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A LARGE BAYESIAN ECONOMY AND INCENTIVE COMPATIBLE CORE CONTRACTS

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1 INTRODUCTION

Our purpose here is to report a result obtained in Yamazaki and Ichiishi (2002) on the incentive compatible core of a Bayesian pure exchange economy with a continuum of population. A general economic model with differential information was introduced by Radner (1968), and R. Wilson (1978) in his seminal paper introduced and analyzed several core concepts in a finite economy with differential information. It was shown that :

- A contract in the (interim) core exists if information is not shared among economic agents;
- However, if information is shared among economic agents, then the core may be empty.

In this setup contracts are executed before the prevailing information state becomes known to economic agents. Thus, in order for a contract to be executed exactly as is written on it, it is necessary that the incentive compatibility condition is imposed on the contract. It should be a part of the feasibility conditions of a contract.

It was shown by Ichiishi and Idzik (1996) that if utility functions are affine linear, a contract in the incentive compatible interim core exists. However, R. Vohra (1999) showed that if utility functions are not affine linear, then the incentive compatible interim core may be empty. The basis of an intrinsic difficulty of existence of an incentive compatible core contract is that the imposition of the incentive compatibility introduces a nonconvexity of feasible sets of contracts. For this reason B. Allen (1999) turned to an economy with a continuum population à la Aumann (1966) and showed that the incentive compatible core is nonempty if there are only finite number of utility types each of which is strictly concave, C^1, differentiably strictly monotone, and satisfies a boundary and a regularity condition, and if the income (or wealth) distribution arising from the initial endowment allocation of the economy is diffused in the sense employed by Yamazaki (1978). The Allen's proof seems to have been made complicated by the fact that the incentive compatibility condition is imposed on individual consumption sets, resulting in nonconvex consumption sets of a specific kind. Our observation is that when there are only finitely many utility types, there is a way to circumvent a difficulty introduced by the incentive compatibility condition without appealing to the diffusion of endowment distribution.

2 MODEL

2.1 Large Bayesian Population and Information Structures

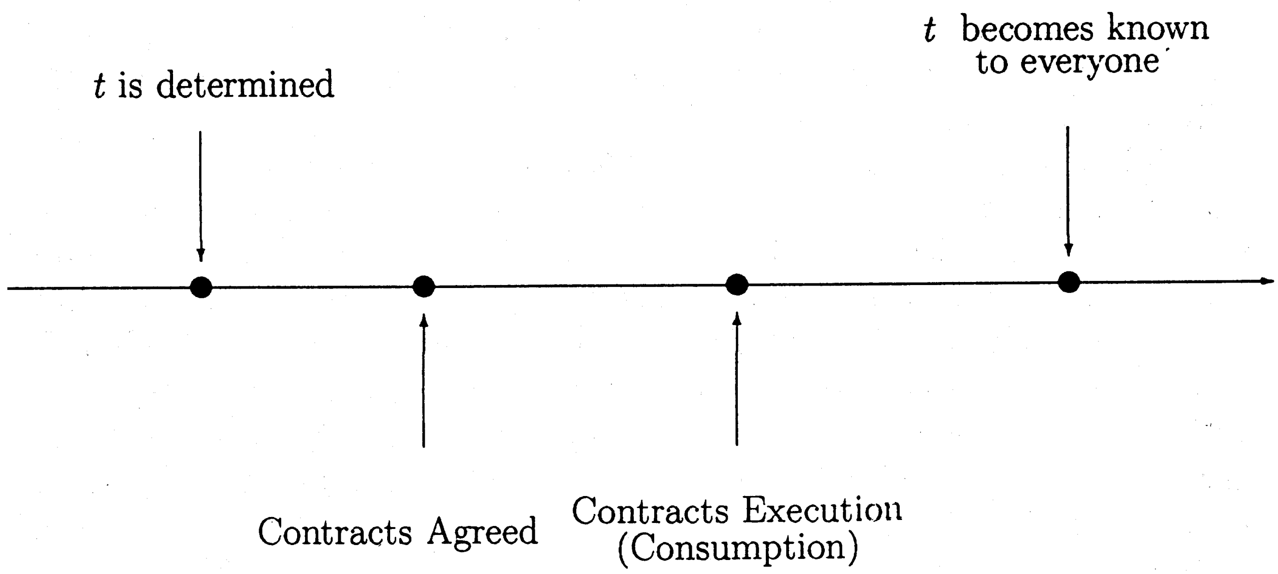
Let $(A, \mathcal{A}, \nu)$ be an atomless measure space (with $\nu(A) = 1$) of economic agents expressing a continuum of population in a large economy as in Aumann (1966) and Hildenbrand(1974). Sets S in $\mathcal{A}$ with positive measures are called *coalitions*. Let $T^1, \ldots, T^m$ be a finite number of agents' type sets. Each $T^i, i = 1, \ldots, m$, is assumed to be nonempty and finite. $I = \{1, \ldots, m\}$ will denote the finite set of indices of type sets of economic agents. At a particular stage of the economic activities agent $a \in A$ of type j alone knows which member of the set T^j is truly realized; in this sense the realized member t^j is called the *private information* of agent a of type j.[1]

[1]Elements of set T^j are *types*. However, with an abuse of words, we will sometimes follow a convention to call $j \in I$ a *type* of agents to avoid a cumbersome wording. Precisely speaking, $j \in I$ is an index of sets of types of agents.

For $J \subset I$, define $T^J := \prod_{j \in J} T^j$ and $T := T^I$ for simplicity. T is the set of possible type profiles and a generic element $t \in T$ is called a *type profile*. It is also called an *information state* in the sense that it is a profile of private information possessed by economic agents.

The *ex ante* period is defined as the period in which each player does not have his private information, but has an *ex ante* probability on the type profiles T, subjective or objective. An *interim* period is defined as a period in which each agent already has his private information but does not know the true type profile. The *ex post* period is defined as the period in which everybody knows the true type profile.

Figure 1: INFORMATION AND A SEQUENCE OF ECONOMIC ACTIVITIES



Given private information t^j, agent a of type j holds his subjective probability $\pi^j(\cdot \mid t^j)$ on the type profiles of other types $T^{I \setminus \{j\}}$. Some works have an *ex ante* probability as a given datum, but here we take *interim* probabilities as given data. It is not necessary to assume that it is derived from an *ex ante* probability π^j on T by the Bayes rule, $\pi^j(t^{I \setminus \{j\}} \mid t^j) = \pi^j(t^{I \setminus \{j\}}, t^j)/\pi^j(T^{I \setminus \{j\}} \times \{t^j\})$.

In addition to the type set T^j, associated with each index $j \in I$ of type sets are a type-profile dependent *von Neumann-Morgenstern utility function* $u^j : \mathbb{R}^\ell_+ \times T \to \mathbb{R}$ and type j's initial endowment vector, $e^j : T^j \to \mathbb{R}^\ell_+$, which depends only upon type j's information state t^j.

For $J \subset I$ the set T^J gives rise to the partition of T: $\{\{t^J\} \times T^{I \setminus J} \mid t^J \in T^J\}$. Denote by $\mathcal{T}^J$ the algebra on T generated by this partition, and set $\mathcal{T}^j := \mathcal{T}^{\{j\}}$. We call $\mathcal{T}^j$ the *private information structure* of agent of type j.

Wilson (1978) pioneered study of the core of a pure exchange economy with incomplete information and a finite population, in which he emphasized the role of revelation of private information. For an expository purpose of this paragraph and the next, assume for a moment that the set I is the finite population of economic agents rather than a finite set of types of agents. When agent j is endowed only with his private information structure, he can distinguish two states (i.e., two type profiles), t and t', iff there exists an event $E \in \mathcal{T}^j$ such that $t \in E$ and $t' \notin E$. Likewise, when coalition J is formed and each member some how fully reveals his information to his colleagues, any member of J can distinguish two states using the pooled information structure $\mathcal{T}^J$. In general, each member j receives only partial information from his colleagues, so the information structure he can use is an algebra $\mathcal{S}^j$ which is finer than his private information structure but is coarser than the fully pooled information structure.

We have followed Harsanyi (1967/1968) in formulating information structures as algebras on the type profile space T. A type profile is a *state* of the nature. Actually, Wilson (1978) and many subsequent authors took a more general approach in which an arbitrary probability space $(\Omega, \mathcal{T}, \pi)$ is given to describe the possible states of the nature, and an arbitrary subalgebra $\mathcal{T}^j$ of $\mathcal{T}$ is also given to describe agent j's private information structure, $j \in I$. Harsanyi's type profile framework ($\Omega = \prod_{j \in I} T^j$) treats the case of extreme asymmetry of information that private information structures $\mathcal{T}^i$ and $\mathcal{T}^j$ are uncorrelated for different agents i and j ($\mathcal{T}^i \cap \mathcal{T}^j = \{\emptyset, T\}$ if $i \neq j$), but Wilson's general approach allows for correlation, that is $\mathcal{T}^i \cap \mathcal{T}^j$ may contain nonempty proper subsets of Ω. For the expository purpose, however, we adopt the type profile framework in this paper.

A *large Bayesian (exchange) economy*

$$\mathcal{E} = \left(\theta : (A, \mathcal{A}, \nu) \to I, \{\mathbb{R}^\ell_+, T^j, \pi^j, u^j, e^j\}_{j \in I}\right)$$

is an economy with l commodities, where $\theta : A \to I$ is a measurable mapping indicating type indices of agents in the economy, $(A, \mathcal{A}, \nu)$ an atomless population of economic agents, I a finite set of agents' type indices, and for each agent a, $\mathbb{R}^\ell_+$ is his consumption set, $T_a := T^{\theta(a)}$ his finite type set, $u_a := u^{\theta(a)} : \mathbb{R}^\ell_+ \times T \to \mathbb{R}$ his type-profile dependent von Neumann-Morgenstern utility function which is continuous and locally nonsatiated for any t in T, $e(a,\cdot) := e^{\theta(a)} : T \longrightarrow \mathbb{R}^{\ell}_{+}$ his initial endowment vector, which depends only on $t^{\theta(a)}$, and $\pi_a := \pi^{\theta(a)}$ an interim subjective probability on $T^{-\theta(a)}$ where, for any j, T may be written as $T = T^j \times T^{-j}$. Note that the measurability of $\theta : A \longrightarrow I$ and the finiteness of I imply that $e(\cdot, t)$ is $\mathcal{A}$-integrable for all $t \in T$.

For each $j \in I$, let $A^j = \theta^{-1}(j)$ $(= \{a \in A \mid \theta(a) = j\})$. We assume that each type $j \in I$ is present in the economy:

Assumption 2.1 $(\forall j \in I)$ $\nu(A^j) > 0$, and $\nu(\cup_{j\in I} A^j) = 1$.

For any coalition S in $\mathcal{A}$, define $I(S) = \{j \in I \mid \nu(S \cap A^j) > 0\}$. $I(S)$ is the set of types of agents belonging to coalition S. For $S \in \mathcal{A}$, define $T_S := T^{I(S)} (= \prod_{j \in I(S)} T^j)$; then, we have $T_A = T^{I(A)} = T^I = T$. The private information structures for individuals $a \in A$ is given by:

$$\mathcal{T}_a := \mathcal{T}^{\theta(a)} \left(= \left\{ \{t^{\theta(a)}\} \times \prod_{j \in I \setminus \{\theta(a)\}} T^j \mid t^{\theta(a)} \in T^{\theta(a)} \right\} \right).$$

Let us define $\mathcal{T}_S$ also for a coalition S by $\mathcal{T}_S := \mathcal{T}^{I(S)}$ which is the algebra on T generated by the partition $\{\{t^{I(S)}\} \times T^{I \setminus I(S)} \mid t^{I(S)} \in T^{I(S)}\}$.

2.2 Contracts on Trades in a Bayesian Economy

Contracts

Let $\mathcal{E} = \left(\theta : (A, \mathcal{A}, \nu) \longrightarrow I, \{\mathbb{R}^{\ell}_{+}, T^j, \pi^j, u^j, e^j\}_{j \in I}\right)$ be a large Bayesian (exchange) economy. We are interested in contracts on trades among economic agents in this economy. We assume that a contract is agreed upon at interim stage, and is executed before prevailing information state $t \in T$ becomes known to everyone. This means that trades and resulting consumptions will take place in general without a complete knowledge of t. (See Figure 1.)

A *contract* (on trades) in $\mathcal{E}$ is a mapping $x : A \times T \longrightarrow \mathbb{R}^{\ell}$ such that $x(\cdot, t)$ is $\mathcal{A}$-integrable for every $t \in T$. Three kinds of *feasibility* requirement are placed on contracts. The first is their resource feasibility.

A contract $x : A \times T \longrightarrow \mathbb{R}^{\ell}$ is *exactly feasible* for $S \in \mathcal{A}$ if we have

$$\int_S x(\cdot, t) d\nu = 0$$

for every $t \in T$.[2] It is *exactly feasible* if it is exactly feasible for A.

The second feasibility requirement is concerned with the discernability of conditions of a contract by individual agents. A contract $x : A \times T \to \mathbb{R}^\ell$ is called a *private information contract* if $x(a, \cdot)$ is $\mathcal{T}_a$–measurable a.e. $a \in A$. If a contract is a private information contract, then any agent can carry out trades required on his part by the contract on the basis of his own information.

For any $t = (t^1, \ldots, t^j, \ldots, t^m) \in T$, $j = 1, \ldots, m$, we write $t = (t^j, t^{-j}) \in T^j \times T^{-j}$. Given a contract $x : A \times T \to \mathbb{R}^\ell$, define for each $a \in A$

$$x(a, t, \mathcal{T}_a) := \{z \in \mathbb{R}^\ell \mid (\exists t'^{\theta(a)} \in T^{\theta(a)})\, z = x_S(a, (t'^{\theta(a)}, t^{-\theta(a)}))\}.$$

The third feasibility requirement is concerned with the feasibility of execution of a contract. A contract $x : A \times T \to \mathbb{R}^\ell$ is *incentive compatible* in $S \in \mathcal{A}$ if

$$(\forall z_a \in x(a, t, \mathcal{T}_a)) E u_a(x(a,t) + e(a,t) | t^{\theta(a)}) \geq E u_a(z_a + e(a,t) | t^{\theta(a)})$$

a.e. $a \in S$ for all $t \in T$. A contract $x : A \times T \to \mathbb{R}^\ell$ is *incentive compatible* if it is incentive compatible in A. An incentive compatible contract does not give incentives for agents to engage in trades which are not in accordance with the agreement of the contract just because others might not be able to catch him of breaching the contract. In this sense its execution is feasible if it is incentive compatible.

Incentive Compatible Core

A pair (S, x_S) consisting of a coalition $S \in \mathcal{A}$ (with $\nu(S) > 0$) and an exactly feasible contract for S, $x_S : A \times T \to \mathbb{R}^\ell$ is an *objection* to another private information contract $x : A \times T \to \mathbb{R}^\ell$ if :

(a) $E\left(u_a(x_S(a,t) + e(a,t)) | t_{i(a)}\right) \geq E\left(u_a(x(a,t) + e(a,t)) | t_{i(a)}\right)$ a.e. $a \in S$;

(b) $E\left(u_a(x_S(a,t) + e(a,t)) | t_{i(a)}\right) > E\left(u_a(x(a,t) + e(a,t)) | t_{i(a)}\right)$ on a subset $S' \subset S$ with a positive measure.

[2] In the literature some of the existing works adopt a weaker form of resource feasibility allowing an inequality in the definition of resource feasibility. If there are economic agents to whom some of the commodities are "bads" in the sense that an increase in their consumption decreases their utility level in some range of their consumption levels, then from an economic point of view it is important to require the exact resource feasibility.

An objection (S, x_S) is a *private information objection* if $x_S : A \times T \to \mathbb{R}^\ell$ is $\mathcal{T}_a$–measurable a.e. $a \in S$. It is an *incentive compatible objection* if $x_S : A \times T \to \mathbb{R}^\ell$ is incentive compatible in S.

Definition 2.1 [Incentive Compatible Core] An exactly feasible incentive compatible private information contract $x : A \times T \to \mathbb{R}^\ell$ is said to be in the *incentive compatible core of the Bayesian economy* if there are no incentive compatible private information objections to the contract x.

3 Statement of a Result

We would like to report an existence result which shows that the interim incentive compatible core is nonempty without assuming affine linearity nor quasiconcavity of utility functions of individual agents. Moreover, we allow commodities to be regarded as bads by many agents. However, what we require is that for each commodity there is a positive weight of agents who regard the commodity to be desirable.

Let us now give a precise statement. Commodity $i \in \{1, \ldots, l\}$ is called a *desirable* commodity for u^j if one has

$$(\forall t \in T)(\forall x \in \mathbb{R}^\ell_+)(\exists k > 0)\ u^j(x + k\iota_i, t) > u^j(x, t)$$

where ι_i is the i-th unit vector.

Theorem 1 *Let* $\mathcal{E} = \left(\theta : (A, \mathcal{A}, \nu) \to I, \{\mathbb{R}^\ell_+, T^j, \pi^j, u^j, e^j\}_{j \in I}\right)$ *be a large Bayesian economy. Then, there exists a private information contract that belongs to the incentive compatible core if the given Bayesian economy* $\mathcal{E}$ *satisfies the following conditions:*

(a) For each commodity i *there is* $j \in I$ *such that commodity* i *is a desirable commodity for* u^j.

(b) For all $t \in T$, $e(a, t) \in \mathbb{R}^\ell_{++}$ *a.e.* $a \in A$.

A proof of this theorem is given in Yamazaki and Ichiishi (2002). Here, we briefly describe steps of its proof. The theorem is proved as a consequence of three propositions. The first step is to define an interim price equilibrium in such a way that corresponding equilibrium contracts belong to the incentive compatible core. Since the main objective is to establish the nonemptiness of the incentive compatible core, it is not necessary to have a reasonable ground for such an equilibrium to be established in competitive markets.

The notion of an interim exact price equilibrium contract to be used is as defined below:

Definition 3.2 [Interim Exact Price Equilibrium Contract] A contract $x : A \times T \to \mathbb{R}^\ell$ is an *interim exact price equilibrium contract* if there is $p : T \to \mathbb{R}^\ell \setminus \{0\}$ such that:

(a) $x : A \times T \to \mathbb{R}^\ell$ is exactly feasible;

(b) $x : A \times T \to \mathbb{R}^\ell$ is a private information contract;

(c) $(\forall t \in T)$ $x(a,t)$ maximizes $E\left(u_a(z + e(a,t))|t_{i(a)}\right)$ subject to $p(t) \cdot z \leq 0$, a.e. $a \in A$.

Then, one can easily check the following result:

Proposition 1 *Given a large Bayesian economy $\mathcal{E}$, an interim exact price contract belongs to the incentive core.*

The second step is to adopt the idea of Hahn and Yannelis (1997) and to show that their result showing that an exactly feasible contract on trades is incentive compatible if it is a private information contract can be extended to a large economy with a continuum of agents if there are only finitely many types.

Proposition 2 *In a large Bayesian economy $\mathcal{E}$, a private information contract $x : A \times T \to \mathbb{R}^\ell$ is incentive compatible if it satisfies:*

$$\int_A x(\cdot, t) d\nu = o$$

for every $t \in T$.

The last but an essential step is to establish the existence of an interim exact price contract. For this we appeal to a recent result on the existence of a Walrasian equilibrium in a large economy by Hara and Yamazaki (2002).

Proposition 3 *Given a large Bayesian economy $\mathcal{E}$, there exists an interim exact price contract if it satisfies the following conditions:*

(a) For each commodity i there is $j \in I$ such that commodity i is a desirable commodity for u^j.

(b) For all $t \in T$, $e(a,t) \in \mathbb{R}^{\ell}_{++}$ a.e. $a \in A$.

We would like to note that the above proposition 3 is not implied by the fundamental Walrasian equilibrium existence results of Aumann (1966) and Schmeidler (1969) because utility functions need not be monotonic for all agents, nor the results of Hildenbrand (1970, 1974) because contracts or allocations must be exactly feasible. In this respect, one may also note that in a setup of economic agents with preferences which are locally nonsatiated but not necessarily monotonic (so that some of the commodities are "bads") such as in Hildenbrand (1970), an equilibrium may fail to exist unless the free disposability of unwanted commodities is assumed.

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