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Rational expectation can preclude trades*

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Abstract. In a pure exchange economy under uncertainty the traders are willing to trade of the amounts of state-contingent commodities and they know their expectations. Common-knowledge about these conditions among all traders can preclude trade if the initial endowments allocation is a rational expectations equilibrium, even when the traders have the non-partition structure of information without the common prior assumption. In the proof it plays essential role to extend the notion of a rational expectations equilibrium and to characterize ex-ante Pareto optimal endowments as the equilibrium. From the epistemic point of view it is emphasized that the partition structure of information for the traders plays no roles in the no trade theorem.

Keywords: Pure exchange economy with knowledge, Rational expectations equilibrium, No trade theorem, Ex-ante Pareto optimal, Common-knowledge.

1. Introduction

One of the purposes of this paper is to introduce a pure exchange economy under generalized information structure and to extend the notion of rational expectations equilibria for the economy. The another purpose is to characterize an ex-ante Pareto optimal endowments allocation as a rational expectations equilibrium, and to extend the no trade theorem of Milgrom and Stokey (1982) in the economy under generalized information structure: It is assumed that (a) the traders are willing to trade of the amounts of state-contingent commodities, and that (b) they have rationality such that they know their expected utilities. We shall show that common-knowledge about (a) and (b) can preclude trade provided that the traders have

- the reflexive and transitive information;
- the subjective priors that are not common for them; and
- the strictly monotone preferences.

In their paper Milgrom and Stokey (1982) show the no trade theorem as follows:[1] Let us consider a pure exchange economy with traders in uncertain environment. Let $\Omega = \Theta \times X$ and the state of Ω consists of a pair (θ, x) where θ ranging over

* The paper is an extended abstract and the final form will be published elsewhere.
[1] See Fudenberg and Tirole (1991), Chapter 14, Subsection(14.3.3), pp. 550-553.

the contingencies on which commodities are defined. The set Θ is interpreted as the set of payoff-relevant events; endowments and utility functions may depend on θ. The set X is interpreted as consisting of payoff-irrelevant events; these events do not affect endowments or taste directly. It is assumed here that the contingent commodities are ex-ante Pareto-optimally allocated, and the traders receive information about the state of Ω representable by information partition, and it is assumed that the traders' beliefs are a common prior distribution; we call it *the common prior assumption*. Now, a trading process takes place where traders try to maximize their expected utilities. We assume that in any equilibrium of this process traders' intended trades are both jointly feasible and common knowledge among them. In this set-up Milgrom and Stokey show that if traders are strictly risk-averse, equilibrium trade is null.

The serious limitations of the analysis in a pure exchange economy under uncertainty such as Milgrom and Stokey's are its use of the information partition structure by which the traders receive information and of the common prior assumption. From the epistemic point of view the information partition structure represents the trades' knowledge: Precisely, the structure is equivalent to the *standard model of knowledge* that includes the 'factivity' of knowledge **T** (what is known is true) and the 'introspection' properties Axioms **4** and **5** (that we know what we do and do not know). The postulate **5** is indeed so strong that describe the hyper-rationality of traders, and thus it is particularly objectionable. Also is the common-knowledge assumption because the common-knowledge operator is defined by an infinite recursion of the knowledge operators. The recent idea of 'bounded rationality' suggests dropping such assumptions since real people are not complete reasoners. The common prior assumption also seems to be problematic.

This raises the question to what extent results as the no trade theorem depend on both common-knowledge and the information partition structure (or the equivalent postulates of knowledge.) The answer is that results strengthen the Milgrom and Stokey 's theorem can be obtained in two ways: First, Tanaka (2000) investigates the theorem on the information partition by iterated elimination reasoning instead of common-knowledge. Secondly, in this paper we drop the hypothesis that the initial endowments are ex-ante Pareto optimal and we extend the no trade theorem to the reflexive and transitive information structure without the traders being risk-aversion and having the common prior assumption. We show the results as follows: In a pure exchange economy under reflexive and transitive information structure, the traders are assumed to have their subjective priors not common and to have strictly monotone preferences. Then

Main Theorem 1. *Any price system for which the initial endowments allocation is a rational expectations equilibrium allocation can preclude trade if all the traders commonly know that they are willing to trade of the amounts of state-contingent commodities and if they know their expectations everywhere with respect to the price.*

To prove it we extend the notion of rational expectations equilibrium for economy under uncertainty to that of economy under reflexive and transitive information structure, and we establish the the existence theorem for the equilibrium: The traders are further assumed to be strictly risk-averse.

Main Theorem 2. *There exists a rational expectations equilibrium allocation relative to a price with respect to which the traders know their expectations everywhere.*

Moreover, we show a generalized version of fundamental theorem of welfare economics, a part of which plays essential role in proving Main Theorem 1:

Main Theorem 3. *The initial endowments allocation is ex-ante Pareto optimal if and only if it is a rational expectations equilibrium allocation relative to a price with respect to which the traders are rational everywhere about their expectations.*

This paper organizes as follows: In Section 2 we first recall a generalized information structure; the RT-information structure, and the knowledge operator model corresponding to it. Secondly we introduce the economy under RT-information structure, called an economy with knowledge, which is a generalization of an economy under uncertainty. In Section 3 we extend the notion of rational expectations equilibrium for economy under uncertainty to that of economy with knowledge, and we establish the fundamental theorem of welfare economics and the existence theorem for the equilibrium. Main Theorem 1 is proved as a consequence of a part of the fundamental theorem. At the end of this section we give the existence theorem for the rational expectations equilibrium. In Section 4 we state the generalized no trade theorem of Milgrom and Stokey. In Section 5 we remark that the RT-information structure plays an essential role in the no trade theorem.

2. The Model

Let Ω be a non-empty *finite* set called a *state space*, $N = \{1, 2, \cdots, n\}$ a set of finitely many *traders*, and let 2^{Ω} denote the field of all subsets of Ω. Each member of 2^{Ω} is called an *event* and each element of Ω called a *state*.

2.1. Information and Knowledge[2]

An *information structure* $(P_i)_{i \in N}$ is a class of mappings P_i of Ω into 2^{Ω}. It is said to be *reflexive* if the following property is true:

Ref $\quad \omega \in P_i(\omega) \quad$ for every $\omega \in \Omega$,

and it is said to be *transitive* if the following property is true:

Trn $\quad \xi \in P_i(\omega)$ implies $P_i(\xi) \subseteqq P_i(\omega)$ for all $\xi, \omega \in \Omega$.

Given our interpretation, an trader i for whom $P_i(\omega) \subseteqq E$ knows, in the state ω, that some state in the event E has occurred. In this case we say that at the state ω the trader i knows E. i's *knowledge operator* K_i on 2^{Ω} is defined by

$$K_i E = \{\omega \in \Omega | P_i(\omega) \subseteqq E\}. \tag{1}$$

The set $P_i(\omega)$ will be interpreted as the set of all the states of nature that i knows to be possible at ω, and $K_i E$ will be interpreted as the set of states of nature for which i knows E to be possible. We will therefore call P_i i's *possibility operator* on Ω and also will call $P_i(\omega)$ i's *possibility set* at ω.

It is noted that i's knowledge operator satisfies the following properties: For every E, F of 2^{Ω},

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[2] See Bacharach (1985), Binmore (1992).

N $\quad K_i\Omega = \Omega \quad$ and $\quad K_i\emptyset = \emptyset$;
K $\quad K_i(E \cap F) = K_iE \cap K_iF$;
T $\quad K_i(E) \subseteqq E \quad$ for every $E \in 2^\Omega$.
4 $\quad K_i(E) \subseteqq K_i(K_i(E)) \quad$ for every $E \in 2^\Omega$.

It is also noted that the possibility operator P_i is uniquely determined by the knowledge operator K_i such as $P_i(\omega) = \bigcap_{\omega \in K_iE} E$.

The *mutual* knowledge operator K_E on 2^Ω is defined by $K_EF = \bigcap_{i \in N} K_iF$. The event K_EF is interpreted as that 'all traders know F.' The *common-knowledge* operator K_C is defined by the infinite recursion of knowledge operators:

$$K_CE := \bigcap_{k=1,2,\ldots} \bigcap_{\{i_1,i_2,\ldots,i_k\} \subset N} K_{i_1}K_{i_2}\cdots K_{i_k}E.^3$$

The *communal* possibility operator is the mapping $M : \Omega \to 2^\Omega$ defined by $M(\omega) = \bigcap_{\omega \in K_CE} E$. All traders *commonly know* E at ω if $\omega \in K_CE$; which is equivalent to that $M(\omega) \subseteqq E$.

2.2. Economy with knowledge

A *pure exchange economy under uncertainty* is a tuple $\langle N, \Omega, (e_i)_{i \in N}, (U_i)_{i \in N}, (\mu_i)_{i \in N} \rangle$ consisting of the following structure and interpretations: There are l commodities in each state of the state space Ω , and it is assumed that Ω is *finite* and that the consumption set of trader i is $\mathbb{R}^l_+$;

- $N = \{1, 2, \cdots, n\}$ is the set of n traders;
- $e_i : \Omega \to \mathbb{R}^l_+$ is i's *endowment*;
- $U_i : \mathbb{R}^l_+ \times \Omega \to \mathbb{R}$ is i's utility function;
- μ_i is a subjective prior on Ω for i.

For simplicity it is assumed that (Ω, μ_i) is a finite probability space with μ_i *full support*[4] for every $i \in N$.

Definition 1. An *economy with knowledge* $\mathcal{E}^K$ is a structure $\langle \mathcal{E}, (P_i)_{i \in N} \rangle$, in which $\mathcal{E}$ is a pure exchange economy under uncertainty with a state-space Ω finite and with (P_i) a reflexive and transitive information structure on Ω.

We denote by $\mathcal{F}_i$ the field generated by $\{P_i(\omega) \mid \omega \in \Omega\}$ and by $\mathcal{F}$ the join of all $\mathcal{F}_i (i \in N)$; i.e. $\mathcal{F} = \vee_{i \in N} \mathcal{F}_i$. It is noted that the atoms $\{A_i(\omega) \mid \omega \in \Omega\}$ of $\mathcal{F}_i$ is the partition induced from P_i. We denote by $\{A(\omega) \mid \omega \in \Omega\}$ the set of all atoms $A(\omega)$ containing ω of the field $\mathcal{F} = \vee_{i \in N} \mathcal{F}_i$.

By an *allocation* we mean a profile $\boldsymbol{a} = (a_i)$ of $\mathcal{F}_i$-measurable functions a_i from Ω into $\mathbb{R}^l_+$ such that for every $\omega \in \Omega$,

$$\sum_{i \in N} a_i(\omega) \leqq \sum_{i \in N} e_i(\omega).$$

[3] That is, when ω occurs then for all k and for all traders $i_1, i_2, \ldots .i_k$, it is true that 'i_1 knows that [i_2 knows that [$\ldots i_{k-1}$ knows that [i_k knows X]] $\ldots$].' This is the *iterated notion* of common-knowledge.

[4] I.e., $\mu_i(\omega) \gneqq 0$ for every $\omega \in \Omega$.

We denote by $\mathcal{A}$ the set of all allocations and denote by $\mathcal{A}_i$ the set of all the i'th components: $\mathcal{A} = \times_{i\in N}\mathcal{A}_i$. A *trade* $\boldsymbol{t} = (t_i)_{i\in N}$ is a profile of $\mathcal{F}_i$-measurable functions t_i from Ω into $\mathbb{R}^l$. It is said to be *feasible* if for all $i \in N$ and for all $\omega \in \Omega$,

$$e_i(\omega) + t_i(\omega) \geqq 0; \quad \text{and} \quad \sum_{i\in N} t_i(\omega) \leqq 0.$$

We shall often refer to the following conditions: For every $i \in N$,

A-1 The function $e_i(\cdot)$ is $\mathcal{F}_i$-measurable with $\sum_{i\in N} e_i(\omega) \gneqq 0$ for all $\omega \in \Omega$.
A-2 For each $x \in \mathbb{R}^l_+$, the function $U_i(x, \cdot)$ is $\mathcal{F}_i$-measurable.
A-3 For each $\omega \in \Omega$, the function $U_i(\cdot, \omega)$ is strictly monotone on $\mathbb{R}^l_+$.
A-4 For each $\omega \in \Omega$, the function $U_i(\cdot, \omega)$ is continuous, strictly quasi-concave and *non-saturated*[5] on $\mathbb{R}^l_+$.

Here it is noted that **A-4** implies to **A-3**.

2.3. Pareto optimality and Acceptability

We set by $\mathbf{E}_i[U_i(a_i)]$ the *ex-ante* expectation defined by

$$\mathbf{E}_i[U_i(a_i)] := \sum_{\omega\in\Omega} U_i(a_i(\omega), \omega)\mu_i(\omega)$$

for each $a_i \in \mathcal{A}_i$.

The endowments $(e_i)_{i\in N}$ are said to be *ex-ante Pareto-optimal* if there is no allocation $(a_i)_{i\in N}$ such that for all $i \in N$,

$$\mathbf{E}_i[U_i(a_i)] \geqq \mathbf{E}_i[U_i(e_i)];$$

and that for some $j \in N$,

$$\mathbf{E}_j[U_j(a_j)] \gneqq \mathbf{E}_j[U_j(e_j)].$$

Let $\mathbf{E}_i[U_i(a_i)|P_i](\omega)$ denote the *interim* expectation defined by

$$\mathbf{E}_i[U_i(a_i)|P_i](\omega) := \sum_{\xi\in\Omega} U_i(a_i(\xi), \xi)\mu_i(\xi|P_i(\omega)).$$

Definition 2. Let $\mathcal{E}^K$ be an economy with knowledge and $\boldsymbol{t} = (t_i)_{i\in N}$ a feasible trade. We say that t_i is *acceptable* for i at state ω provided that

$$\mathbf{E}_i[U_i(t_i + e_i)|P_i](\omega) \geqq \mathbf{E}_i[U_i(e_i)|P_i](\omega).$$

Denote by $Acp(t_i)$ the set of all the states in which t_i is acceptable for i, and by $Act(\boldsymbol{t})$ the intersection $\bigcap_{i\in N} Acp(t_i)$.

3. Rational Expectations Equilibrium

In this section we extend the notion of rational expectations equilibrium for an economy under uncertainty to that for an economy with knowledge. We show the fundamental theorem of welfare economics concerning about the relationship between ex-ante Pareto optimal allocations and rational expectations equilibria.

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[5] I.e.; For any $x \in \mathbb{R}^l_+$ there exists an $x' \in \mathbb{R}^l_+$ such that $U_i(x', \omega) \gneqq U_i(x, \omega)$.

3.1. Price system and rational expectations equilibrium

Let $\mathcal{E}^K = \langle N, \Omega, (e_i)_{i \in N}, (U_i)_{i \in N}, (\mu_i)_{i \in N}, (P_i)_{i \in N} \rangle$ be a pure exchange economy with knowledge. A *price system* is a non-zero function $p : \Omega \to \mathbb{R}^l_+$. We denote by $\sigma(p)$ the set of all atoms of the smallest field that p is measurable, and by $\sigma(p)(\omega)$ the component containing ω. The *budget set* of a trader i at a state ω for a price system p is defined by

$$B_i(\omega, p) = \{\, a \in \mathbb{R}^l_+ \mid p(\omega) \cdot a \leqq p(\omega) \cdot e_i(\omega) \,\}.$$

Let $\sigma(p) \cap P_i : \Omega \to 2^\Omega$ be defined by $(\sigma(p) \cap P_i)(\omega) := \sigma(p)(\omega) \cap P_i(\omega)$; it is plainly observed that $\sigma(p) \cap P_i$ is a reflexive and transitive information structure of trader i. We denote by $\sigma(p) \vee \mathcal{F}_i$ the field generated by $(\sigma(p) \cap P_i)$ and denote by $A_i(p)(\omega) = \sigma(p) \cap A_i(\omega)$ the atom containing ω.

Definition 3. A *rational expectations equilibrium* for an economy $\mathcal{E}^K$ with knowledge is a pair $(p, \boldsymbol{x})$, in which p is a price system and $\boldsymbol{x} = (x_i)_{i \in N}$ is an allocation satisfying the following conditions:

RE 1 For every $i \in N$ x_i is $\sigma(p) \vee \mathcal{F}_i$-measurable.
RE 2 For every $i \in N$ and for every $\omega \in \Omega$, $x_i(\omega) \in B_i(\omega, p)$.
RE 3 For all $i \in N$, if $y_i : \Omega \to \mathbb{R}^l_+$ is $\sigma(p) \vee \mathcal{F}_i$-measurable with $y_i(\omega) \in B_i(\omega, p)$ for all $\omega \in \Omega$, then

$$\mathbf{E}_i[U_i(x_i) | \sigma(p) \cap P_i](\omega) \geqq \mathbf{E}_i[U_i(y_i) | \sigma(p) \cap P_i](\omega)$$

pointwise on Ω.

The profile $\boldsymbol{x} = (x_i)_{i \in N}$ is called a *rational expectations equilibrium allocation.*

We denote by $R_i(p)$ the event that i is rational about his expectation; i.e.,

$$R_i(p) = \{\omega \in \Omega \mid (\sigma(p) \cap P_i)(\omega) \subseteqq [\mathbf{E}_i[U_i(\cdot) | \sigma(p) \cap P_i](\omega)] \,\}$$

and denote by $R(p)$ the event that all traders are rational: i.e., $R(p) = \bigcap_{i \in N} R_i(p)$.

Definition 4. A trader i is said to be *rational* about his expectation *with respect to a price system* p at ω if $\omega \in R_i(p)$. And all traders are *rational everywhere* about their expectations if $R(p) = \Omega$.

3.2. Fundamental Theorem in Welfare Economics

We establish a generalized version of the fundamental theorem of welfare economics for initial endowments in the economy with knowledge (Propositions 2 and 3), and Proposition 1 below is also a key to proving Main Theorem 1:

Proposition 1. *Let $\mathcal{E}^K$ be an economy with knowledge satisfying the conditions A-1, A-2 and A-3. Then the initial endowments allocation $\boldsymbol{e} = (e_i)_{i \in N}$ is ex-ante Pareto optimal if it is a rational expectations equilibrium allocation relative to some price system p with respect to which all traders are rational everywhere about their expectations.*

The next proposition states that the converse in Proposition 1 is also valid under the additional assumption that the traders are strictly risk-averse for traders:

Proposition 2. *Let $\mathcal{E}^K$ be an economy with knowledge satisfying the conditions A-1, A-2 and A-4. If the initial endowments allocation $\mathbf{e} = (e_i)_{i \in N}$ is ex-ante-Pareto optimal then it is a rational expectations equilibrium allocation relative to some price system p with respect to which all traders are rational every where about their expectations.*

Proof. For each $\omega \in \Omega$ we denote by $G(\omega)$ the set of all vectors $\sum_{i \in N} e_i(\omega) - \sum_{i \in N} y_i$ such that $y_i \in \mathbb{R}^l_+$ and $U_i(y_i, \omega) \geqq U_i(e_i(\omega), \omega)$ for all $i \in N$.

First, in view of the conditions **A-1**, **A-2** and **A-4** we note that that $G(\omega)$ is convex and closed in $\mathbb{R}^l_+$. We can establish the proposition in observing the following three points: First

Claim 1: For each $\omega \in \Omega$ there exists $p^*(\omega) \in \mathbb{R}^l_+$ such that $p^*(\omega) \cdot v \leqq 0$ for all $v \in G(\omega)$.

Secondly, let p be the price system defined as follows: For each $\omega \in \Omega$ and for all $\xi \in A(\omega)$, $p(\xi) := p^*(\omega)$. We can show

Claim 2: The pair $(p, (e_i)_{i \in N})$ is a rational expectations equilibrium for $\mathcal{E}^K$.

Finally, it is observed that all traders are rational with respect to the price p. □

3.3. Main Theorem 3

We now state Main Theorem 3 explicitly as follows:

Theorem 1. *Let $\mathcal{E}^K$ be an economy with knowledge satisfying the conditions A-1, A-2 and A-4. The initial endowments allocation is ex-ante Pareto optimal if and only if it is a rational expectations equilibrium allocation relative to a price with respect to which the traders are rational everywhere about their expectations.*

Proof. Follows immediately from Propositions 1 and 2.

The following remark has been already proved in the proof of Proposition 1:

Remark 1. Let $\mathcal{E}^K$ be a pure exchange economy with knowledge satisfying the conditions **A-1**, **A-2** and **A-3**. If the allocation of initial endowments $\mathbf{e} = (e_i)_{i \in N}$ is a rational expectations equilibrium allocation relative to some price system p with respect to which all traders are rational every where about their expectations then the pair $(p(\omega), (e_i(\omega))_{i \in N})$ constitutes an *ex-post* competitive equilibrium for the pure exchange economy $\mathcal{E}^K(\omega)$ with complete information for each $\omega \in \Omega$.

3.4. Existence Theorem

It will well end this section in giving the explicit statement of Main Theorem 2: The existence theorem of rational expectations equilibrium for an economy with knowledge.

Theorem 2. *Suppose a pure exchange economy with knowledge satisfies the conditions A-1, A-2 and A-4. If the initial endowments allocation $\mathbf{e} = (e_i)_{i \in N}$ satisfies the additional condition that $e_i(\omega) \gneqq 0$ for all $\omega \in \Omega$ and for each $i \in N$ then there exists a rational expectations equilibrium for the economy such that all traders are rational about their expectations with respect to the price.*

4. No Trade Theorem

In this section we shall give two extensions of the no trade theorem of Milgrom and Stokey (1982): First we give the below theorem that directly extends the no trade theorem to an economy with knowledge, and secondly we give Main Theorem 1.

4.1. Theorem of Milgrom and Stocky

Theorem 3. *Let $\mathcal{E}^K$ be an economy with knowledge satisfying the conditions A-1, A-2 and A-3, and let $\boldsymbol{t} = (t_i)_{i\in N}$ be a feasible trade. Suppose that the initial endowments allocation $(e_i)_{i\in N}$ is ex-ante Pareto-optimal. Then the traders can never agree to any non null trade at each state where they commonly know both the acceptable trade $\boldsymbol{t} = (t_i)$ and rationality of their expectations; that is, $\boldsymbol{t}(\omega) = \mathbf{0}$ at every $\omega \in K_C(Act(\boldsymbol{t}) \cap R)$.*

Proof. Follows from the key lemma below.

Lemma 1. *Let $\mathcal{E}^K$, $\boldsymbol{t} = (t_i)_{i\in N}$ and $(e_i)_{i\in N}$ be the same as in Theorem 3. If $\omega \in K_C(Acp(t_i) \cap R_i)$ for each $i \in N$ then the equality is true:*

$$\mathbf{E}_i[U_i(t_i^* + e_i)|P_i](\omega) = \mathbf{E}_i[U_i(e_i)|P_i](\omega), \tag{2}$$

where the trade $\boldsymbol{t}^* = (t_i^*)_{i\in N}$ is defined by

$$t_i^*(\xi) := \begin{cases} t_i(\xi) & \text{if } \xi \in M(\omega), \\ 0 & \text{if not.} \end{cases} \tag{3}$$

4.2. Rational expectations equilibrium and No trade theorem

It is interesting to consider what can be said if we drop the hypothesis that the endowments are ex-ante Pareto optimal in Theorem 3. Is the no trade theorem still true if the endowments allocation is rational expectations equilibrium allocations? We shall give an affirmative answer. To state it explicitly we introduce the knowledge operator $K_i^{(p)}$ on 2^Ω induced from the information structure $\sigma(p) \cap P_i$; which is defined by

$$K_i^{(p)}(E) = \{\omega \in \Omega \mid (\sigma(p) \cap P_i)(\omega) \subseteqq E\},$$

and let $K_C^{(p)}$ be the common-knowledge operator defined by the infinite recursion of the operators $\{K_i^{(p)}\}_{i\in N}$.[6] We can now explicitly state Main Theorem 1 as follows:

Theorem 4. *Let $\mathcal{E}^K$ be an economy with knowledge satisfying the conditions A-1, A-2 and A-3. If $\mathbf{e} = (e_i)_{i\in N}$ is a rational expectations equilibrium allocation relative to some price system p with respect to which all traders are rational everywhere about their expectations, then the traders can never agree to any non null trade at each state where they commonly know both the acceptable feasible trade $\boldsymbol{t} = (t_i)_{i\in N}$; that is, $\boldsymbol{t}(\omega) = \mathbf{0}$ at every $\omega \in K_C^{(p)}(Act(\boldsymbol{t}))$.*

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[6] That is, $K_C^{(p)}E := \bigcap_{k=1,2,\dots} \bigcap_{\{i_1,i_2,\dots,i_k\}\subset N} K_{i_1}^{(p)} K_{i_2}^{(p)} \cdots K_{i_k}^{(p)} E$.

Proof. Consider now the economic with knowledge

$$\mathcal{E}^{K(p)} = \langle N, \Omega, (e_i)_{i\in N}, (U_i)_{i\in N}, (\mu_i)_{i\in N}, (\sigma(p)\cap P_i)_{i\in N}\rangle.$$

By the similar argument in the proof of Theorem 3 it can be plainly observed that $t(\omega) = \mathbf{0}$ at every $\omega \in K_C^{(p)}(Act(\boldsymbol{t}))$ if $\boldsymbol{e}$ is ex-ante Pareto optimal, and thus Theorem 4 follows from Proposition 1. □

5. Concluding Remarks

Our real concern is to what extent the no trade theorem of Milgrom and Stokey (1982) depends on the information partition and on the hypothesis that the initial endowments are ex-ante Pareto optimal. As we have observed, the reflexivity and transitivity of information structure can preclude trade if the traders commonly know that they are willing to trade of the amounts of state-contingent commodities. Both the information partition and the strictly risk-aversion for the traders of the amounts of commodities play no roles in the no trade theorem.

Could we prove the theorem under the generalized information structure jettisoning the reflexivity or the transitivity? The following two examples show that the reflexivity **Ref** and the transitivity **Trn** of the information structure (or the equivalent postulates Axioms **4** and **T**) do play an essential role.

Example 1. Let $\mathcal{E}^K = \langle N, \Omega, (e_i)_{i\in N}, (U_i)_{i\in N}, \mu, (P_i)_{i\in N}\rangle$ the economy with knowledge in which N, Ω, e_i, U_i are the same in Section ??, and

- $\mu(\omega) = \frac{1}{2}$ for each $\omega \in \Omega$;
- P_i is defined by

$$P_1(\omega) := \{\omega_2\} \quad \text{and} \quad P_2(\omega) := \{\omega_1\}$$

for each $\omega \in \Omega$.

It is plainly observed the two points: First that both P_i $(i = 1, 2)$ are not reflexive but transitive, and second that the endowments $(e_i)_{i=1,2}$ are both ex-ante Pareto optimal. Let $\boldsymbol{t} = (t_i)_{i=1,2}$ be the feasible non-zero trade defined by

$$t_1(\omega) := \begin{cases} -2 & \text{if} \quad \omega = \omega_1 \\ 0 & \text{if} \quad \omega = \omega_2 \end{cases} \quad \text{and} \quad t_2(\omega) := \begin{cases} 2 & \text{if} \quad \omega = \omega_1 \\ 0 & \text{if} \quad \omega = \omega_2. \end{cases}$$

Then it can be verified that $Act(\boldsymbol{t}) = R = \Omega$ and thus $K_C(Act(\boldsymbol{t})\cap R) = \Omega$. However the trade $\boldsymbol{t}$ is not null at $\omega_1 \in K_C(Act(\boldsymbol{t})\cap R)$. □

Example 2. Let $\mathcal{E}^K = \langle N, \Omega, (e_i)_{i\in N}, (U_i)_{i\in N}, \mu, (P_i)_{i\in N}\rangle$ the economy with knowledge in which N, e_i are the same in Section ??, and

- $\Omega = \{\omega_1, \omega_2, \omega_3\}$
- $\mu(\omega) = \frac{1}{3}$ for each $\omega \in \Omega$;
- $U_i : \mathbb{R}^l_+ \times \Omega \to \mathbb{R}$ is defined by

$$U_1(x, \omega) = (x+1)^2 \quad \text{and} \quad U_2(x, \omega) = \sqrt{x+3};$$

– P_i is defined by

$$P_1(\omega) := \begin{cases} \{\omega_1\} & \text{if } \omega = \omega_1 \\ \{\omega_2, \omega_3\} & \text{if } \omega = \omega_2 \text{ or } \omega_3 \end{cases} \qquad P_2(\omega) := \begin{cases} \{\omega_1, \omega_3\} & \text{if } \omega = \omega_1 \\ \{\omega_2, \omega_3\} & \text{if } \omega = \omega_2 \text{ or } \omega_3. \end{cases}$$

It is plainly observed that P_2 are reflexive and not transitive. Let $\boldsymbol{t} = (t_i)_{i=1,2}$ be the feasible non-zero trade defined by

$$t_1(\omega) := \begin{cases} 1 & \text{if } \omega = \omega_1 \text{ or } \omega_2 \\ -1.5 & \text{if } \omega = \omega_3 \end{cases} \qquad t_2(\omega) := \begin{cases} -1 & \text{if } \omega = \omega_1 \text{ or } \omega_2 \\ 1.5 & \text{if } \omega = \omega_3. \end{cases}$$

Then it follows that $Act(\boldsymbol{t}) = R = \Omega$ and $K_C(Act(\boldsymbol{t}) \cap R) = \Omega$. However the trade $\boldsymbol{t}$ is not null at any $\omega \in K_C(Act(\boldsymbol{t}) \cap R)$. □

Nevertheless, common-knowledge of the acceptance of feasible trades seems a rather strong assumption. Could not we get away with less, say with mutual knowledge? The answer is no again: For the counter example see Fudenberg and Tirole (1991, p.552).

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