A Model for Occurrence of Turbulence in Circular Pipe Flows: Experimental Definition of the Problem

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Abstract

A conceptual model was constructed for the problem of determining the conditions under which the transition from laminar to turbulent flow in circular pipes and between parallel plates occurs, so that it becomes possible to calculate the critical Reynolds number (Rc). Up until now this problem has been investigated by stability theory with disturbances. However, the minimum critical Reynolds number (Rc(min)) has not yet been obtained theoretically. Accordingly, the author took up the problem directly from many previous experimental investigations and found that (i) plots of the transition length versus the Reynolds number (Re) show that the transition occurs in the entrance region under the conditions of natural disturbances, and (ii) plots of Rc versus the ratio (γ) of bellmouth diameter (BD) to the pipe diameter (D) show that with larger shapes of bellmouths, laminar flow will persist to higher Res. The problem is thus defined clearly as: *Entrance shape determines the critical Reynolds number.*

1 Kanda's Transition Model

A layout of the procedure for the modeling and simulation of this problem to determine Rc is illustrated in Fig. 1 on the 7th page. Hence, we shall focus on the transition length, which is the distance between the pipe inlet and the point where transition from laminar to turbulent flow occurs, and on the shape of bellmouths fitted at the pipe inlet. The objectives of this study are to derive and verify the concepts of the model from previous experimental investigations, especially (1) - (3), and (6) below.

(1) Transition occurs in the entrance region under the conditions of natural disturbances (Kanda and Oshima, 1987).

(2) Entrance shape determines Rc apparently: with larger shapes of bellmouths, laminar flow will persist up to higher Res (Kanda, 1999a).

(3) Rc(min) of approximately 2000 is obtained in the case of a straight circular pipe, i.e., when no bellmouth is fitted on the pipe inlet (Kanda, 1999a and 1999b).

(4) The model holds for flows in circular pipes and between parallel plates, i.e., for internal flows (Kanda, 2001).

(5) For external flow such as boundary-layer flow past a flat plate, transition occurs necessarily further downstream since its steady state does not exist (Kanda, 2000a).

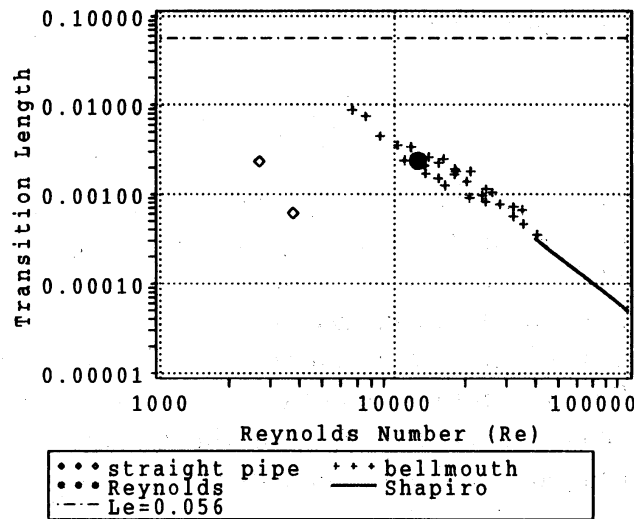(6) Disturbances are not considered in the current version of the model (Kanda, 2000b).

Fig. 2   Transition length vs Reynolds number.

2 Entrance Length and Transition Length

2.1 Entrance Length

The transition length should be compared to the entrance length. The uniform velocity profile at the pipe inlet is gradually transformed further downstream into the parabolic, Poiseuille-type distribution by the action of viscous forces on the wall. The entrance length (Ze) is the distance between the pipe inlet and the point where the velocity profile grows into the fully developed, parabolic distribution. The downstream region after the point Ze is called the fully developed region. The dimensionless entrance length (Le) is usually expressed as

$$Le \equiv \frac{Ze}{D \times Re}$$

Shah and London (1978) defined Le as the point where the developing centerline velocity equals 99% of the Poiseuille value u_{max}, and recommended the following correlation for Le:

$$Le = \frac{0.6}{Re(1 + 0.035Re)} + 0.056 \tag{1}$$

From Eq. (1), Le varies for Res below about 100; however, it approaches a constant value of 0.056 for Res above 600 (see the constant line of Le = 0.056 in Fig. 2).

2.2 Transition Length

When the transition length (Zt) is compared to the entrance length, the same dimensionless unit is desirable and the dimensionless transition length (Lt) is thus defined by

$$Lt \equiv \frac{Zt}{D \times Re}$$

For Reynolds' color-dye experiments (1883), Lt can be estimated as

$$Lt \approx \frac{30}{12,600} = 0.00238 \tag{2}$$

Figure 2 shows the experimental data of Lt, where the diamond and plus symbols show the transition length for flow in the straight pipe and through the bellmouth entrance, respectively (Kanda and Oshima, 1987). The black dot is the result of Reynolds' color-dye experiments. The straight line is drawn according to the result of Shapiro and Smith's experiments (1948), in which Re (based on the pipe diameter) ranged from 39,000 to 590,000. Shapiro and Smith found that transition from a laminar to turbulent boundary layer occurs at a Reynolds number (Rz, based on the distance (z) from the pipe inlet) of about 500,000, which compares well with the corresponding figure for a flat plate, i.e.,

$$Rz \equiv \frac{z u_0}{\nu} = 500,000 \tag{3}$$

From Shapiro and Smith's experimental results, Lt is expressed as

$$Lt = \frac{z}{DRe} = \frac{Rz}{(Re)^2} = \frac{500,000}{(Re)^2} \tag{4}$$

If Reynolds' critical value of 12,600 is applied to Eq. (4), Lt becomes

$$Lt = \frac{500,000}{12,600 * 12,600} = 0.00315 \tag{5}$$

Although this value of 0.00315 is to some extent larger than the value of 0.00238, which is calculated using Eq.(2), they are of the same order of magnitude.

The major conclusion for Lt is that under the conditions of ordinary disturbances, the transition should necessarily take place in the entrance region.

$$Lt \ll Le \ (\approx 0.056) \tag{6}$$

3 Effects of Bellmouth

3.1 Previous Assumptions on the Bellmouth

Bellmouths are designed to have the following effects on the entrance flow:

(1) The entrance to the pipe is well rounded, and the fluid enters smoothly from a reservoir, having an almost uniform velocity over the pipe inlet cross section.

(2) The entrance region begins at the pipe inlet, and not at the bellmouth inlet. Accordingly, the entrance length is measured as the distance between the pipe inlet and the point at which the velocity profile grows into a fully developed parabolic distribution.

(3) The fluid will always have some residual disturbances carried along with it. Bellmouths are used to minimize disturbances prior to flow entering the pipe.

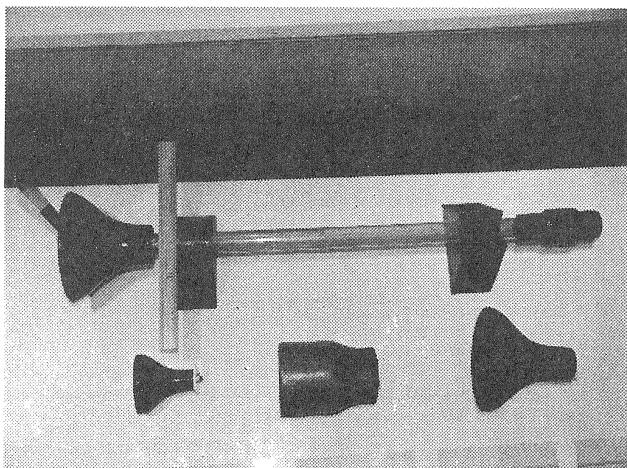The author, however, showed that a bellmouth creates a flow field similar to that in
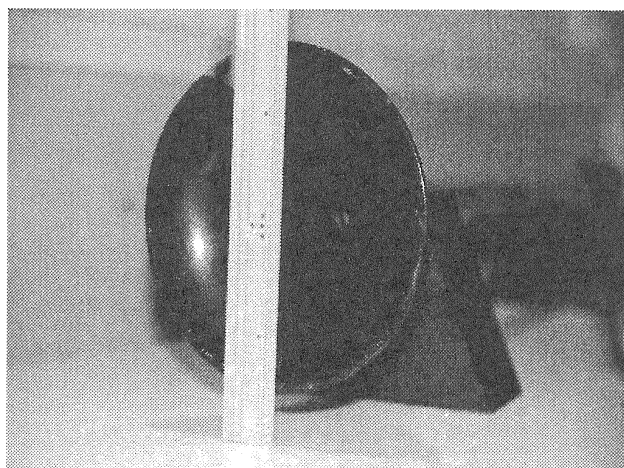
Fig. 3 Reynolds' bellmouth (a).

Fig. 4 Reynolds' bellmouth (b).

the entrance region (Kanda, 1998): (i) At the bellmouth outlet, the axial velocity is not uniform but develops a profile somewhat similar to Poiseuille's parabolic profile, because large vorticities occur on the bellmouth wall and then spread from the wall into the fluid; (ii) Since radial pressure distributions exist, Prandtl's boundary-layer assumption for pressure does not hold for the entire bellmouth region.

3.2 Bellmouth Shapes and Critical Re

We shall focus on the shape of bellmouths, especially on the ratio (γ) of bellmouth diameter to pipe diameter and consider what determines Rc. Figures 3 and 4 show the Reynolds' bellmouths in his color-dye experiments. Results of previous experimental investigations are listed in Table 1. Figure 5 is drawn by selecting entrances whose sizes are well described: Nos. 1 · 10, and 16 in Table 1.

The major conclusions for the relation between Rc and γ are as follows:
(1) Rc takes the minimum value Rc(min) when γ approaches a limit of one.

$$Rc(min) = \lim_{\gamma \to 1} Rc \approx 2000 \tag{7}$$

(2) With the same shape as the Reynolds' bellmouth, Rc increases proportionally to γ as Eq. (8).

$$Rc \approx \gamma \cdot Rc(min) \tag{8}$$

For Kanda and Oshima's value of 5790 < Rc < 6690, Rc is estimated using $\gamma = 2.9$,
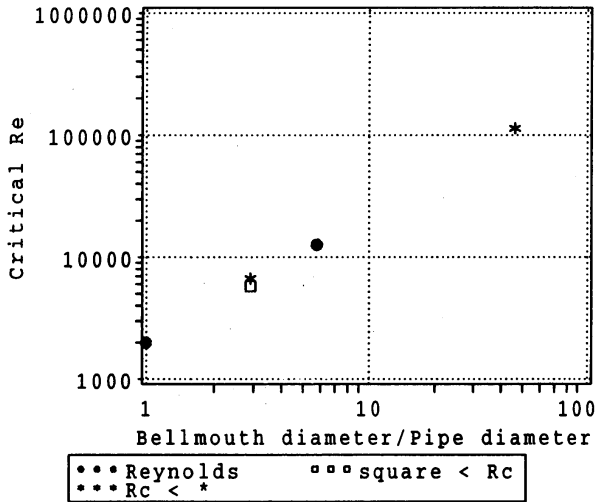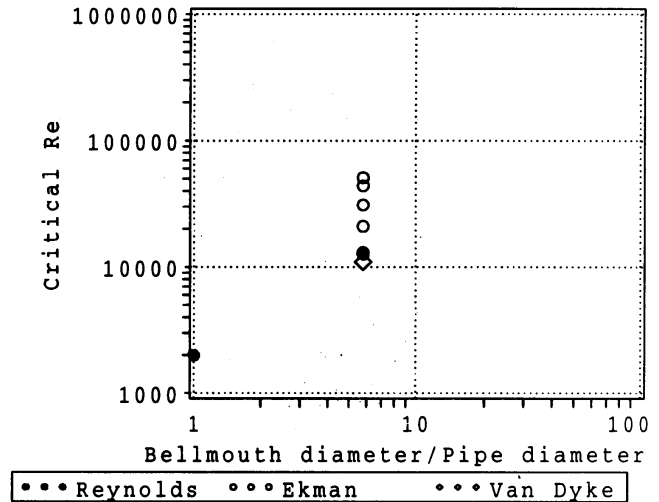
$$Rc \approx 2.9 \times 2000 \approx 5,800$$

For Reynolds' critical value of 12,600, Rc is estimated using $\gamma = 5.78$,

$$Rc \approx 5.78 \times 2000 \approx 11,560$$

For Shapiro' critical value of Rc < 113,800, Rc is estimated using $\gamma = 46.2$,

$$Rc \approx 46.2 \times 2000 \approx 92,400$$

The values calculated above are fairly close to their experimental values of Rc.

Fig. 5 Rc vs bellmouth shape γ.

Fig. 6 Rc using Reynolds' apparatus.

4 General Questions on Disturbances

The present situation of the study of the transition in pipe flows is not obvious. The obscure comprehension may be caused partly by Ekman's experimental results and partly by Van Dyke's introduction of a current critical value when using the same Reynolds' color-dye experimental apparatus. The three different critical Reynolds numbers (Rcs) were presented (see Fig. 6):

(i) Reynolds: $11.800 < Rc < 14,100$;   (ii) Ekman: $12,900 < Rc < 51,200$;
(iii) Van Dyke: $Rc < 13,000$.

It is thought that this difference is due to different disturbances in flows. It, however, may be natural to obtain nearly the same results anywhere and anytime if fluid dynamics is scientifically based, such as Rc(min) of approximately 2000.

(1) Ekman's case (Ekman, 1910)
Kanda (2000b) noted that in the first section of Ekman's paper, the word *wax* was used five times: "After the trumpet mouth had been rigidly attached to the glass tube, both were covered inside, in the neighborhood of the joint, by a layer of soft wax. ... The trumpet mouth was now fastened more rigidly, and the wax joint was improved. A preliminary experiment (No. 4) ... gave a much higher value of the critical Reynolds number ...." Apparently, the application of *wax* made Rc increase to 51,000. Then, concerning the wax, is it true that the wax coated on the joint could directly decrease disturbances?

The author considers as follows using the normal wall strength (Kanda, 1999b), which is the radial component of the curl of vorticity multiplied by (2/Re) (see Eq.(9)).

$$ normal\ wall\ strength \ \equiv\ \frac{2}{Re}[\nabla \times \omega]_r\Big|_{r=R} \ =\ -\frac{2}{Re}\frac{\partial \omega}{\partial z}\Big|_{r=R} \ > \ 0 \qquad (9) $$

where ω is the vorticity, r the radial coordinate, and R the pipe radius:

(i) the viscosity of the wax is higher than that of water; (ii) Re is inversely proportional to viscosity; (iii) the normal wall strength varies inversely to Re; (iv) the higher viscosity of the wax caused the normal wall strength to be much higher than in the case of without wax (Reynolds' experiments) and thus Rc increased, and; (v) in the case of without wax (No. 2 and 3), Ekman's results are nearly equal to that of Reynolds: Rc = approximately 13,000.

(2) Van Dyke's case

Van Dyke (1982) states: " ... the original apparatus has survived at the University of Manchester. Using it a century later, N. H. Johannesen and C. Lowe have taken this sequence of photographs. ... Modern traffic in the streets of Manchester made Rc lower than the value 13,000 found by Reynolds."

Professor J. D. Jackson of the University of Manchester kindly allowed the author to take photographs of the original apparatus on May 25, 1994 (see Figs. 3 and 4). *Of the original apparatus, the bellmouths (trumpet mouths) have been safely kept in the glass case there.* The bellmouth currently used at the University of Manchester differs from the original ones, so that Rc observed by Johannesen and Lowe differed from the results of Reynolds' experiments.

Conclusions

The following conclusions are derived under the condition of an ordinary disturbance or a natural one in flow.

(1) The transition from laminar to turbulent flow occurs in the entrance region since the dimensionless transition length (Lt) is less than 0.01 for most experiments.

(2) It is the shape of pipe entrances that determines the critical Reynolds number (Rc). With larger shapes of bellmouths, laminar flow will persist up to higher Reynolds numbers.

(3) The minimum critical Reynolds number (Rc(min)) of approximately 2000 is obtained in the following two cases:

(i) The first case is from the Reynolds' pressure experiments; i.e., when fluid is initially admitted in a high state of disturbance, as the fluid proceeds along the pipe, the turbulent flow settles down to a stable condition. Above Rc(min), the turbulent flow never settles down to a stable condition.

(ii) The second is from the experimental data plotted in Fig. 5; in the case of a straight circular pipe only, i.e., when no bellmouth is fitted on the pipe inlet.

(4) Although there is apparently a marked difference in phenomena between (3-i) and (3-ii), the theory of the occurrence of transition and the theory of the settlement of turbulence should be the same (Kanda, 2000a).

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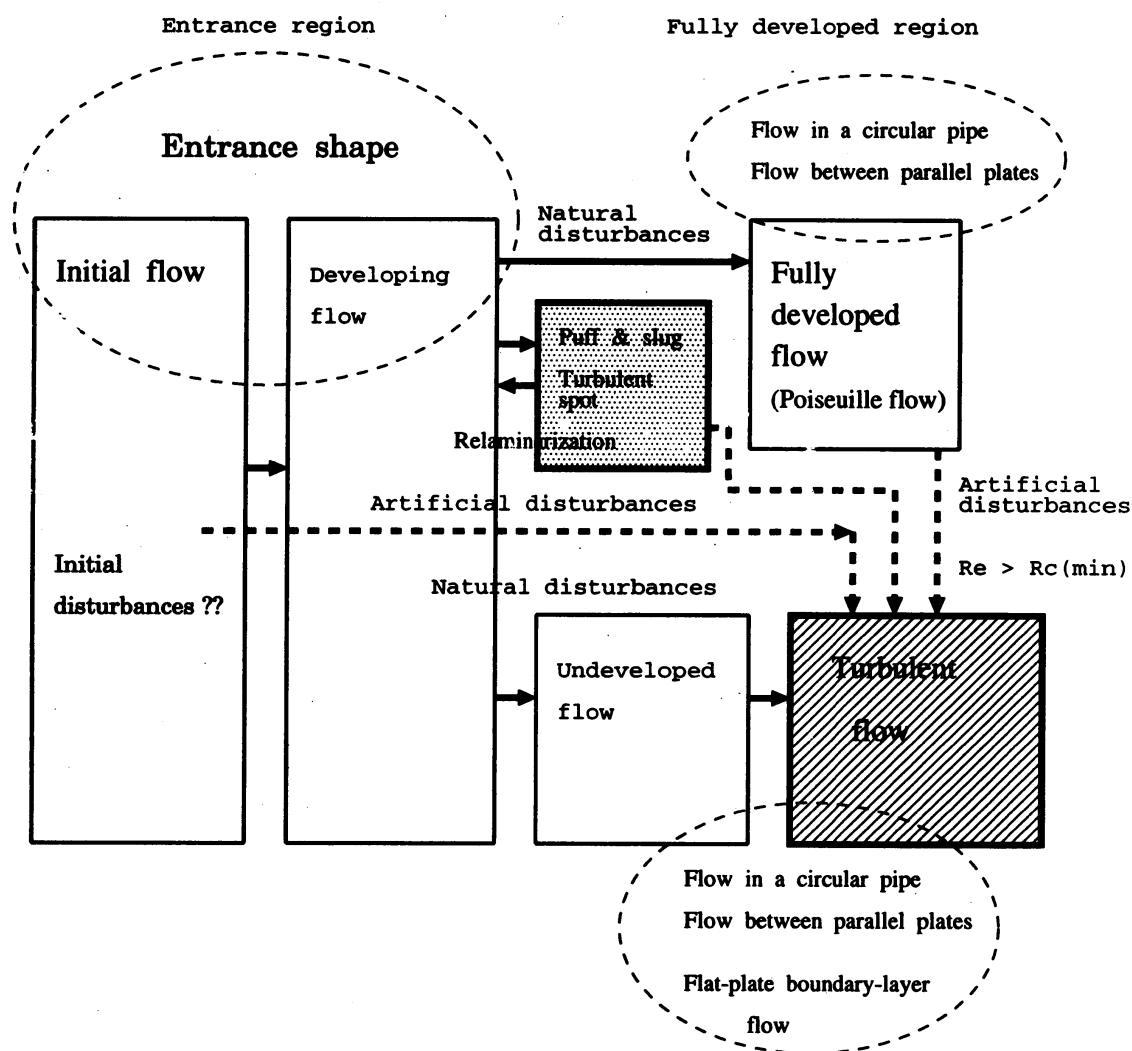
Fig. 1   Modeling and simulation.

Table 1   Critical Reynolds numbers and entrance shapes

No	Rc	Bellmouth		Pipe diameter (D)	BD/D & (γ)	Author
		diameter (BD)	length			
1	2030 & 2080	-	-		1	Reynolds, 1883
2	1900	-	-		1	Goldstein, 1965
3	2100	-	-		1	Goldstein, 1965
4	2500	-	-		1	Stanton & Pannel (i), 1914
5	2100	-	-		1	Stanton & Pannel (ii), 1914
6	2100	-	-	1.27, 1.91	1	Senecal & Rothfus, 1953
7	1930 <Rc < 2702	-	-	3	1	Kanda & Oshima, 1987
8	5790 <Rc < 6690	8.7	4	3	2.9	Kanda & Oshima, 1987
9	12,600	15.5	< 10	2.68	5.78	Reynolds, 1883
10	Rc <113,800	86.4	10.16	1.87	46.2	Shapiro et al. (pipe 4), 1948
11	6000			5.13		Reshotko, 1958
		Calming chamber			CD/D	
		diameter (CD)	length			
12	Rc <78,000	6.35	548.6	0.947	6.70	Shapiro et al. (pipe 1), 1948
13	Rc <38,900	25.4		0.947	26.8	Shapiro et al. (pipe 2), 1948
14	Rc <130,000	25.4		1.87	13.6	Shapiro et al. (pipe 3), 1948
15	23,000 <Rc	40.8	177.5	5.13	7.95	Reshotko, 1958
16	13,000 <Rc	25.4	127	3.175	8	Leite, 1959

[note]   Length is in units of cm.