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INTRODUCTION TO COOPERATIVE EXTENSIONS OF THE BAYESIAN GAME

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1 INTRODUCTION

The present paper provides an introductory summary of some materials taken from Ichiishi and Yamazaki (2002), a survey paper on the Bayesian cooperative game theory.

The Bayesian cooperative game theory provides foundations of analysis of an economy with organizations as production units, in particular analysis of resource allocation mechanisms instituted in organizations as superior alternatives to the market mechanism. Firms (organizations) in the present-day free societies are interdependent, so we emphasize a general game-theoretical model in which the feasibility and implications of coordinated strategy choice within a coalition are influenced by the outsiders' strategy choice.

While the conventional noncooperative Bayesian analyses sometimes have assumed the presence of a mediator for the firm activities, there is no need for a mediator in the cooperative Bayesian analysis. Indeed, in reality, corporations are operated without consulting with a mediator; the managers at various levels of corporate hierarchy are not mediators but players in a coalition pursuing their own interests.

The first part (section 2) provides the key ingredients. After formulating the basic one-shot model, which synthesizes Harsanyi's (1967/1968) Bayesian game and Aumann and Peleg's (1960) non-side-payment game (NTU game), and illustrating economic examples, two conditions that an endogenously determined strategy is required to satisfy are discussed: measurability with respect to an information structure, and Bayesian incentive compatibility. Several descriptive solution concepts that have been proposed to date are discussed.

The second part (section 3) addresses two of the issues studied in the literature. The first issue is the existence of the descriptive solutions. The second issue is explanation of information revelation, that is, a process through which private information turns into public information. Two approaches have been taken in the past. One approach explains it as a consequence of taking actions, and the other approach explains it as credible information-transmission at the contract negotiation (e.g., credible talk). The former approach is classified into two more specific approaches: information revelation by actions during the contract execution, and information revelation by choosing an *interim* contract.

2 Basic Ingredients

2.1 One-shot model

We construct a general one-shot model appropriate for analyzing cooperative behavior in an environment in which the players are endowed with differentiated information (private information). The required model needs to embody both Harsanyi's Bayesian game and Aumann and Peleg's non-side-payment game (NTU game). We start fixing basic notation.

N: finite set of players.
$\mathcal{N} := 2^N \setminus \{\emptyset\}$: nonempty coalitions.
C^j: choice set (action set).
T^j: finite type set.
$C^S := \prod_{j \in S} C^j$, $T^S := \prod_{j \in S} T^j$, $C := C^N$, $T := T^N$.

$u^j : C\times T \to \mathbf{R}$: type-profile-dependent von Neumann-Morgenstern utility function.

The *ex ante* period is defined as the period in which the players do not have their private information yet, that is, they do not know their own types yet, but have probabilities on the type-profile space, subjective or objective. The *interim* period (or *in mediis* period) is the period in which each player has his private information, that is, he knows his own type, but does not know the others' types. The *ex post* period is the period in which every player knows the exact realized type profile.

Let $\pi^j(\cdot \mid t^j)$ be the probability on $T^{N\setminus\{j\}}$ that player j has in the *interim* period, given his type t^j. If he has an *ex ante* probability π^j on T, the *interim* probability is derived from it by the Bayes' rule,

$$\pi^j(t^{N\setminus\{j\}} \mid t^j) = \frac{\pi^j(t^{N\setminus\{j\}}, t^j)}{\pi^j(T^{N\setminus\{j\}} \times \{t^j\})}.$$

DEFINITION 2.1.1 (Harsanyi, 1967/1968) A *Bayesian game* is a list of specified data, $\{C^j,\ T^j,\ u^j,\ \{\pi^j(\cdot \mid t^j)\}_{t^j\in T^j}\}_{j\in N}$.

An *information structure* is an algebra $\mathcal{A}$ on T. Let $\mathcal{T}^S$ be the information structure generated by the events, $\{t^S\} \times T^{N\setminus S}$, $t^S \in T^S$. Player j's *private information structure* is then given as $\mathcal{T}^j := \mathcal{T}^{\{j\}}$. Notice that a function on T is $\mathcal{T}^S$ -measurable, iff it is a function only of t^S.

For sensible analyses of the workings of a society in which cooperative behavior takes place, we need to introduce the feasibility concept: Let $\mathbf{C}_0^S(t)(\subset C^S)$ be the set of feasible joint choices that coalition S can make when the type profile is t. Notice that

$$\begin{aligned}\mathbf{C}_0^S(t) &\neq \prod_{j\in S} \mathbf{C}_0^j(t) \ \text{ if } \#S \geq 2,\\ \mathbf{C}_0^S(t) &\neq \mathbf{C}_0^S(t') \ \text{ if } t \neq t'.\end{aligned}$$

Complete information is defined as the situation in which there is no private information, that is, $\#T = 1$; in this situation, notation t may be suppressed. In the complete information case, if each player's utility depends only on his choice, that is, if $u^j(c) = u^j(c^j)$, then the set of utility allocations attainable in coalition S is the set,

$$V(S) := \{u \in \mathbf{R}^N \mid \exists\, c^S \in \mathbf{C}_0^S : \forall\, j \in S : u_j \leq u^j(c^j)\}.$$

Set $V(S)$ is a cylinder, based on a subset of $\mathbf{R}^S$.

DEFINITION 2.1.2 (Aumann and Peleg, 1960) A *non-side-payment game* is a correspondence V from $\mathcal{N}$ to $\mathbf{R}^N$, such that $[u, v \in \mathbf{R}^N, \forall\, j \in S : u_j = v_j]$ implies $[u \in V(S)$ iff $v \in V(S)]$.

Player j's *strategy* $x^j : T \to C^j$ is a plan of his choices contingent upon type profiles. Define

$$X^j := \{x^j : T \to C^j\}, \text{ strategy space.}$$
$$X^S := \prod_{j \in S} X^j, \;\; X := X^N.$$

When the members of coalition S jointly choose their strategies, their set of feasible strategies may be constrained by the outsiders' strategy choice. This fact is described by the *feasible-strategy correspondence* $F^S : X \twoheadrightarrow X^S$. Clearly, for each $\bar{x} \in X$,

$$F^S(\bar{x}) \subset \{\mathcal{T}^S\text{-measurable selections of } \mathbf{C}_0^S\}.$$

Many of the works on Bayesian cooperative games done to date assume the objective *ex ante* probability π on the type profile space.

DEFINITION 2.1.3 (Ichiishi and Idzik, 1996) A *Bayesian society* is a list of specified data

$$\mathcal{S} := (\{C^j, T^j, u^j\}_{j \in N},\; \{\mathbf{C}_0^S, F^S\}_{S \in \mathcal{N}},\; \pi).$$

In his pioneering paper, Wilson paid particular attention to the information that each player can use at the time of action.

DEFINITION 2.1.4 (Wilson, 1978) A *communication system* for coalition S is an $\#S$-tuple of algebras $\{\mathcal{A}^j\}_{j \in S}$ on T such that

$$\forall\, j \in S : \mathcal{T}^j \subset \mathcal{A}^j \subset \mathcal{T}^S.$$

It is called *null*, if $\mathcal{A}^j = \mathcal{T}^j$ for all $j \in S$. It is called *full*, if $\mathcal{A}^j = \mathcal{T}^S$ for all

2.2 Examples

EXAMPLE 2.2.1 A *Bayesian pure exchange economy* is a list of specified data,

$$\mathcal{E}_{pe} := (\{\mathbf{R}^l_+, T^j, u^j, e^j, \}_{j \in N}, \pi),$$

where $\mathbf{R}^l_+$ is the consumption set, T^j is the type space, $u^j : \mathbf{R}^l_+ \times T \to \mathbf{R}$ is the utility function, and $e^j : T^j \to \mathbf{R}^l_+$ is the initial endowment of consumer $j \in N$.

The associated Bayesian society,

$$(\{C^j, T^j, u^j\}_{j \in N}, \ \{\mathbf{C}^S_0, F^S\}_{S \in \mathcal{N}}, \ \pi),$$

is defined as follows: N, T^j, u^j and π are given in economy $\mathcal{E}_{pe}$.

$$\begin{aligned} C^j &:= \mathbf{R}^l_+, \\ F^S(\bar{x}) &:= \left\{ x^S : T \to C^S \;\middle|\; \begin{array}{l} x^S \text{ is } \mathcal{T}^S\text{-measurable}, \\ \forall\, t : \sum_{j \in S} x^j(t) \le \sum_{j \in S} e^j(t^j) \end{array} \right\}. \end{aligned}$$

Some works (e.g., Hahn and Yannelis (1997), Vohra (1999) and Yazar (2001)) re-formulate the model so that j's strategy is an excess demand plan, $z^j : t \mapsto x^j(t) - e^j(t^j)$. Demand plan x^j is $\mathcal{T}^S$-measurable iff excess demand plan z^j is $\mathcal{T}^S$-measurable. But choice of demand plan versus excess demand plan as a strategy affects some results. □

EXAMPLE 2.2.2 A *Bayesian coalition production economy* is a list of specified data,

$$\mathcal{E}_{cp} := \left(\{\mathbf{R}^l_+, T^j, u^j, e^j\}_{j \in N}, \pi, \{Y^S\}_{S \in \mathcal{N}}\right).$$

The associated Bayesian society,

$$(\{C^j, T^j, u^j\}_{j \in N}, \ \{\mathbf{C}^S_0, F^S\}_{S \in \mathcal{N}}, \ \pi),$$

is defined as follows: N, T^j, u^j and π are given in economy $\mathcal{E}_{cp}$.

$$\begin{aligned} C^j &:= \mathbf{R}^l_+, \\ F^S(\bar{x}) &:= \left\{ x^S : T \to C^S \;\middle|\; \begin{array}{l} x^S \text{ is } \mathcal{T}^S\text{-measurable}, \\ \exists\, y : T \to \mathbf{R}^l,\ \mathcal{T}^S\text{-measurable}, \\ \forall\, t \in T : y(t) \in Y^S(t), \\ \sum_{j \in S} x^j(t) \le y(t) + \sum_{j \in S} e^j(t^j) \end{array} \right\}. \end{aligned}$$

EXAMPLE 2.2.3 Chandler's (1962) firm in multidivisional form (M-form firms) is formulated as a particular instance of the Bayesian coalition production economy, which has a more specific structure (to be specified in subsection 3.2.1). Here, distinction of a marketed commodity and a nonmarketed commodity is essential: While the former has a price established in the market outside the firm, the latter is produced or traded only for internal use, so does not have a price. Various technological states are considered the possible types of each division. Define

N: profit-centers (semiautonomous decisionmakers) in the M-form firm.
k_m: # of the marketed commodities.
k_n: # of the nonmarketed commodities.
$r^j : T^j \to \mathbf{R}^{k_n}$: the resource function.

A *profit center game with incomplete information* of Ichiishi and Radner (1999) is a list of specified data $\mathcal{D} := (\mathcal{E}_{cp}, p)$ of Bayesian coalition production economy $\mathcal{E}_{cp}$ and price vector p for the marketed commodities. Here,

$$\begin{aligned} \mathcal{E}_{cp} &:= \left(\{\mathbf{R}^{k_m+k_n}, T^j, \text{ profit function}, r^j\}_{j\in N},\ \{\pi^j\}_{j\in N}, \{Y^j\}_{j\in N}\right), \\ p &\in \mathbf{R}^{k_m}_{+}, \end{aligned}$$

so the *ex ante* probability on T is the product probability of π^j's.

The technology of each division, which determines the production set $Y^j(t)$ is embodied in its assets (resources $r^j(t)$); this fact is called the *asset specificity*. Therefore, it makes sense to postulate that $r^j(\cdot)$ is 1-1 on T^j.

A *transfer payment problem* is how to determine prices of nonmarketed commodities transferred from one division to another. The cooperative game played by the divisions determines these prices. □

2.3 Measurability as a feasibility requirement

Suppose that the grand coalition N is entertaining a strategy bundle $\bar{x} : T \to C$, but that S may defect and take $x^S : T \to C^S$. The following condition says that a player cannot take different actions contigent on two type profiles which he cannot discern.

CONDITION 2.3.1 (Radner, 1967) Suppose that communication system $\{\mathcal{A}^j\}_{j\in S}$ will be available at the time of action (strategy execution). Then, members of S can take only those strategies $x^S \in F^S(\bar{x})$ such that x^j is $\mathcal{A}^j$-measurable for every $j \in S$.

The *private information case* is defined as the situation in which the null communication system $\{\mathcal{T}^j\}_{j\in S}$ is available at the time of action. In accordance with the private information case, Yannelis (1991) introduced the *private measurability* condition on strategies that j's strategy x^j be $\mathcal{T}^j$-measurable for every $j \in S$. Define

$$F'^S(\bar{x}) := \left\{x^S \in F^S(\bar{x}) \;\middle|\; x^j \text{ is } \mathcal{T}^j\text{-measurable for all } j \in S\right\}.$$

2.4 Bayesian incentive compatibility

2.4.1 Private information case

We present the other basic requirement (Bayesian incentive compatibility) for the private information case first. Suppose that the grand coalition N is entertaining a strategy bundle $\bar{x} : T \to C$, but that S may defect and take $x^S : T \to C^S$. Let $\{\bar{t}^j\}_{j\in S}$ be S's true type profile.

If player j takes honest action $x^j(\bar{t}^j)$, his conditional expected utility at that time (i.e., given $\bar{t}^j$) is

$$\begin{aligned} &Eu^j(x^S, \bar{x}^{N\setminus S} \mid \bar{t}^j) \\ &:= \sum_{t^{N\setminus\{j\}}\in T^{N\setminus\{j\}}} u^j(x^j(\bar{t}^j), x^{S\setminus\{j\}}(t^{S\setminus\{j\}}), \bar{x}^{N\setminus S}(t^{N\setminus S}), (\bar{t}^j, t^{N\setminus\{j\}}))\pi(t^{N\setminus\{j\}} \mid \bar{t}^j). \end{aligned}$$

If, on the other hand, he takes a wrong action $c^j \in x^j(T^j) \setminus \{x^j(\bar{t}^j)\}$, then, assuming the others' honest action, his conditional expected utility given $\bar{t}^j$ becomes

$$\begin{aligned} &Eu^j(c^j, x^{S\setminus\{j\}}, \bar{x}^{N\setminus S} \mid \bar{t}^j) \\ &:= \sum_{t^{N\setminus\{j\}}\in T^{N\setminus\{j\}}} u^j(c^j, x^{S\setminus\{j\}}(t^{S\setminus\{j\}}), \bar{x}^{N\setminus S}(t^{N\setminus S}), (\bar{t}^j, t^{N\setminus\{j\}}))\pi(t^{N\setminus\{j\}} \mid \bar{t}^j). \end{aligned}$$

For truthful execution of the plan, the members in S agree only on a strategy bundle such that taking a wrong action is not worthwhile. Ichiishi and Idzik (1996), whose first draft had been circulated since Summer 1991, introduced the following Bayesian incentive compatibility condition to the Bayesian core analysis (or more generally, to the Bayesian strong equilibrium analysis).

CONDITION 2.4.1 (d'Aspremont and Gérard-Varet, 1979) In the private information case, members of S agree only on those strategies $x^S \in F'^S(\bar{x})$ that are *Bayesian incentive-compatible*, that is,

$$\begin{aligned} &\forall\, j \in S : \forall\, \bar{t}^j \in T^j : \forall\, c^j \in x^j(T^j) : \\ &Eu^j(x^S, \bar{x}^{N\setminus S} \mid \bar{t}^j) \geq Eu^j(c^j, x^{S\setminus\{j\}}, \bar{x}^{N\setminus S} \mid \bar{t}^j). \end{aligned}$$

Define

$$\hat{F}^S(\bar{x}) := \{x^S \in F'^S(\bar{x}) \mid x^S \text{ is Bayesian incentive-compatible.}\}.$$

Nonemptiness of the set $\hat{F}^S(\bar{x})$ is easily guaranteed. Indeed, if $x^S \in F^S(\bar{x})$ is a constant function, then $x^S \in \hat{F}^S(\bar{x})$.

Within the framework of Bayesian pure exchange economy, in which an excess demand plan is a strategy, Hahn and Yannelis observed that private measurability implies Bayesian incentive compatibility.

PROPOSITION 2.4.2 (Hahn and Yannelis, 1997) *Let $\mathcal{E}_{pe}$ be a Bayesian pure exchange economy in the private information case, in which player j's strategy is j's excess demand plan z^j, and the coalitional feasibility is defined by the exact equality,*

$$\forall\, t \in T : \sum_{j \in S} z^j(t) = \mathbf{0}.$$

Then, private measurability implies Bayesian incentive compatibility.

This proposition is no longer valid if a demand plan x^j is used as a strategy, as the following example shows:

EXAMPLE 2.4.3 Consider the Bayesian pure exchange economy with one commodity ($l = 1$), two types for each consumer ($T^j = \{a^j, b^j\}$), and risk-neutral utility function ($u^j(c^j, t) = c^j$), and suppose that the initial endowment is given as

$$e^j(t^j) = \begin{cases} 1, & \text{if } t^j = a^j, \\ 2, & \text{if } t^j = b^j. \end{cases}$$

Then the initial endowment bundle e is attainable with equality in N and satisfies measurability. But

$$Eu^j(e^j(b^j) \mid a^j) = 2 > 1 = Eu^j(e^j \mid a^j),$$

so strategy e^j is not Bayesian incentive-compatible. □

Moreover, proposition 2.4.2 is not valid either in the general model of Bayesian society $\mathcal{S}$.

REMARK 2.4.4 The revelation principle does not work here, that is, Bayesian incentive compatibility cannot be assumed without loss of generality. To see this point, consider example 2.4.3. The initial endowment bundle e as a strategy bundle satisfies feasibility with exact equality and private measurability. Player j's best action is to always report b^j. Let $\sigma^j : t^j \mapsto b^j$ the constant pretension function. The resulting strategy $e^j \circ \sigma^j : t^j \mapsto 2$ is private measurable and Bayesian incentive-compatible, yet it is not feasible. □

2.4.2 Non-private information case

In an attempt to endogenously determine Wilson's communications system (definition 2.1.4), Yazar (2001) considered another cooperative game played in the Bayesian pure exchange economy $\mathcal{E}_{pe}$. Each player j's strategy is a pair $(z^j, \mathcal{C}^j)$ of a net trade plan $z^j : T \to \mathbf{R}^l$ and a communication plan $\mathcal{C}^j$ $(\subset \mathcal{T}^j)$. By choosing this strategy, he makes the information structure $\mathcal{C}^j$ available to everybody in his coalition. Choice of $\{z^j, \mathcal{C}^j\}_{j\in S}$ results in the communication system $\{\mathcal{A}^j\}_{j\in S}$ defined by $\mathcal{A}^j := \mathcal{T}^j \vee (\vee_{i\in S}\mathcal{C}^i)$. The plan z^j is $\vee_{i\in S}\mathcal{C}^i$-measurable. Denote by $C^j(t^j)$ the minimal element of $\mathcal{C}^j$ that contains t^j.

Suppose that the members of coalition S have chosen strategy bundle $\{z^j, \mathcal{C}^j\}_{j\in S}$, and let $\bar{t} := \{\bar{t}^j\}_{j\in S}$ be the true type profile. Player j knows realization of $E := \{\bar{t}^j\} \times T^{N\setminus\{j\}}$. Suppose j thinks that $t \in E$ occurred. Assuming that his colleagues i are sending information honestly through the promised communication plan $\mathcal{C}^i$, player j receives the information that event $\prod_{i\in S} C^i(t^i)$ has occurred. By $\vee_{i\in S}\mathcal{C}^i$-measurability, plan z^j is constant on $\prod_{i\in S} C^i(t^i)$. His conditional expected utility of the honest action given $\bar{t}^j$, $Eu^j(z^j(t) + e^j(\bar{t}^j) \mid \bar{t}^j)$ is thus determined.

If player j sends false information $C'^j \times T^{N\setminus\{j\}} \in \mathcal{C}^j$, then (assuming that his colleagues i are sending information honestly) he receives the information that event $E' := C'^j \times \prod_{i\in S\setminus\{j\}} C^i(t^i)$ has occurred. Plan $z^j(t)$ is constant on E'. His conditional expected utility of the wrong action given $\bar{t}^j$, $Eu^j(z^j(E') + e^j(\bar{t}^j) \mid \bar{t}^j)$ is thus determined. Yazar's Bayesian incentive compatibility condition guarantees that j's offer of communication plan $\mathcal{C}^j$ to his colleagues is credible, and that j acts honestly.

DEFINITION 2.4.5 (Yazar, 2001) Coalition S's strategy bundle $\{z^j, C^j\}_{j \in S}$ in the Bayesian pure exchange economy is *Bayesian incentive-compatible*, in the sense that

$$\neg \exists j \in S : \exists \tilde{t}^j \in T^j : \exists C' \in \mathcal{C}^j : \forall t \in \{\tilde{t}^j\} \times T^{N \setminus \{j\}} :$$
$$Eu^j(z^j(E') + e^j(\tilde{t}^j) \mid \tilde{t}^j) > Eu^j(z^j(t) + e^j(\tilde{t}^j) \mid \tilde{t}^j).$$

where $E' := C' \times \prod_{i \in S \setminus \{j\}} C^i(t^i)$.

We present Vohra's (1999) *mediator-based approach* to $\mathcal{E}_{pe}$. The role of a mediator is essentially to remove the need for private measurability. Here is the scenario:

1. S designs $z^S + e^S \in F^S$. Notice $\mathcal{T}^S$-measurability of each z^j (rather than $\mathcal{T}^j$-measurability).
2. Player j confidentially reports t^j to the mediator.
3. The mediator has reports t^S.
4. The mediator tells j to make choice $z^j(t^S)$.

Let $\bar{t}^S$ be the true type profile. Honest report in stage 2 gives him the conditional expected utility, $Eu^j(z^j + e^j \mid \bar{t}^j)$. On the other hand, dishonest report of $\tilde{t}^j$ in stage 2 gives him the conditional expected utility, $Eu^j(z^j(\tilde{t}^j, \cdot) + e^j \mid \bar{t}^j)$. The mediator is simply an enforcement agency; he does not know the true type profile. So the members of coalition S designs a plan z^S in stage 1 which induces honest report in stage 2.

DEFINITION 2.4.6 (Vohra, 1999) Strategy bundle $z^S \in F^S - \{e^S\}$ is *Bayesian incentive-compatible*, in the sense that

$$\neg \exists j \in S : \exists \bar{t}^j : \exists \tilde{t}^j :$$
$$Eu^j(z^j(\tilde{t}^j, \cdot) + e^j \mid \bar{t}^j) > Eu^j(z^j + e^j \mid \bar{t}^j).$$

In spirit, Yazar's strategy with the full communication plan $(z^j, \mathcal{T}^j)$ is identical to Vohra's strategy z^j (although the detailed formulation is different). Thus, Yazar's model may be considered an extension of Vohra's model; the former allows for an arbitrary communication plan.

To assume presence of a mediator is a step backward, since there is no mediator in reality. We present an alternative scenario for Vohra's approach; it is intended to eliminate the mediator.

1. S designs $z^S + e^S \in F^S$.
2. Players independently and simultaneously report t^j's each other.
3. The players have updated information t^S.
4. Player j makes the promised choice $z^j(t^S)$, using the full communication plan.

There is a vital problem about this scenario: In stage 2, decision is made at the *interim* stage in which player has only the *interim* conditional probability $\pi^j(\cdot \mid \bar{t}^j)$ given his true type $\bar{t}^j$. At this time he has not made action yet. In stage 4, decision is made at the *ex post* stage in which player knows the information $\bar{t}^S$. Upon receiving a sharper information in stage 4, he may realize that his decision in stage 2 turns out to be suboptimal. This problem is illustrated in the following example.

EXAMPLE 2.4.7 Consider the Bayesian pure exchange economy, given by $l = 1$, $\#N = \#T^j = 2$, $e^j(t^j) = 1$, $u^j(c^j, t) = c^j$, $\pi(t) = 1/4$ for all t. Consider the plan $z^N := \{(z^1(t), z^2(t))\}_{t \in T} \in F^N - \{e^N\}$ given in the following table:

	t_1^2	t_2^2
t_1^1	(-1, 1)	(1, -1)
t_2^1	(1, -1)	(-1, 1)

This strategy satisfies Vohra's Bayesian incentive compatibility (condition 2.4.6). Let $\bar{t}$ be any true type profile, say $\bar{t} = (t_1^1, t_1^2)$. In stage 4, consumer 1 regrets his report in stage 2, breaks off from the coalition, taking back his initial endowment. □

The mediator-based approach without a mediator thus postulates a corporate (coalitional) atmosphere which forces its members to always act according to an agreed upon strategy bundle. It is this invisible enforcement atmosphere that we label as the "mediator." In reality, however, the effectiveness of this kind of mediator is questionable.

The private information case, together with the associated private measurability condition, postulates the safe attitude of each coalition that it avoids to design those mechanisms that could make its member reluctant to act at the time of contract execution.

2.5 Descriptive solution concepts

Most works in this area postulated that each player plays both the role of principal and the role of agent: Players get together to make coordinated strategy choice as principals. They decide on their self-sustaining strategy bundles (descriptive solution of the game). Each player execute his agreed strategy as an agent in the *interim* period. The solution is called *ex ante* (*interim*, resp.), if it is agreed upon in the *ex ante* period (in an *interim* period, resp.). The theory intends to endogenously determine a mechanism (solution) without a mediator.

2.5.1 *Interim* solution concepts

Given Bayesian pure exchange economy $\mathcal{E}_{pe}$ and any $E \subset T$, define

$$F_E^S := \left\{ x^S : E \to \mathbf{R}^{l \cdot \# S} \;\middle|\; \forall\, t \in E : \sum_{j \in S} x^j(t) = \sum_{j \in S} e^j(t) \right\}.$$

In the first solution concept, players use only of the null communication system. It is easy to incorporate private measurability and Bayesian incentive compatibility in this solution concept.

DEFINITION 2.5.1 (Wilson, 1978) A commodity allocation plan x^* is said to be in the *coarse core* of $\mathcal{E}_{pe}$, if
(i) $x^* \in F_T^N$; and
(ii) it is not true that

$$\begin{aligned} &\exists\, S \in \mathcal{N} : \exists\, E \in \wedge_{j \in S} \mathcal{T}^j : \exists\, x^S \in F_E^S : \forall\, j \in S : \forall\, t \in E : \\ &\quad Eu^j(x^j \mid \mathcal{T}^j)(t) > Eu^j(x^{*j} \mid \mathcal{T}^j)(t). \end{aligned}$$

In the present extreme case ($\mathcal{T}^i \wedge \mathcal{T}^j = \{\emptyset, T\}$ if $i \neq j$), condition (ii) becomes:

$$\begin{aligned} &\neg\, \exists\, j \in N : \exists\, t^j \in T^j : \exists\, x^j \in F^j_{\{t^j\} \times T^{N \setminus \{j\}}} : \\ &\quad Eu^j(x^j \mid t^j) > Eu^j(x^{*j} \mid t^j), \\ &\neg\, (\exists S : \# S \geq 2) : \exists\, x^S \in F_T^S : \forall\, j \in S : \forall\, t^j \in T^j : \\ &\quad Eu^j(x^j \mid t^j) > Eu^j(x^{*j} \mid t^j). \end{aligned}$$

Notice the subtle difference from the *ex ante* coalitional stability condition,

$$\neg\, \exists S : \exists\, x^S \in F_T^S : \forall\, j \in S : \forall\, t^j \in T^j : Eu^j(x^j \mid t^j) > Eu^j(x^{*j} \mid t^j).$$

In the second solution, players are given a set of feasible communication systems, and use them in determining their strategy bundle.

DEFINITION 2.5.2 (Wilson, 1978) A commodity allocation plan x^* is said to be in the *fine core* of $\mathcal{E}_{pe}$ with families $C(S)$ of feasible communication systems, $S \in \mathcal{N}$, if
(i) $x^* \in F_T^N$; and
(ii) if it is not true that

$$\begin{aligned} &\exists\, S \in \mathcal{N} : \exists\, \{\mathcal{A}^j\}_{j \in S} \in C(S) : \exists\, E \in \wedge_{j \in S} \mathcal{A}^j : \\ &\exists\, x^S \in F_E^S : \forall\, j \in S : \forall\, t \in E \\ &\qquad Eu^j(x^j \mid \mathcal{A}^j)(t) > Eu^j(x^{*j} \mid \mathcal{A}^j)(t). \end{aligned}$$

Here, $C(S) \ni$ the full communication system.

Wilson left open the important question: to clarify the process according to which the members of coalition S come to be endowed with communication systems $C(S)$. Since $C(S)$ contains the full communication system, condition (ii) for the fine core includes:

$$\begin{aligned} &\neg\, \exists\, S \in \mathcal{N} : \exists\, t^S \in T^S : \exists\, x^S \in F^S_{\{t^S\} \times T^{N \setminus S}} : \\ &\forall\, j \in S : \quad Eu^j(x^j \mid t^S) > Eu^j(x^{*j} \mid t^S). \end{aligned}$$

Notice the asymmetry in the grand coalition and the blocking coalition in this condition; while the grand coalition needs to design a choice bundle for every possible type profile, the blocking coalition S needs only to make a choice given t^S. If we remove this asymmetry and incorporate the two basic requirements on the strategy, private measurability and Bayesian incentive compatibility, we obtain the *interim* Bayesian incentive-compatible core concept. Because of its importance, we present the concept extended to the Bayesian society.

DEFINITION 2.5.3 Let $\mathcal{S}$ be a Bayesian society in the private information case. A strategy bundle $x^* \in X$ is called an *interim* Bayesian incentive-compatible strong equilibrium, if
(i) $x^* \in \hat{F}^N(x^*)$; and
(ii) it is not true that

$$\exists\, S \in \mathcal{N} : \exists\, t^S \in T^S : \exists\, x^S \in \hat{F}^S(x^*) :$$
$$\forall\, j \in S : \quad Eu^j(x^S, x^{*N \setminus S} \mid t^j) > Eu^j(x^* \mid t^j).$$

A very specific instance of this concept was used in Ichiishi and Sertel's (1998) study of a profit-center game. No general existence theorem has been established for the *interim* Bayesian incentive-compatible strong equilibrium.

For completeness, we present a solution concept based on the mediator-based approach to the Bayesian pure exchange economy. Define for each $E \in \mathcal{T}^S$:

$$F_E^{ic,S} := \left\{ x^S \in F_E^S \;\middle|\; \begin{array}{l} x^S \text{ is } \mathcal{T}^S\text{-measurable, and} \\ \forall\, j \in S : x^j - e^j \text{ is Bayesian incentive-} \\ \text{compatible (condition 2.4.6).} \end{array} \right\}.$$

DEFINITION 2.5.4 (Vohra, 1999) A commodity allocation plan z^* is said to be in the *coarse core* of $\mathcal{E}_{pe}$, if
(i) $z^* \in F_T^{ic,N} - \{e^N\}$; and
(ii) if it is not true that there exist $S \in \mathcal{N}$, $E \in \wedge_{j \in S} \mathcal{T}^j$, and $z^S \in F_E^{ic,S} - \{e^S\}$ such that

$$\forall\, j \in S : \forall\, t \in E :$$
$$Eu^j(z^j + e^j \mid \mathcal{T}^j)(t) > Eu^j(x^{*j} + e^j \mid \mathcal{T}^j)(t).$$

2.5.2 *Ex ante* solution concepts

We turn to *ex ante* solution concepts. For Bayesian pure exchange economy $\mathcal{E}_{pe}$, define:

$$F'^S := \left\{ x^S : T \to \mathbf{R}^{l \cdot \# S} \;\middle|\; \begin{array}{l} \forall\, j \in S : \\ x^j \text{ is } \mathcal{T}^j\text{-measurable,} \\ \forall\, t : \sum_{j \in S} x^j(t) \le \sum_{j \in S} e^j(t) \end{array} \right\}.$$

DEFINITION 2.5.5 (Yannelis, 1991) A commodity allocation plan x^* is called a *private information core allocation* of $\mathcal{E}_{pe}$ in the private information case, if
(i) $x^* \in F'^N$: and
(ii) it is not true that

$$\exists\, S \in \mathcal{N} : \exists\, x^S \in F'^S : \forall\, j \in S :$$
$$Eu^j(x^j) > Eu^j(x^{*j}),$$

where $Eu^j(x^j)$ is the *ex ante* expected utility of x^j.

DEFINITION 2.5.6 (Ichiishi and Idzik, 1996) A strategy bundle $x^* \in X$ of $\mathcal{S}$ is called an *ex ante Bayesian incentive-compatible strong equilibrium*, if
(i) $x^* \in \hat{F}^N(x^*)$; and
(ii) it is not true that

$$\exists\, S \in \mathcal{N} : \exists\, x^S \in \hat{F}^S(x^*) : \forall\, j \in S :$$
$$Eu^j(x^S, x^{*N \setminus S}) > Eu^j(x^*).$$

REMARK 2.5.7 In both the original definitions of the *ex ante* private core (definition 2.5.5) and the *ex ante* Bayesian incentive-compatible strong equilibrium (definition 2.5.6), the inequality in (ii) was replaced by:

$$\forall\, t^j \in T^j : \ Eu^j(x^S, x^{*N \setminus S} \mid t^j) \geq Eu^j(x^* \mid t^j),$$

with strict inequality for at least one t^j. The present condition (ii) is stronger. The existence proofs of Yannelis (1991) and Ichiishi and Idzik (1996) actually establish the existence of these stronger solutions. □

Another interactive mode studied to date is a multi-principal, multi-agent relationship. While there is no general theory of this mode, Ichiishi and Koray (2000) studied a specific model of education, a version of Spence' model. In their model, the first-stage game played by the principals have the same feature as the prisoner's dilemma game, so there exists no cooperative equilibrium.

3 Issues to Address

3.1 Existence

Wilson (1978) established a coarse core nonemptiness theorem, by applying Scarf's core nonemptiness theorem for balanced games (see, e.g., Scarf (1973, theorem 8.3.6, p. 211)). In regard to emptiness of the fine core, Wilson made the following observation on a specific numerical example of $\mathcal{E}_{pe}$: By considering the blocking behavior of coalitions using the full communication system, the initial endowment is shown to be the only candidate for an unblocked allocation. But the initial endowment is blocked by the grand coalition using its null communication system. Yannelis (1991) established a private core nonemptiness theorem for $\mathcal{E}_{pe}$.

THEOREM 3.1.1 (Ichiishi and Idzik, 1996) *Let $\mathcal{S}$ be a Bayesian society in the private information case. Assume:*
(i) C^j*: a nonempty, compact, convex, and metrizable subset of a Hausdorff locally convex topological vector space over* $\mathbf{R}$*;*
(ii) $u^j(\cdot, t)$*: continuous and linear affine in* C^j*;*
(iii) $\mathbf{C}_0^S(t)$*: nonempty, closed and convex;*
(iv) F^S*: both upper and lower semicontinuous in X, and has nonempty, closed and convex values;*
(v) *for any $\bar{x} \in X$ and any balanced family $\mathcal{B}$ with the associated balancing coefficients $\{\lambda_S\}_{S \in \mathcal{B}}$, it follows that*

$$\sum_{S \in \mathcal{B}} \lambda_S \tilde{F}^S(\bar{x}) \subset F^N(\bar{x}),$$

where $\tilde{F}^S(\bar{x}) := \{x \mid x^S \in F^S(\bar{x}),\ x^{N \setminus S} = \mathbf{0}\}$*;*
(vi) *either F^S is a constant correspondence, or for any $\bar{x} \in X$, there exists $\hat{x}^S \in F^S(\bar{x})$, such that for all $j \in S$ and all $\bar{t}^j, \tilde{t}^j \in T^j$ for which $\bar{t}^j \neq \tilde{t}^j$,*

$$Eu^j(\hat{x}^j \mid \bar{t}^j) > Eu^j(\hat{x}^j(\tilde{t}^j) \mid \bar{t}^j).$$

Then, there exists a Bayesian incentive-compatible strong equilibrium of $\mathcal{S}$.

The affine linearity condition (ii) on $u^j(\cdot, t)$ means risk-neutrality, but of course it is automatically satisfied if the players use mixed choices. It is known to be crucial when Bayesian incentive compatibility is involved (the story is different for the pure exchange economy in the private information case in which the players decide on excess demand plans as strategies). Indeed, the main result of Vohra (1999) is an example of an empty mediator-based Bayesian incentive-compatible coarse core.

Forges, Mertens and Vohra (2000) studied the mediator-based Bayesian pure exchange economy with correlated choices (probabilities on choice bundles). While their framework cannot be directly included in the Bayesian society (since no individually taken strategy here, we can imbed it in a certain Bayesian society. The imbedded model, while satisfying assumption (ii) of theorem 3.1.1, does not satisfy assumption (v) of theorem 3.1.1. Indeed, Forges, Mertens and Vohra (2000) provided an example which has no Bayesian incentive-compatible core allocation.

3.2 Approaches to information revelation

Each player j is endowed with his private information structure $\mathcal{T}^j$, so he knows his true type $\bar{t}^j$ at the beginning of the *interim* period. By the time the strategy execution is over, player j will have narrowed down the range of his colleague i's possible true types to a subset A_i^j of T^i. In other words, while the players start with the null communication system $\{\mathcal{T}^j\}_{j\in N}$, they end up with an endogenously determined finer communication system $\{\mathcal{A}^j\}_{j\in N}$. This information revelation process is not easy to analyze, since a player j may not want to pass on his private information to his colleagues, and even if j decides to do so, his colleagues may think that j is not truthfully passing on his information but is trying to manipulate them with false information. This subsection will review two approaches taken in the literature for endogenous determination of an information structure: passive information revelation by action; and active information revelation by credible transmission of information (e.g., by credible talking). The first approach is classified into two specific approaches: information revelation by contract execution, and information revelation by choosing a contract.

3.2.1 By actions during the contract execution

This approach borrows the idea from the rational expectations equilibrium. In order to see before and after the information processing, Ichiishi, Idzik and Zhao's (1994) constructed a two-*interim*-period model of Bayesian society $\mathcal{S}$.

Let $C^j = C_1^j \times C_2^j$. Set C_k^j is the choice set for *interim* period $k = 1, 2$. For any function $f : T \to Z$, let $\mathcal{A}(f)$ be the smallest algebra that contains $\{f^{-1}(z) \mid z \in Z\}$.

POSTULATE 3.2.1 (Information-Revelation Process) Given any strategy bundle $\bar{x} \in X$, coalition S designs only those $x^S \in F^S(\bar{x})$ such that for all $j \in S$
(i) x_1^j is measurable with respect to $\mathcal{T}^j$, and
(ii) x_2^j is measurable with respect to $\mathcal{T}^j \vee \mathcal{A}(x_1^S)$.

Denote by $F'^S(\bar{x})$ the set of $x^S \in F^S(\bar{x})$ that satisfy the information-revelation process (postulate 3.2.1). We can define Bayesian incentive compatibility in this two-period framework, although precise definition is a little involved. Denote by $\hat{F}^S(\bar{x})$ the set of $x^S \in F'^S(\bar{x})$ that satisfy Bayesian incentive compatibility.

Define an *ex ante Bayesian incentive-compatible strong equilibrium* of $\mathcal{S}$ using correspondences $\hat{F}^S$ as in definition 2.5.6.

A Bayesian society studied here is a specified list of data,

$$\mathcal{S} := (\{C^j, T^j, u^j\}_{j \in N}, \{C_0^S, F^S\}_{S \in \mathcal{N}},\ \pi).$$

Fix $(\{C^j, T^j, u^j\}_{j \in N},\ \{C_0^S\}_{S \in \mathcal{N}},\ \pi)$, and vary $\{F^S\}_{S \in \mathcal{N}}$. We then have different Bayesian societies, hence the space of Bayesian societies. Endow $\mathrm{SPACE_{ne}}$ with a natural pseudo-metric d. The following a generic existence theorem for a full-information revealing Bayesian incentive-compatible strong equilibrium.

THEOREM 3.2.2 (Ichiishi, Idzik and Zhao, 1994) *There exists an open and dense subset $SPACE'_{\mathrm{ne}}$ of $SPACE_{\mathrm{ne}}$ such that for each $\mathcal{S} \in SPACE'_{\mathrm{ne}}$ there exist multitude of Bayesian incentive compatible strong equilibria, and there exists a Bayesian incentive compatible strong equilibrium x^* such that x_1^{*j} is 1-1 on T^j.*

Ichiishi and Radner (1999) studies information revelation problem within the two-*interim*-period framework of profit-center game with incomplete information (example 2.2.3) $(\mathcal{E}_{cp}, p)$. They established exact existence theorems for a full-information revealing Bayesian incentive compatible core strategy of $(\mathcal{E}_{cp},\ p)$. Their three theorems postulate (1) a convex technology, (2) a specific instance of increasing returns to scale, that is, a stronger version of Scarf's (1986) distributiveness condition, and (3) for a specific supplier-customer relationship among the divisions, respectively.

3.2.2 By choosing a contract

We present the idea of information revelation by choosing a contract. This idea has been floating around the literature as a folklore; there is no explicit written work. The point here is that agreeing or refusing to sign a contract reveals private information. It applies to *interim* contracting in the private information case. We will see the idea by two examples. Both examples are a simple Bayesian society, in which

$$\begin{aligned}
&C^j = \mathbf{R},\\
&u^j(c,t) = c^j,\\
&S := \{1,2\},\\
&T^i = \{H^i, L^i\},\\
&\pi^i(H^i) = \pi^i(L^i) = 1/2 \text{ for each } i \in S,
\end{aligned}$$

and the grand coalition is deliberating on the constant strategy bundle x:

$$\forall\, j \in N : \forall\, t \in T : x^j(t) = 1.$$

EXAMPLE 3.2.3 This example shows that coalition formation could be made more difficult as a result of the information revelation. Suppose that S finds the following strategy bundle $x'^S \in F^S(x)$:

	H^2	L^2
H^1	(0, 0)	(4, 0)
L^1	(0, 4)	(0, 0)

Then,

$$\forall\, i \in S : E(x'^i \mid H^i) = 2 > 1 = E(x^i \mid H^i).$$

So S can improve upon x using x'^S when the true type-profile is $\bar{t}^S = (H^1, H^2)$.

However, player 1 knows that player 2 agrees to the joint strategy x'^S only when 2's true type is H^2, since

$$E(x'^2 \mid L^2) = 0 < 1 = E(x^2 \mid L^2).$$

Then player 2's agreement to x'^S reveals the information to player 1 that 2's true type is H^2. Given this information, player 1 does not agree to x'^S since

$$x'^1(t^1, H^2) = 0 < 1 = x^1(t^1, H^2), \text{ for } t^1 = H^1, L^1.$$

Thus, strategy x'^S cannot serve as a "blocking" strategy against x^S. □

EXAMPLE 3.2.4 This example shows that coalition formation could be made easier as a result of the information revelation. Suppose that S finds the following strategy bundle $x''^S \in F^S(x)$:

	H^2	L^2
H^1	(1.5, 1.5)	(0, 0)
L^1	(1.5, 1.5)	(0, 0)

Then,

$$\begin{aligned} E(x''^1 \mid H^1) &= E(x''^1 \mid L^1) = 0.75 \\ &< 1 = E(x^1 \mid H^1) = E(x^1 \mid L^1). \end{aligned}$$

So S cannot improve upon x using x''^S according to the traditional "blocking" criterion. However, when the true type profile is $\bar{t}^S = (H^1, H^2)$, player 2 wants to agree to the joint strategy x''^S. When this happens, player 1 infers that 2's true type is H^2, so 1 also wants to agree to x''^S. Thus, strategy x''^S serves as a "blocking" strategy against x^S. □

This kind of information revelation occurs within a "blocking" coalition. The scenario does not address how the private information is revealed only through the original strategy bundle x of the grand coalition N. In particular, given a strong equilibrium strategy bundle (or a core strategy bundle) x^*, this kind of information revelation does not occur, since there are no "blocking" coalitions.

3.2.3 By credible information-transmission (e.g., by credible talking) at the contract negotiation

We will see how the mediator-based approach, with all its problematic nature, has provided an idea about endogenous determination of a communication system. Recall Yazar's (2001) formulation of a strategy in $\mathcal{E}_{pe}$ and her Bayesian incentive compatibility condition (condition 2.4.5). Define

$$\hat{F}^S := \left\{ \{z^j, \mathcal{C}^j\}_{j\in S} \;\middle|\; \begin{array}{l} \forall\, j \in S : z^j \text{ is } \bigvee_{i\in S} \mathcal{C}^i\text{-measurable,} \\ \mathcal{C}^j \subset \mathcal{T}^j, \\ \{z^j, \mathcal{C}^j\}_{j\in S} \text{ is Bayesian incentive-compatible} \\ \forall\, t : \sum_{j\in S} z^j(t) = \mathbf{0} \end{array} \right\}.$$

A strategy bundle $\{z^{*j}, \mathcal{C}^{*j}\}_{j\in N}$ of the grand coalition in $\mathcal{E}_{pe}$ is said to be in the *EC-core* (endogenous communication plan core), if (i) $\{z^{*j}, \mathcal{C}^{*j}\}_{j\in N} \in \hat{F}^N$, and (ii) if it is not true that

$$\begin{array}{l} \exists\, S \in \mathcal{N} : \exists\, \{z^j, \mathcal{C}^j\}_{j\in S} \in \hat{F}^S : \\ \quad \forall\, ij \in S : \;\; Eu^j(z^j + e^j) > Eu^j(z^{*j} + e^j). \end{array}$$

The communication system $\{\mathcal{A}^{*j}\}_{j\in N}$, $\mathcal{A}^{*j} := \mathcal{T}^j \vee (\vee_{i\in N} \mathcal{C}^{*i})$, sustains as a result of credible talk at the contract negotiation.

The following lemma is the key step in Yazar's result:

LEMMA 3.2.5 (Yazar, 2001) *For any coalition $S \in \mathcal{N}$, let $\{\mathcal{C}^j\}_{j\in S}$ and $\{\mathcal{C}'^j\}_{j\in S}$ be two communication plan bundles, and let $\{z^j\}_{j\in S}$ be a net trade bundle. If $\mathcal{C}'^j \subset \mathcal{C}^j$ for every $j \in S$ and if $\{z^j, \mathcal{C}'^j\}_{j\in S} \in \hat{F}^S$, then $\{z^j, \mathcal{C}^j\}_{j\in S} \in \hat{F}^S$.*

THEOREM 3.2.6 (Yazar, 2001) *Let $\{\mathcal{C}^j\}_{j\in N}$ and $\{\mathcal{C}'^j\}_{j\in N}$ be two communication plan bundles, and let $\{z^j\}_{j\in N}$ be a net trade bundle for the grand coalition. If $\mathcal{C}'^j \subset \mathcal{C}^j$ for every $j \in N$ and if $\{z^j, \mathcal{C}'^j\}_{j\in N}$ is in the EC-core, then $\{z^j, \mathcal{C}^j\}_{j\in N}$ is also in the EC-core.*

In particular, if the EC-core is nonempty at all, then there exists a strategy bundle in the EC-core which gives rise to the full communication system.

For the special case in which each utility function $u^j(\cdot, t)$ is affine linear on the consumption set $\mathbf{R}^l_+$, Yazar (2001) also established nonemptiness of the EC-core.

For Vohra's Bayesian incentive compatibility (condition 2.4.6) applied to an arbitrary communication plan, the analogue of Yazar's lemma is trivially

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