数理解析研究所講究録 1276 巻 2002 年 54-56

Estimate of Real Part of Analytic Functions

Mamoru Nunokawa

At first, the author shows the following theorem,

Theorem 1. Let $p(z)$ be analytic in the unit disk $E = \{ z : |z| < 1 \}$

$p(0)=1$ and suppose that

(1) $$-zp'(z)/(\alpha - p(z)) \prec 2z/(1-z^2) \quad \text{in } E$$

where $1 < \alpha$ and $\prec$ means the symbol of subordination.

Then we have

$$\operatorname{Re} p(z) < \alpha \quad \text{in } E.$$

Proof. Putting

$$q(z) = (\alpha - p(z))/(\alpha - 1), \quad q(0) = 1,$$

then $q(z)$ is analytic in E. If there exists a point z_0 ($z_0 \in E$)

such that

$$\operatorname{Re} q(z) > 0 \quad \text{for } |z| < |z_0|$$

$$\mathrm{Re}\ q(z_0) = 0$$

Then from Nunokawa's result [1], we have

$$z_0 q'(z_0)/q(z_0) = z_0 p'(z_0)/(\ p(z_0) - \alpha\ )$$

$$= ik$$

where k is real and

$$k \geqq (\ a + 1/a\ )/2 \qquad \text{when } \arg q(z_0) = \pi/2$$

and

$$k \leqq -(\ a + 1/a\ )/2 \qquad \text{when } \arg q(z_0) = \pi/2$$

$q(z_0) = \pm ia$ and $0 < a$.

This contradicts the hypothesis of the theorem and so it

completes the proof.

Applying the same method as the proof of Theorem 1, we ca

obtain the following theorem.

Theorem 2. Let p(z) be analytic in E and suppose that

$$(2) \qquad z P'(z)/(p(z) - \beta) \prec 2z/(1 - z^2) \quad \text{in } E.$$

where $\beta < 1$ and $\prec$ denote the symbol of subordination.

Then we have

$$\beta < \operatorname{Re} p(z) \quad \text{in } E.$$

Reference

[1] M. Nunokawa, On properties of Non-Carathéodory functions, Proceedings of the Japan Academy, Vol. 68, Ser. A, No. 6, 152-153 (1992).

Department of Mathematics
University of Gunma
Aramaki, Maebashi, 371-8510, JAPAN
e-mail: nunokaw@edu.gunma-u.ac.jp