symbols $d_p^{(j)}$ and code symbols $X_q^{(j)}$ take on values $+1$ or -1 only. The transmitted signal for the j-th user is $s^{(j)}(t) = d^{(j)}(t)X^{(j)}(t)$. For asynchronous systems the received signal $r(t)$ is given by

$$r(t) = \sum_{j=1}^{J} s^{(j)}(t - t^{(j)}) + n(t), \tag{4}$$

where $t^{(j)}$ is the time delay of j-th user's signal and $n(t)$ is the channel noise process which we assume to be a white Gaussian process with two-sided spectral density $N_0/2$. The output of the i-th correlation receiver during the p-th time interval is given by

$$\begin{aligned} Z_p^{(i)} &= \int_{pT+\tau^{(i)}}^{(p+1)T+\tau^{(i)}} r(t)X^{(i)}(t - \tau^{(i)})dt \\ &= S_p^{(i)} + I_{J,p}^{(i)} + \eta^{(i)}, \end{aligned} \tag{5}$$

where

$$S_p^{(i)} = \int_{pT+\tau^{(i)}}^{(p+1)T+\tau^{(i)}} s^{(i)}(t - t^{(i)})X^{(i)}(t - \tau^{(i)})dt \tag{6}$$

is the signal component or the self-interference component,

$$I_{J,p}^{(i)} = \int_{pT+\tau^{(i)}}^{(p+1)T+\tau^{(i)}} \sum_{\substack{j=1 \\ j\neq i}}^{J} s^{(j)}(t - t^{(j)})X^{(i)}(t - \tau^{(i)})dt \tag{7}$$

is the multiple-access interference (MAI) from the other $J-1$ channels, and $\eta^{(i)}$ is the noise component. $\tau^{(i)}$ is the time delay of i-th correlation receiver. If $t^{(i)} = \tau^{(i)}$, then $S_p^{(i)}$ is equal to $d_p^{(i)}N$. Note that $S_p^{(i)}$ is called the *autocorrelation function*, when regarded as a function of a relative time delay $t^{(i)} - \tau^{(i)}$.

The aim of this paper is to give the distribution of the self-interference $S_p^{(i)}$.

3 Spreading Sequences Generated by Markov Chains

Let $\boldsymbol{X} = \{X_n\}_{n=0}^{\infty}$ and $\boldsymbol{Y} = \{Y_n\}_{n=0}^{\infty}$ be sequences of $\{-1,1\}$-valued stationary random variables. Suppose that $\boldsymbol{X}$ and $\boldsymbol{Y}$ are stationary 2-state Markov chains with 2-dimensional transition matrix P, and mutually independent. Let their probabilities be $\text{Prob}\{X_n = -1\} = \text{Prob}\{Y_n = -1\} = \text{Prob}\{X_n = 1\} = \text{Prob}\{Y_n = 1\} = \frac{1}{2}$. Let λ be the eigenvalue of P other than 1. Then the transition matrix P is given by

$$P = \frac{1}{2}\begin{pmatrix} 1+\lambda & 1-\lambda \\ 1-\lambda & 1+\lambda \end{pmatrix}. \tag{8}$$

For simplicity, suppose irreducible, aperiodic Markov chains, then for $\ell, m, k \geq 0$ we have

$$\begin{aligned}
&\mathrm{E}_X[X_n] = \mathrm{E}_Y[Y_n] = 0,\ \mathrm{E}_{XY}[X_n Y_{n+\ell}] = 0,\\
&\mathrm{E}_X[X_n X_{n+\ell}] = \lambda^\ell, \mathrm{E}_Y[Y_n Y_{n+\ell}] = \lambda^\ell,\\
&\mathrm{E}_X[X_n X_{n+\ell} X_{n+\ell+k}] = 0,\\
&\mathrm{E}_X[X_n X_{n+\ell} X_{n+\ell+k} X_{n+\ell+k+m}] = \lambda^{l+m},
\end{aligned} \tag{9}$$

where $\mathrm{E}_Z[\cdot]$ denotes the expected value with respect to the distribution of a random variable Z.

4 Distribution of the Self-Interference

The *aperiodic cross-correlation function* between two binary sequences $\boldsymbol{X}$ and $\boldsymbol{Y}$ is given by

$$R_N^A(\ell; \boldsymbol{X}, \boldsymbol{Y}) = \sum_{n=0}^{N-1-\ell} X_n Y_{n+\ell} \quad (\ell = 0, 1, \cdots, N-1), \tag{10}$$

which is introduced by Pursley[7]. Using this, we get its even and odd cross-correlation functions are respectively defined by

$$R_N^E(\ell; \boldsymbol{X}, \boldsymbol{Y}) = R_N^A(\ell; \boldsymbol{X}, \boldsymbol{Y}) + R_N^A(N-\ell; \boldsymbol{Y}, \boldsymbol{X}), \tag{11}$$
$$R_N^O(\ell; \boldsymbol{X}, \boldsymbol{Y}) = R_N^A(\ell; \boldsymbol{X}, \boldsymbol{Y}) - R_N^A(N-\ell; \boldsymbol{Y}, \boldsymbol{X}). \tag{12}$$

We assume that the relative time delay $t^{(j)} - \tau^{(i)}$ is expressed as $\ell_{ij} + \frac{k_{ij}}{M}$, $\ell_{ij} \in \{0, 1, \cdots, N-1\}$, $k_{ij} \in \{0, 1, \cdots, M-1\}$, ($M$ is some positive integer). Using the *up-sampled sequence by a factor of M* for $\boldsymbol{X}$, defined by

$$\widehat{\boldsymbol{X}} = \{\underbrace{X_0, \cdots, X_0}_{M}, \underbrace{X_1, \cdots, X_1}_{M}, \cdots, \underbrace{X_{N-1}, \cdots, X_{N-1}}_{M}\}, \tag{13}$$

which is regarded as a special kind of Kronecker sequences [8], we have

$$\begin{aligned}
S_p^{(i)} = &\frac{d_p^{(i)} + d_{p+1}^{(i)}}{2} \frac{1}{M} R_{NM}^E(\ell_{ii} M + k_{ii}; \widehat{\boldsymbol{X}}^{(i)}, \widehat{\boldsymbol{X}}^{(i)})\\
&+ \frac{d_p^{(i)} - d_{p+1}^{(i)}}{2} \frac{1}{M} R_{NM}^O(\ell_{ii} M + k_{ii}; \widehat{\boldsymbol{X}}^{(i)}, \widehat{\boldsymbol{X}}^{(i)}).
\end{aligned} \tag{14}$$

For simplicity, we denote ℓ_{ii}, k_{ii}, and $\widehat{\boldsymbol{X}}^{(i)}$ as ℓ, k, and $\widehat{\boldsymbol{X}}$, respectively. $\frac{1}{M} R_{NM}^{E/O}(\ell M + k; \widehat{\boldsymbol{X}}, \widehat{\boldsymbol{Y}})$ has the relation

$$\begin{aligned}
&\frac{1}{M} R_{NM}^{E/O}(\ell M + k; \widehat{\boldsymbol{X}}, \widehat{\boldsymbol{Y}})\\
&\qquad = \left(1 - \frac{k}{M}\right) R_N^{E/O}(\ell; \boldsymbol{X}, \boldsymbol{Y}) + \frac{k}{M} R_N^{E/O}(\ell + 1; \boldsymbol{X}, \boldsymbol{Y}),
\end{aligned} \tag{15}$$

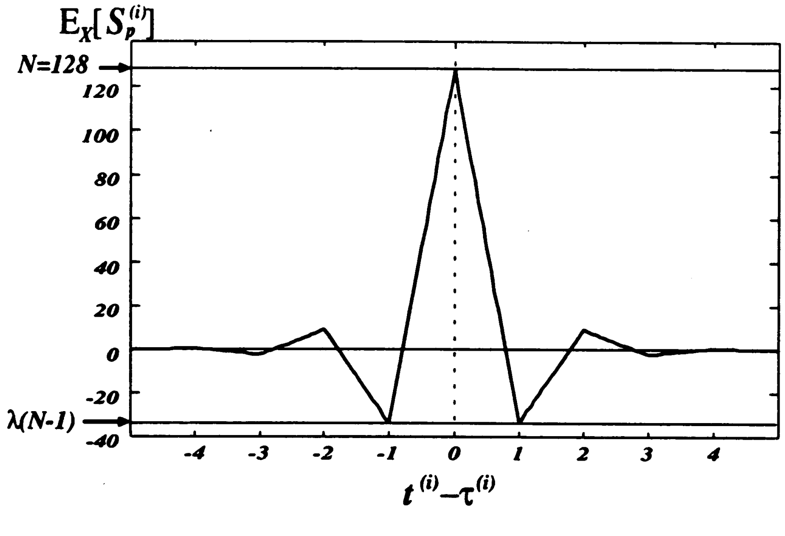

Figure 2: Expectation of Self-Interference

where the superscript E/O denotes either even or odd. Using (9)-(15), we get the expectation of $S_p^{(i)}$ with respect to SS codes $\boldsymbol{X}$ as

$$\mathrm{E}_X[S_p^{(i)}] = d_p^{(i)}\left[\left(1-\frac{k}{M}\right)(N-\ell)\lambda^{\ell}+\frac{k}{M}(N-\ell-1)\lambda^{\ell+1}\right] + d_{p+1}^{(i)}\left[\left(1-\frac{k}{M}\right)\ell\lambda^{N-\ell}+\frac{k}{M}(\ell+1)\lambda^{N-\ell-1}\right]. \quad (16)$$

Figure 2 shows the expectation of $S_p^{(i)}$ for $N = 128$ and $d_p^{(i)} = d_{p+1}^{(i)} = 1$ with the eigenvalue of transition matrix of Markov chain $\lambda = -2+\sqrt{3}$, which minimize MAI[5] and, hence, is regarded as a candidate of SS codes.

Applying the central limit theorem [9] to $\frac{1}{\sqrt{N}}R_N^{E/O}(\ell,\mathbf{X},\mathbf{X})$ in σ_S^2, together with using (15), we get the variance of $S_p^{(i)}/\sqrt{N}$ with respect to $\boldsymbol{X}$, denoted by σ_S^2, as

$$\sigma_S^2 = \left(1-\frac{k}{M}\right)^2\mathcal{G}_+(\ell)+2\frac{k}{M}\left(1-\frac{k}{M}\right)\mathcal{H}_+(\ell)+\left(\frac{k}{M}\right)^2\mathcal{G}_+(\ell+1)$$
$$+d_p^{(i)}d_{p+1}^{(i)}\left[\left(1-\frac{k}{M}\right)^2\mathcal{G}_-(\ell)+2\frac{k}{M}\left(1-\frac{k}{M}\right)\mathcal{H}_-(\ell)+\left(\frac{k}{M}\right)^2\mathcal{G}_-(\ell+1)\right], \quad (17)$$

where $\mathcal{G}_\pm(\ell)$ and $\mathcal{H}_\pm(\ell)$ are given by $\mathcal{G}_\pm(\ell) = (\mathcal{G}^E(\ell)\pm\mathcal{G}^O(\ell))/2$ and $\mathcal{H}_\pm(\ell) = (\mathcal{H}^E(\ell)\pm\mathcal{H}^O(\ell))/2$, where

$$\mathcal{G}^{E/O}(\ell) = \frac{1}{N}\mathrm{E}[R_N^{E/O}(\ell;\boldsymbol{X},\boldsymbol{X})^2]-\frac{1}{N}\mathrm{E}[R_N^{E/O}(\ell;\boldsymbol{X},\boldsymbol{X})]^2, \quad (18)$$

$$\mathcal{H}^{E/O}(\ell) = \frac{1}{N}\mathrm{E}[R_N^{E/O}(\ell;\boldsymbol{X},\boldsymbol{X})R_N^{E/O}(\ell+1,\boldsymbol{X},\boldsymbol{X})] - \frac{1}{N}\mathrm{E}[R_N^{E/O}(\ell;\boldsymbol{X},\boldsymbol{X})]\cdot\mathrm{E}[R_N^{E/O}(\ell+1,\boldsymbol{X},\boldsymbol{X})], \quad (19)$$

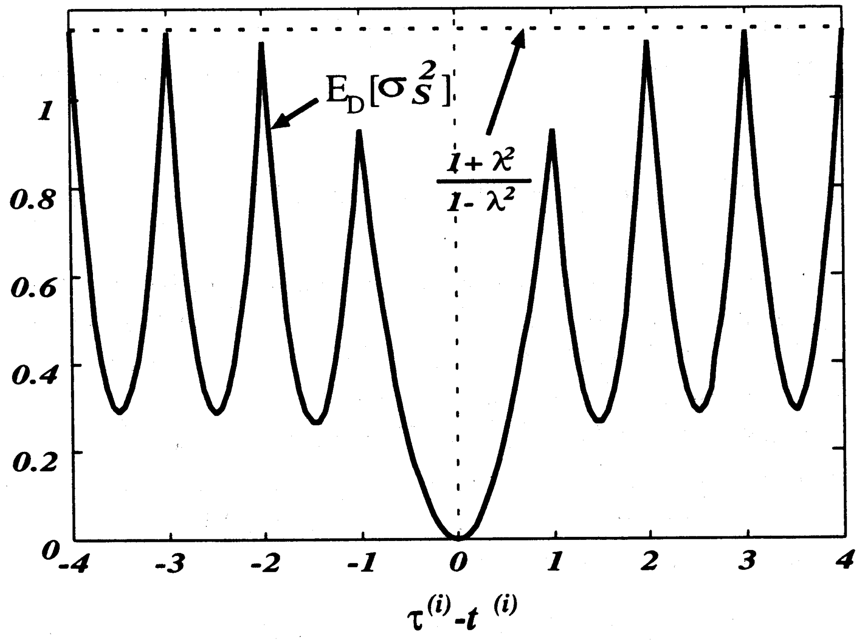

Figure 3: Variance of self-interference

Though precise calculation results are considerably complicated (See Appendix), we can approximate $\mathcal{G}_{\pm}(\ell)$ and $\mathcal{H}_{\pm}(\ell)$ for a sufficiently large N as follows:

$$\mathcal{G}_{+}(\ell) \approx -\left(2\ell + \frac{1+\lambda^2}{1-\lambda^2}\right)\lambda^{2\ell} + \frac{1+\lambda^2}{1-\lambda^2}, \tag{20}$$

$$\mathcal{G}_{-}(\ell) \approx \frac{2\ell}{N}\left((N-2\ell) + \frac{1+\lambda^2}{1-\lambda^2}\right)\lambda^{N-2\ell} \qquad \text{for} \quad 0 \le \ell \le \lfloor \tfrac{N}{2} \rfloor, \tag{21}$$

$$\mathcal{H}_{+}(\ell) \approx -\left((2\ell+1) + \frac{1+\lambda^2}{1-\lambda^2}\right)\lambda^{2\ell+1} + \frac{2\lambda}{1-\lambda^2}, \tag{22}$$

$$\mathcal{H}_{-}(\ell) \approx \frac{2\ell}{N}\left((N-2\ell-1) + \frac{1+\lambda^2}{1-\lambda^2}\right)\lambda^{N-2\ell-1} \quad \text{for} \quad 0 \le \ell \le \lfloor \tfrac{N-1}{2} \rfloor. \tag{23}$$

By (11), (12), (18), and (19), it is obvious that there exist symmetric relations $\mathcal{G}(\ell) = G(N-\ell)$ and $\mathcal{H}(\ell) = H(N-1-\ell)$. Using these relations, we obtain the expressions for $\frac{N}{2} \le \ell \le N$.

5 Bit Error Rate

Finally, we will briefly mention the bit error ratio (BER) of a nearly synchronized correlator, *i.e.*, $\ell = 0$ or $N-1$. Let the time-delay $\varepsilon = \frac{k}{M}$ for $\ell = 0$, and $\varepsilon = 1 - \frac{k}{M}$ for $\ell = N-1$, then we obtain the expectation and the variance of $S_p^{(i)}/\sqrt{N}$ by $\sqrt{N}(1-\varepsilon(1-\lambda))$ and $\varepsilon^2(1-\lambda^2)$, respectively, as N approaches to infinity.

For a 2-user multiple access system, Kohda and Fujisaki [5] gave a simple expression of the variance of $I_{2,p}^{(i)}/\sqrt{N}$ with respect to spreading codes $\boldsymbol{X}$ and $\boldsymbol{Y}$ as the following:

$$\sigma^2 \stackrel{\text{def}}{=} \mathrm{E}_K[\mathrm{E}_D[\mathrm{Var}_{X,Y}[I_{2,p}^{(i)}/\sqrt{N}]]] = \frac{2}{3}\frac{1+\lambda+\lambda^2}{1-\lambda^2}, \tag{24}$$

for $M \gg 1$, where $E_K[\cdot]$ denotes the average with respect to the time delays k_i, which is assumed to take values on $\{0, 1, \cdots, M-1\}$ with equal probability. For general J-user systems, the variance of MAI is given by $(J-1)\sigma^2$ because of the additive property of the Gaussian distribution.

The bit error occurs when $Z_p^{(i)}$ in (5) is positive if $d_p^{(i)} = -1$ (or $Z_p^{(i)} < 0$ if $d_p^{(i)} = +1$). Since $Z_p^{(i)}/\sqrt{N}$ tends to Gaussian distribution with mean $\sqrt{N}(1-\varepsilon(1-\lambda))$ and variance $\sigma_T(\varepsilon,\lambda)^2 = \varepsilon^2(1-\lambda^2) + (J-1)\sigma^2 + \frac{N_0}{2}$ as N approaches to infinity, the BER is given by

$$P_e(\varepsilon;\lambda) = Q\left(\frac{\sqrt{N}(1-\varepsilon(1-\lambda))}{\sigma_T(\varepsilon,\lambda)}\right), \tag{25}$$

where

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{\omega^2}{2}\right] d\omega. \tag{26}$$

Hence we get the expected BER: $\overline{P_e}(\lambda) = 2\int_0^{\frac{1}{2}} P_e(\varepsilon;\lambda)d\varepsilon$. Lastly note that since the classic LFSR-based SS codes are assumed to be approximately identified with sequences of i.i.d. binary random variables, SS codes generated by Markov chains show great promising.

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Appendix Details of $\mathcal{G}_{\pm}(\ell)$ and $\mathcal{H}_{\pm}(\ell)$

Using (9)-(15), we obtain

$$\begin{aligned}\mathcal{G}_+(\ell) &= \left(-2\ell + \frac{3\ell^2}{N}\right)\lambda^{2\ell} + \frac{1+\lambda^2}{1-\lambda^2} - \frac{(N-2\ell)}{N}\frac{1+\lambda^2}{1-\lambda^2}\lambda^{2\ell} \\ &\quad - \frac{4}{N}\frac{\lambda^2(1-\lambda^{2\ell})}{(1-\lambda^2)^2} - \frac{\ell^2}{N}\lambda^{2(N-\ell)} \end{aligned} \tag{27}$$

$$\begin{aligned}\mathcal{G}_-(\ell) &= \frac{2\ell}{N}\frac{1+\lambda^2}{1-\lambda^2}\lambda^{N-2\ell} + 2\frac{(N-2\ell)\ell}{N}\lambda^{N-2\ell} \\ &\quad - \frac{4}{N}\frac{\lambda^{N-2\ell+2}(1-\lambda^{2\ell})}{(1-\lambda^2)^2} - \frac{2(N-\ell)\ell}{N}\lambda^N, \end{aligned} \tag{28}$$

for $1 \le \ell \le \frac{\lfloor N \rfloor}{2}$, and

$$\begin{aligned}\mathcal{H}_+(\ell) &= \left(-(2\ell+1) + \frac{3\ell^2+3\ell+1}{N}\right)\lambda^{2\ell+1} - \frac{(N-2\ell-1)}{N}\frac{1+\lambda^2}{1-\lambda^2}\lambda^{2\ell+1} \\ &\quad + \frac{(N-1)}{N}\frac{2\lambda}{1-\lambda^2} - \frac{4}{N}\frac{\lambda^3(1-\lambda^{2\ell})}{(1-\lambda^2)^2} - \frac{\ell(\ell+1)}{N}\lambda^{2N-2\ell-1} \end{aligned} \tag{29}$$

$$\begin{aligned}\mathcal{H}_-(\ell) &= \frac{(N-2\ell)(2\ell-1)}{N}\lambda^{N-2\ell-1} + \frac{4\ell}{N}\frac{\lambda^{N-2\ell+1}}{1-\lambda^2} - \frac{1}{N}\frac{1+\lambda^2}{1-\lambda^2}\lambda^{N-2\ell+1} \\ &\quad - \frac{(N-\ell)(\ell+1)}{N}\lambda^{N-1} + \frac{\ell(N-\ell-1)}{N}\lambda^{N+1} \end{aligned} \tag{30}$$

for $1 \le \ell \le \frac{\lfloor N-1 \rfloor}{2}$.