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# Remarks on Shimura's oracle cut elimination and Kripke sheaf semantics for modal predicate logics (An interim report)

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**Abstract**

Shimura [*Cut-free systems for some modal logics containing* **S4**, Reports on Mathematical Logic 26(1992), 39–65.] introduced an operator $I$ which sends a modal (propositional) logic $\mathbf{L}$ containing **S4** to a modal logic $I(\mathbf{L})$. He introduced a Gentzen-style formal system for $I(\mathbf{L})$ with *oracles* and showed some interesting results on $I$. He stated just brief words on modal *predicate* logics in a short section, and left detailed studies uncultivated. In the present article, we make remarks on this topic for modal predicate logics, especially on completeness with respect to Kripke-type (possible world) semantics. We present an example of strongly Kripke-frame complete $\mathbf{L}$ whose $I$ image $I(\mathbf{L})$ is Kripke-frame incomplete. We also show a positive result that if we take the Kripke sheaf semantics instead of the Kripke frame semantics, the operator $I$ preserves the strong Kripke-sheaf completeness.

**Keywords:** modal predicate logics, oracle sequent systems for modal logics, Kripke completeness, Kripke sheaf semantics.

## Introduction

In [11], Shimura defined a modal propositional logic $I(\mathbf{L})$ based on a modal (propositional) logic $\mathbf{L}$ containing **S4**.[1] His definition induces an operator $I$ which sends an arbitrary normal (propositional) extension $\mathbf{L}$ of **S4** to a normal extension $\mathbf{L}$) of **S4**. It

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[1]Shimura [11] introduces six logics $I(\mathbf{L})$, $II(\mathbf{L})$, $III(\mathbf{L})$, $IV(\mathbf{L})$, $V(\mathbf{L})$ and $VI(\mathbf{L})$ based on $\mathbf{L}$. In this paper, we deal only with his first logic $I(\mathbf{L})$. We can show similar results for other $I(\mathbf{L})$ and $V(\mathbf{L})$, as well.

is a natural and fascinating subject to study whether or not a property of **L** is preserved under $I$, and which property of **L** makes $I(\mathbf{L})$ to have some interesting property.

Shimura [11] introduced a Gentzen-style formal system for $I(\mathbf{L})$ with *oracles* given by **L** and showed the cut-elimination theorem with the presence of oracles. He proved some results on $I$ by making use of his oracle cut elimination, especially on the preservation of completeness with respect to Kripke-type (possible world) semantics.

He dealt only with modal propositional logics, and stated just brief words on modal *predicate* logics in a short section, saying 'analogues hold for the predicate logic,' and left detailed studies uncultivated. In the present article, we make remarks on non-preservation and preservation of the (strong) completeness with respect to Kripke-type (possible world) semantics for modal predicate logics. Indeed, we present an example of **L** which is strongly complete with respect to Kripke frame semantics, but whose $I$ image $I(\mathbf{L})$ is Kripke frame incomplete.

However, we can make the situation much better, if we take the Kripke sheaf semantics instead of the Kripke frame semantics. That is, the operator $I$ preserves the strong completeness of **L** with respect to Kripke sheaf semantics.

In Section 1, we give some preliminaries to make this article rather self-contained. Shimura's $I(\mathbf{L})$ and its Gentzen-type formal system with oracles are also presented here in the setting of modal predicate logics. In Section 2, we give preliminaries of Kripke frame semantics and present a modal *predicate* logic which is a counter example of a simple predicate analogue of Shimura's preservation result on modal propositional logics. The Kripke sheaf semantics is briefly explained in Section 3. We show here a Claim which is left to be proved in Section 2, and complete the proof of the counter example. In Section 4, we show an affirmative result which is a modal predicate analogue of Shimura's Theorem in [11] by making use of the Kripke sheaf semantics. Section 5 is devoted to make concluding remarks.

# 1 Preliminaries

We fix a pure first-order modal language $\mathcal{L}$, which consists of logical connectives $\vee$ (disjunction), $\wedge$ (conjunction), $\supset$ (implication), $\neg$ (negation), a modal operator $\Box$ (necessity), and quantifiers $\exists$ (existential quantifier) and $\forall$ (universal quantifier), a denumerable list of individual variables and a denumerable list of $m$-ary predicate variables for each $m < \omega$. As usual, 0-ary predicate variables are identified with propositional variables. Note that $\mathcal{L}$ contains neither individual constants nor function symbols.

Our basic modal logic is the first-order modal predicate logic $\mathbf{S4}_*$. Here we define $\mathbf{S4}_*$ in the Gentzen-style formal system **GS4**.

**Definition 1.1 (GS4: Gentzen-Style System for $\mathbf{S4}_*$)**
As usual, upper case Greek letters $\Gamma$, $\Sigma$, ... stand for finite (possibly empty) sequences of formulas. Let **LK** be Gentzen's sequent calculus for first-order classical logic. The

**GS4** is defined by adding to **LK** two rules for $\Box$.

$$\frac{A, \Gamma \to \Theta}{\Box A, \Gamma \to \Theta}\ (\Box \to) \qquad\qquad \frac{\Box\Gamma \to A}{\Box\Gamma \to \Box A}\ (\mathbf{S4} \to \Box)$$

where $\Box\Gamma$ is the sequence of formulas $\Box B_1$, $\Box B_2$, ..., $\Box B_n$ with $\Gamma$ being $B_1, B_2, \ldots, B_n$. A formula $A$ is said to be *provable* in $\mathbf{S4}_*$, if the sequent $\to A$ is provable in **GS4**.

It is well-known that **GS4** enjoys the cut-elimination theorem. That is,

**Fact 1.2** *Each proof $P$ of* **GS4** *can be transformed into a cut-free proof $P'$ with the same end-sequent of $P$.*

In this article, a *modal predicate logic* is understood as a set **L** of formulas of $\mathcal{L}$ which satisfies the following five conditions:

(1) **L** contains all formulas provable in $\mathbf{S4}_*$,

(2) **L** is closed under the rule of modus ponens (from $A$ and $A \supset B$, infer $B$),

(3) **L** is closed under the rule of necessitation (from $A$, infer $\Box A$),

(4) **L** is closed under the rule of generalization (from $A$, infer $\forall x A$),

(5) **L** is closed under the rule of substitution (from $A$, infer $\check{S}_B^{p(u_1,\ldots,u_n)} A|$).[2]

Following these terminologies, we identify $\mathbf{S4}_*$ with the set of formulas provable in it. For a set $S$ of formulas of $\mathcal{L}$, we denote by $\mathbf{S4}_* + S$ the smallest modal predicate logic containing $\mathbf{S4}_* \cup S$. If $S = \{X_1, \ldots, X_n\}$, we write $\mathbf{S4}_* + X_1 + \cdots + X_n$ instead of $\mathbf{S4}_* + \{X_1, \ldots, X_n\}$. Note that modal predicate logics are all normal extensions of $\mathbf{S4}_*$. We denote by $\mathsf{NExtS4}_*$ the set of all modal predicate logics. Now we define the operator $I : \mathsf{NExtS4}_* \to \mathsf{NExtS4}_*$.

**Definition 1.3 (Cf. Shimura [11])** Let **L** be an arbitrary normal modal logic containing $\mathbf{S4}_*$. We define $I(\mathbf{L})$ by putting:

$$I(\mathbf{L}) = \mathbf{S4}_* + \{B \vee \Box(B \supset A)\ ;\ A \in \mathbf{L}\}.$$

Note that as an axiom schema, $B \vee \Box(B \supset A)$ is equivalent to $p \vee \Box(p \supset A)$, where $p$ is a propositional variable not occurring in $A$. It is obvious that $\mathbf{S4}_* \subseteq I(\mathbf{L}) \subseteq \mathbf{L}$ for every $\mathbf{L} \in \mathsf{NExtS4}_*$.

[2]For the precise definition of $\check{S}_B^{p(u_1,\ldots,u_n)} A|$, see Church [1].

**Definition 1.4 (G$I$(L): Sequent system for $I$(L))** Shimura's oracle sequent system for $I(\mathbf{L})$ is built on the base of Gentzen's **LK** by adding two inference rules for modal operator $\Box$. One of these rules is the left-$\Box$ rule ($\Box \rightarrow$) for **GS4**;

$$\frac{A, \Gamma \rightarrow \Theta}{\Box A, \Gamma \rightarrow \Theta}\ (\Box \rightarrow)$$

and another one is the right-$\Box$ rule which is made applicable by consulting *oracles* given by **L**;

$$\frac{\Box\Gamma, \Pi \rightarrow \Lambda, A \qquad [\Box\Gamma \rightarrow \Box A]}{\Box\Gamma, \Pi \rightarrow \Lambda, \Box A}\ (\mathbf{G}I(\mathbf{L}) \rightarrow \Box),$$

where $[\Box\Gamma \rightarrow \Box A]$ means '$\Box\Gamma \rightarrow \Box A$ is provable in **L**'.

**Fact 1.5** G$I$(L) *is equivalent to* $I(\mathbf{L})$. *That is, for every formula $A$, $A$ is in $I(\mathbf{L})$ if and only if the sequent $\rightarrow A$ is provable in* G$I$(L).

One of the interesting achievement in Shimura [11] is that G$I$(L) enjoys the cut-elimination theorem. That is,

**Fact 1.6 (Shimura [11])** *Each proof $P$ of* G$I$(L) *can be transformed into a cut-free proof $P'$ with the same end-sequent of $P$.*

# 2 Kripke frame semantics for modal predicate logics

In this section we recall basics of the Kripke frame semantics for modal propositional and predicate logics containing $\mathbf{S4}_*$. Shimura's preservation result on modal propositional logics is also recalled here. We present a modal *predicate* logic which is a counterexample of a simple predicate analogue of Shimura's preservation result.

For each non-empty set $U$, we denote by $\mathcal{L}[U]$ the language obtained from $\mathcal{L}$ by adding the name $\overline{u}$ of each $u \in U$. In what follows, we sometimes use the same letter $u$ for the name of $u$. We sometimes identify $\mathcal{L}[U]$ with the set of all sentences of $\mathcal{L}[U]$.

**Definition 2.1** A quasi-ordered set $\mathbf{M} = \langle M, R\rangle$ with the $R$-least element $0_{\mathbf{M}}$ is said to be a *Kripke base*. That is, $R$ is a reflexive and transitive relation on $M$, and $0_{\mathbf{M}}Ra$ for every $a \in \mathbf{M}$. A pair $\langle \mathbf{M}, U\rangle$ of a Kripke base $\mathbf{M} = \langle M, R\rangle$ and a mapping $U$ of $M$ to the power set $2^S$ of some nonempty set $S$ is said to be a *Kripke frame*, if (1) $U(a) \neq \emptyset$ for every $a \in M$, and (2) for every $a, b \in M$, $aRb$ implies $U(a) \subseteq U(b)$.

A binary relation $\models$ between each $a \in M$ and each atomic sentence of $\mathcal{L}[U(a)]$ is said to be a *valuation* on $\langle \mathbf{M}, U\rangle$. We extend $\models$ to a relation between each $a \in M$ and each sentence of $\mathcal{L}[U(a)]$ inductively as follows:

- $a \models A \wedge B$ if and only if $a \models A$ and $a \models B$,
- $a \models A \vee B$ if and only if $a \models A$ or $a \models B$,
- $a \models A \supset B$ if and only if $a \not\models A$ or $a \models B$,
- $a \models \Box A$ if and only if $b \models B$ for every $b \in M$ with $aRb$,
- $a \models \neg A$ if and only if $a \not\models A$,
- $a \models \forall x A(x)$ if and only if for every $u \in U(a)$, $a \models A(\overline{u})$,
- $a \models \exists x A(x)$ if and only if there exists $u \in U(a)$ such that $a \models A(\overline{u})$.

A pair $(\mathcal{F}, \models)$ of a Kripke frame $\mathcal{F}$ and a valuation $\models$ on it is said to be a *Kripke-frame model*. A formula $A$ of $\mathcal{L}$ is said to be *true* in a Kripke-frame model $(\mathcal{F}, \models)$ if $a \models \overline{A}$ for every $a \in \mathbf{M}$, where $\overline{A}$ is the universal closure of $A$. A formula $A$ of $\mathcal{L}$ is said to be *valid* in a Kripke frame $\mathcal{F}$ if for every valuation $\models$ on $\mathcal{F}$, $A$ is true in $(\mathcal{F}, \models)$. The set of formulas of $\mathcal{L}$ valid in $\mathcal{F} = \langle \mathbf{M}, U \rangle$ is denoted by $L(\mathcal{F})$ or $L\langle \mathbf{M}, U \rangle$. The following proposition is a fundamental property of Kripke frame semantics.

**Proposition 2.2** *For each Kripke frame $\mathcal{F}$, the set $L(\mathcal{F})$ contains all formulas provable in* $\mathbf{S4}_*$, *and is closed under the modus ponens, the rule of necessitation, the rule of generalization and the rule of substitution. Namely, $L(\mathcal{F})$ is a modal predicate logic.*

By the above Proposition 2.2, the set $\bigcap_{\mathcal{F} \in \mathcal{C}} L(\mathcal{F})$ is always a modal predicate logic for every class $\mathcal{C}$ of Kripke frames. Suppose that we have a class $\mathcal{C}$ of Kripke frames such that $\mathbf{L} = \bigcap_{\mathcal{F} \in \mathcal{C}} L(\mathcal{F})$. Then $\mathbf{L}$ is said to be *complete* with respect to $\mathcal{C}$, or $\mathcal{C}$ *characterizes* $\mathbf{L}$. We have a stronger concept.

**Definition 2.3** Let $\mathbf{L}$ be a modal predicate logic. A pair $(S, T)$ of sets of formulas of $\mathcal{L}$ is said to be $\mathbf{L}$-*inconsistent*, if there exists $A_1, A_2, \ldots, A_k \in S$ and $B_1, B_2, \ldots, B_l \in T$ such that $A_1 \wedge A_2 \wedge \ldots \wedge A_k \supset B_1 \vee B_2 \vee \ldots \vee B_l$ is provable in $\mathbf{L}$. A pair $(S, T)$ of sets of formulas of $\mathcal{L}$ is said to be $\mathbf{L}$-*consistent*, if $(S, T)$ is not inconsistent.

Let $\mathcal{C}$ be a class of Kripke frames. A modal predicate logic $\mathbf{L}$ is said to be *strongly complete* with respect to $\mathcal{C}$, if

(1) $\mathbf{L} \subseteq L(\mathcal{F})$ for every $\mathcal{F} \in \mathcal{C}$,

(2) for every $\mathbf{L}$-consistent pair $(S, T)$, there exits a Kripke frame $\mathcal{F} = \langle \mathbf{M}, U \rangle \in \mathcal{C}$, a mapping $f$ of the set $FV$ of all free individual variables to $U(0_{\mathbf{M}})$, and a valuation $\models$ such that (a) $0_{\mathbf{M}} \models A^f$ for every $A \in S$, (b) $0_{\mathbf{M}} \not\models B^f$ for every $B \in T$.

Here $A^f$ (and $B^f$) is the sentence obtained from $A$ ($B$, respectively) by replacing all free occurrences of each free individual variable $x \in FV$ by the name $\overline{f(x)}$ of $f(x)$.

**Definition 2.4** Let $\{\mathcal{F}_i \,;\, i \in I\}$ be a set of Kripke frames with $\mathcal{F}_i = \langle \mathbf{M}_i, U_i \rangle$ and $\mathbf{M}_i = \langle M_i, R_i \rangle$ for each $i \in I$. We may assume that $M_i \cap M_j = \emptyset$ $(i \neq j)$. By $\sum_{i \in I} \mathbf{M}_i$, we mean the quasi-ordered set obtained as the disjoint union of $\{\mathbf{M}_i \,;\, i \in I\}$. Suppose we have a new element $0 \notin \bigcup_{i \in I} M_i$. We define $0 \uparrow \sum_{i \in I} \mathbf{M}_i$ as the quasi-ordered set obtained from $\sum_{i \in I} \mathbf{M}_i$ by adding the new $R$-least element 0. Note that if $I = \emptyset$, then $0 \uparrow \sum_{i \in I} \mathbf{M}_i$ is the singleton $\{0\}$.

Suppose next that $V$ is a non-empty set such that $V \subseteq \cap_{i \in I} U(0_{\mathbf{M}_i})$. We define $(0, V) \uparrow \sum_{i \in I} \mathcal{F}_i$ as the Kripke frame $\langle 0 \uparrow \sum_{i \in I} \mathbf{M}_i, U \rangle$ whose base is $0 \uparrow \sum_{i \in I} \mathbf{M}_i$ and for every $a \in 0 \uparrow \sum_{i \in I} \mathbf{M}_i$,

$$U(a) = \begin{cases} V & (a = 0), \\ U_i(a) & (a \in \mathbf{M}_i). \end{cases}$$

If $I = \{1, 2, \ldots, n\}$, we write $0 \uparrow \sum_{i \in I} \mathbf{M}_i$ by $0 \uparrow (\mathbf{M}_1, \ldots, \mathbf{M}_n)$, and $(0, V) \uparrow \sum_{i \in I} \mathcal{F}_i$ by $(0, V) \uparrow (\mathcal{F}_1, \ldots, \mathcal{F}_n)$.

Now we recall Shimura's completeness result in [11]. Note that Kripke bases are the Kripke frames for modal *propositional* logics.

**Fact 2.5 (Theorem 3.2 in [11])** *Let* **L** *be a modal propositional logic containing* **S4** *characterized by a class $C$ of Kripke bases. Then, propositional $I(\mathbf{L})$ is characterized by the Kripke bases of the form $0 \uparrow (\mathbf{M}_1, \ldots, \mathbf{M}_n)$ where $\mathbf{M}_1, \ldots, \mathbf{M}_n \in C$ and $n \geq 1$.*

Shimura [11] stated that an analogue of this Fact holds for the predicate logics. Here is a very simple analogue of Fact 2.5 due to Shimura (personal communication).

**Shimura's Analogue for the Predicate Logics** *Let* **L** *be a modal predicate logic characterized by a class $C$ of Kripke frames. Then, $I(\mathbf{L})$ is characterized by to the class of Kripke frames of the form $(0, V) \uparrow \sum_{i \in I} \mathcal{F}_i$ where $\mathcal{F}_i \in C$ $(i \in I)$.*

Here we have a counterexample to this statement. Let $\mathbf{S4}^*$ be the logic $\mathbf{S4}_* + \exists x q(x) \supset \forall x q(x)$, where $q$ is a unary predicate variable.

**Lemma 2.6** $\mathbf{S4}^*$ *is strongly characterized by a class of Kripke frames.*

*Proof.* For every Kripke base $\mathbf{M} = \langle M, R \rangle$, we denote by $\mathbf{M}^o$ the Kripke frame $\langle \mathbf{M}, U^o \rangle$ with the constant mapping $U$ whose image is a singleton i.e., $U(a) = \{0\}$ for every $a \in M$. Then $\mathbf{S4}^*$ is strongly characterized by the class of Kripke frames of the form $\mathbf{M}^o$, since propositional **S4** is strongly complete with respect to the class of Kripke frames for modal propositional logics. (Recall that Kripke bases are the Kripke frames for modal propositional logics.) □

**Lemma 2.7** *No class of Kripke frames characterizes $I(\mathbf{S4}^*)$.*

*Proof.* Note that $\exists x q(x) \vee \Box(\exists x q(x) \supset \forall x q(x))$ is provable in $I(\mathbf{S4}^*)$, since $p \vee \Box(p \supset (\exists x q(x) \supset \forall x q(x)))$ is provable in $I(\mathbf{S4}^*)$, where $p$ is a new propositional variable. Then we have the following two claims.

**Claim 1.** *If* $\exists x q(x) \vee \Box(\exists x q(x) \supset \forall x q(x))$ *is valid in a Kripke frame, then so is* $\Box p \vee \Box\neg\Box p \vee \Box(\exists x q(x) \supset \forall x q(x))$.

**Claim 2.** $\Box p \vee \Box\neg\Box p \vee \Box(\exists x q(x) \supset \forall x q(x))$ *is not provable in* $I(\mathbf{S4}^*)$.

Claim 2 will be shown in the next section by making use of the Kripke sheaf semantics. We show here Claim 1. Let $\mathcal{F} = \langle \mathbf{M}, U \rangle$ be a Kripke frame such that $\Box p \vee \Box\neg\Box p \vee (\exists x q(x) \supset \forall x q(x)) \notin L(\mathcal{F})$, We prove that $\exists x q(x) \vee \Box(\exists x q(x) \supset \forall x q(x)) \notin L(\mathcal{F})$. By the assumption, there is an element $a \in \mathbf{M} = \langle M, R \rangle$ and a valuation $\models$ on $\mathcal{F}$ such that:

(1) $a \not\models \Box p$,

(2) $a \not\models \Box\neg\Box p$,

(3) $a \not\models \Box(\exists x q(x) \supset \forall x q(x))$.

By (1) and (2), we have an element $b \in \mathbf{M}$ with $aRb$ and $a \neq b$. By (3), we have a $c \in \mathbf{M}$ and $\alpha$, $\beta \in U(c)$ with $aRc$ and $\alpha \neq \beta$. If $a \neq c$, then define a valuation $\models^1$ by:

$$x \models^1 q(u) \text{ if and only if } x = c \text{ and } u = \alpha,$$

for every $x \in \mathbf{M}$ and every $u \in U(x)$. Then, we have $a \not\models^1 \exists x q(x) \vee \Box(\exists x q(x) \supset \forall x q(x))$. If $a = c$, then $\{\alpha, \beta\} \subseteq U(a) \subseteq U(b)$. Define a valuation $\models^2$ by:

$$x \models^2 q(u) \text{ if and only if } x = b \text{ and } u = \alpha,$$

for every $x \in \mathbf{M}$ and every $u \in U(x)$. Then, we have $a \not\models^2 \exists x q(x) \vee \Box(\exists x q(x) \supset \forall x q(x))$. Hence, $\exists x q(x) \vee \Box(\exists x q(x) \supset \forall x q(x))$ is not valid in $\mathcal{F}$. This completes the proof of Claim 1.

Now our Lemma directly follows from these two Claims. □

This example shows the following.

**Corollary 2.8** *There is a strongly Kripke-frame complete modal predicate logic* $\mathbf{L}$ *such that* $I(\mathbf{L})$ *fails to be Kripke-frame complete.*

Now we know that Shimura's analogue of Fact 2.5 does not work well. Moreover, even if we put strong assumption of Fact 2.5 that $\mathbf{L}$ is strong Kripke-frame complete, and even if we relax the conclusion that $I(\mathbf{L})$ is just Kripke-frame complete, the statement is not true. In the next Section, we introduce the Kripke sheaf semantics to get rid of this difficulty.

# 3 Kripke sheaf semantics for predicate logics

In this section we prepare the Kripke sheaf semantics to make this article self-contained. We refer readers to [14] for details[3].

**Definition 3.1** We can regard a Kripke base $\mathbf{M} = \langle M, R \rangle$ as a category in the usual way. Let $\mathcal{S}$ denote the category of all non-empty sets. A covariant functor $D$ from a Kripke base $\mathbf{M}$ to $\mathcal{S}$ is called a *domain-sheaf* over $\mathbf{M}$. That is,

DS1) $D(a)$ is a non-empty set for every $a \in M$,
DS2) for every $a, b \in M$ with $aRb$, there exists a mapping $D_{ab} : D(a) \to D(b)$,
DS3) $D_{aa}$ is the identity mapping $id_{D(a)}$ of $D(a)$ for every $a \in M$,
DS4) $D_{ac} = D_{bc} \circ D_{ab}$ for every $a, b, c \in M$ with $aRb$ and $bRc$.

A pair $\mathcal{K} = \langle \mathbf{M}, D \rangle$ of a Kripke base $\mathbf{M}$ and a domain-sheaf $D$ over $\mathbf{M}$ is called a *Kripke sheaf*. If every $D_{ab}$ ($aRb$) is the set-theoretic inclusion, $\langle \mathbf{M}, D \rangle$ is said to be a *Kripke frame*.

For each $d \in D(a)$ and each $b \in M$ with $aRb$, $D_{ab}(d)$ is said to be the *inheritor* of $d$ at $b$. For each formula $A$ of $\mathcal{L}[D(a)]$ and each $b \in M$ with $aRb$, the *inheritor* $A_{a,b}$ of $A$ at $b$ is a formula of $\mathcal{L}[D(b)]$ obtained from $A$ by replacing occurrences of $\overline{u}$ ($u \in D(a)$) by the name $\overline{v}$ of the inheritor $v$ of $u$ at $b$.

A binary relation $\models$ between each $a \in M$ and each atomic sentence of $\mathcal{L}[D(a)]$ is said to be a *valuation* on $\langle \mathbf{M}, D \rangle$. We extend $\models$ to a relation between each $a \in M$ and each sentence of $\mathcal{L}[D(a)]$ inductively as follows:

- $a \models A \wedge B$ if and only if $a \models A$ and $a \models B$,
- $a \models A \vee B$ if and only if $a \models A$ or $a \models B$,
- $a \models A \supset B$ if and only if $a \not\models A$ or $a \models B$,
- $a \models \Box A$ if and only if $b \models B_{a,b}$ for every $b \in M$ with $aRb$,
- $a \models \neg A$ if and only if $a \not\models A$,
- $a \models \forall x A(x)$ if and only if for every $u \in D(a)$, $a \models A(\overline{u})$,
- $a \models \exists x A(x)$ if and only if there exists $u \in D(a)$ such that $a \models A(\overline{u})$.

A pair $(\mathcal{K}, \models)$ of a Kripke sheaf $\mathcal{K}$ and a valuation $\models$ on it is said to be a *Kripke-sheaf model*. A formula $A$ of $\mathcal{L}$ is said to be *true* in a Kripke-sheaf model $(\mathcal{K}, \models)$ if $a \models \overline{A}$ for every $a \in \mathbf{M}$, where $\overline{A}$ is the universal closure of $A$. A formula $A$ of $\mathcal{L}$ is said to be

[3]The paper [14] dealt mainly with *superintuitionistic* predicate logics, not modal predicate logics. However, the reader can find basic information on the Kripke sheaf semantics for modal predicate logics in Section 5 of [14].

*valid* in a Kripke sheaf $\mathcal{K}$ if for every valuation $\models$ on $\mathcal{K}$, $A$ is true in $(\mathcal{K}, \models)$. The set of formulas of $\mathcal{L}$ valid in $\mathcal{K} = \langle \mathbf{M}, D\rangle$ is denoted by $L(\mathcal{K})$ or $L\langle \mathbf{M}, D\rangle$. The following proposition is a fundamental property of Kripke-sheaf semantics.

**Proposition 3.2** *For each Kripke-sheaf $\mathcal{K}$, the set $L(\mathcal{K})$ contains all formulas provable in* $\mathbf{S4}_*$, *and is closed under the modus ponens, the rule of necessitation, the rule of generalization and the rule of substitution. Namely, $L(\mathcal{K})$ is a modal predicate logic.*

This property ensures that Kripke sheaves can be used for the study of modal predicate logics. Suppose for example that we have given a given formula $A$ and a modal predicate logic $\mathbf{L} = \mathbf{S4}_* + X_1 + \cdots + X_n$. If we can construct a Kripke sheaf $\langle M, D\rangle$ such that 1) $X_1, \ldots, X_n$ are valid in $\langle M, D\rangle$, and 2) $A$ is not valid in $\langle M, D\rangle$. Then, by the virtue of this Proposition, we have that $A \notin \mathbf{L}$.

We define completeness and strong completeness of a modal predicate logic with respect to the Kripke sheaf semantics. The definitions are just the same as those for Kripke frames, except replacing 'frame(frames)' by 'sheaf(sheaves)'.

**Definition 3.3** Let $\{\mathcal{K}_i \ ; \ i \in I\}$ be a set of Kripke sheaves with $\mathcal{K}_i = \langle \mathbf{M}_i, D_i\rangle$ and $\mathbf{M}_i = \langle M_i, R_i\rangle$ for each $i \in I$. Suppose next that we have a non-empty set $V$ and a family $f = \{f_i : V \to D_i(0_{\mathbf{M}_i}) \ ; \ i \in I\}$. We define $(0, V) \uparrow_f \sum_{i\in I} \mathcal{F}_i$ as the Kripke sheaf $\langle 0 \uparrow \sum_{i\in I} \mathbf{M}_i, D\rangle$ whose base is $0 \uparrow \sum_{i\in I} \mathbf{M}_i$ and for every $a, b \in 0 \uparrow \sum_{i\in I} \mathbf{M}_i$ with $aRb$,

$$D(a) = \begin{cases} V & (a = 0), \\ D_i(a) & (a \in \mathbf{M}_i), \end{cases} \qquad D_{ab} = \begin{cases} Id_V & (a = b = 0), \\ (D_i)_{0_{\mathbf{M}_i} b} \circ f_i & (a = 0 \text{ and } b \in \mathbf{M}_i), \\ (D_i)_{ab} & (a, b \in \mathbf{M}_i). \end{cases}$$

Now we show the Claim 2 presented in the previous section. First we have to mention the following Lemma, which was originally proved in Shimura [11] for modal predicate logics. The proof can be carried out essentially in the same way in [11].

**Lemma 3.4** *Let* $\mathbf{L}$ *be a modal predicate logic sound with respect to a class $\mathcal{C}$ of Kripke frames. That is,* $\mathbf{L} \subseteq L(\mathcal{F})$ *for every $\mathcal{F} \in \mathcal{C}$. Then, $I(\mathbf{L})$ is sound with respect to the class of Kripke frames of the form $(0, V) \uparrow_f \sum_{i\in I} \mathcal{F}_i$ where $\mathcal{F}_i \in \mathcal{C}$ $(i \in I)$.*

Let $\mathcal{F}_1 = \langle \{1\}, \{1\}\rangle$ be the Kripke frame with the trivial Kripke base $\{1\}$ whose individual domain is the singleton $\{1\}$. Let $\omega$ be the set $\{0, 1, \ldots\}$. There is a unique mapping $\pi : \omega \to \{1\}$. Since $\mathbf{S4}^* \subset L(\mathcal{F}_1)$, we have $I(\mathbf{S4}^*) \subset L(\,(0, \omega) \uparrow_\pi \mathcal{F}_1)$ by Lemma 3.4.

**Lemma 3.5 (Claim 2)** $\Box p \vee \Box\neg\Box p \vee \Box(\exists x q(x) \supset \forall x q(x)) \notin I(\mathbf{S4}^*)$.

*Proof.* Let us define a valuation $\models$ on $(0, \omega) \uparrow_\pi \mathcal{F}_1$ by:

$a \models p$ if and only if $a = 1$, $\quad a \models q(u)$ if and only if $u = 1$.

Then we have (1) $0 \not\models \Box p$, (2) $0 \not\models \Box\neg\Box p$, and (3) $0 \not\models \Box(\exists x q(x) \supset \forall x q(x))$. Hence $I(\mathbf{S4}^*) \subseteq L(\,(0, \omega) \uparrow_\pi \mathcal{F}_1) \not\ni \Box p \vee \Box\neg\Box p \vee \Box(\exists x q(x) \supset \forall x q(x))$. $\Box$

# 4 Affirmative result on predicate logics

In this section, we show an affirmative result which is a modal predicate analogue of Shimura's Theorem (Fact 2.5) by making use of the Kripke sheaf semantics. The aim of this section is to show the following.

**Theorem 4.1 (predicate version)** *Let* **L** *be a normal extension of* **S4** *strongly characterized by a class* $\mathcal{C}$ *of Kripke sheaves. Then,* $I(\mathbf{L})$ *is strongly characterized by the Kripke sheaves of the form* $(0, V) \uparrow_f \sum_{j \in J} \mathcal{K}_j$ *where* $V$ *is a non-empty set and* $\{\mathcal{K}_j \,;\, j \in J\}$ *is a countable (possibly finite) subset of* $\mathcal{C}$.

**Definition 4.2 (Cf. Komori [7], Fitting [8])** Let $P$ be a set of individual variables. A pair of $(S, T)$ is said to be $I(\mathbf{L})$*-saturated with respect to* $P$, if $(S, T)$ is $I(\mathbf{L})$-consistent, every individual variable occurring in $S \cup T$ is in $P$, and

- $A \wedge B \in S \Rightarrow A \in S$ and $B \in S$,
- $A \wedge B \in T \Rightarrow A \in T$ or $B \in T$,
- $A \vee B \in S \Rightarrow A \in S$ or $B \in S$,
- $A \vee B \in T \Rightarrow A \in T$ and $B \in T$,
- $\neg A \in S \Rightarrow A \in T$,
- $\neg A \in T \Rightarrow A \in S$,
- $A \supset B \in S \Rightarrow A \in T$ or $B \in S$,
- $A \supset B \in T \Rightarrow A \in S$ and $B \in T$,
- $\Box A \in S \Rightarrow A \in S$,
- $\Box A \in T$ and $(S^{\Box}, \{\Box A\})$ is **L**-inconsistent $\Rightarrow A \in T$, where $S^{\Box} = \{\Box B; \Box B \in S\}$
- $\forall x A(x) \in S \Rightarrow A(v) \in S$ for every $v \in P$,
- $\forall x A(x) \in T \Rightarrow A(v) \in T$ for some $v \in P$,
- $\exists x A(x) \in S \Rightarrow A(v) \in S$ for some $v \in P$,
- $\exists x A(x) \in T \Rightarrow A(v) \in T$ for every $v \in P$.

Then we can show the following Lemma in the quite similar way that is used in Fitting [8, Theorem 4.2, Ch. 5].

**Lemma 4.3** *Let $(S,T)$ be an $I(\mathbf{L})$-consistent pair. Let $Q$ be the set of all individual variables occurring freely in $S \cup T$. Take a denumerable list $v_1, v_2, \ldots$ of new individual variables not in $S$, and put $P = S \cup \{v_1, v_2, \ldots\}$. Then, there exists a $I(\mathbf{L})$-saturated pair $(S^*, T^*)$ with respect to $P$ such that $S \subseteq S^*$ and $T \subseteq T^*$.*

The above $(S^*, T^*)$ is said to be a $I(\mathbf{L})$-*saturated extension* of $(S,T)$.

The following Lemma can be shown essentially in the similar way that is used in Shimura [11, Theorem 3.2] and Komori [7, Lemma 3.12]. Shimura's Theorem deals with modal *propositional* logics, and Komori's Lemma concerns with *superintuitionistic* predicate logics and is described in the Kripke frame semantics with $\mathbf{L}$ being taken from special sequence of logics. Here we have to carry out our proof in more general setting. However, by the virtue of Kripke sheaves, we can apply Shimura's and Komori's idea more directly.

**Lemma 4.4** *Suppose that $\mathbf{L}$ is strongly complete with respect to a class $\mathcal{C}$ of Kripke sheaves. Let $(S,T)$ be a $I(\mathbf{L})$-saturated pair with respect to $P$. Then there exist a countable subset $\{\mathcal{K}_j = \langle \mathbf{M}_j, D_j \rangle \ ; \ j \in J\}$ of $\mathcal{C}$, a family $f = \{f_j : \omega \to D_j(0_{\mathbf{M}_j}) \ ; \ j \in J\}$ of mappings, and a valuation on $(0,\omega) \uparrow_f \sum_{j \in J} \mathcal{K}_j$ such that (1) for every $A \in S$, $0 \models A^f$, (2) for every $B \in T$, $0 \not\models B^f$.*

*Proof.* Let $J$ be the set $\{\Box A \in T \ ; \ (\Box S, \{A\})$ is $\mathbf{L}$-consistent $\}$. Then $J$ is at most countable. For each $\Box A \in J$, There are a Kripke-sheaf model $\langle \mathcal{K}_{\Box A}, \models_{\Box A} \rangle$ with $\mathcal{K}_{\Box A} = \langle \mathbf{M}_{\Box A}, D_{\Box A} \rangle \in \mathcal{C}$ and a mapping $f_{\Box A} : FV \to D_{\Box A}(0_{\mathbf{M}_{\Box A}})$ such that $0_{\mathbf{M}_{\Box A}} \not\models_{\Box A} \Box A^{f_{\Box A}}$ and $0_{\mathbf{M}_{\Box A}} \models_{\Box A} \Box X^{f_{\Box A}}$ for every $\Box X \in S^{\Box}$. Since $0_{\mathbf{M}_{\Box A}} \not\models_{\Box A} \Box A^{f_{\Box A}}$, there is an element $a_{\Box A} \in \mathbf{M}_{\Box A}$ such that $a_{\Box A} \not\models_{\Box A} (A^{f_{\Box A}})_{0_{\mathbf{M}_{\Box A}} a_{\Box A}}$.

Let $f$ be the family $\{f_{\Box A} : FV \to D_{\Box A}(0_{\mathbf{M}_{\Box A}}) \ ; \ \Box A \in J\}$, and put the Kripke sheaf $(0, FV) \uparrow_f \sum_{\Box A \in J} \mathcal{K}_{\Box A} = \langle 0 \uparrow \sum_{\Box A \in J} \mathbf{M}_{\Box A}, D \rangle$. Define a valuation $\models$ on $\mathcal{K}$ by:

$0 \models X$ if and only if $X \in S$

for every atomic formula $X$ of $\mathcal{L}$,

$a \models X$ if and only if $a \in \mathbf{M}_{\Box A}$ and $a \models_{\Box A} X$

for every $a \in \sum_{\Box A \in J} \mathbf{M}_{\Box A}$ and every atomic formula $X \in [D(a)]$. Note that $\mathcal{L}[FV]$ is identified with the set of all formulas of $\mathcal{L}$. Clearly, we have that for every $a \in \sum_{\Box A \in J} \mathbf{M}_{\Box A}$ and every formula $X \in [D(a)]$, $a \models X$ if and only if $a \in \mathbf{M}_{\Box A}$ and $a \models_{\Box A} X$. Then, by induction on the length of $X$, we can show that

$0 \models X$ if $X \in S$,

$0 \models X$ if $X \in T$.

We sketch here the most essential case that $\Box Y \in T$ implies $0 \not\models \Box Y$. Suppose that $\Box Y \in T$. If $(S^{\Box}, \{\Box Y\})$ is $\mathbf{L}$-inconsistent, then $A \in T$ by the $I(\mathbf{L})$-saturatedness of $(S,T)$. Hence, by the induction hypothesis, we have $0 \not\models Y$. Therefore $0 \not\models \Box Y$.

If $(S^{\Box}, \{\Box Y\})$ is **L**-consistent, then $\Box Y \in J$ and $a_{\Box Y} \not\models_{\Box Y} Y_{0_{\mathbf{M}_{\Box Y}} a_{\Box Y}}$. Verify that $Y_{0_{\mathbf{M}_{\Box Y}} a_{\Box Y}}$ is just the same one with $Y_0 a_{\Box Y}$. Hence $0 \not\models \Box Y$. $\Box$

Now we show Theorem 4.1. From Lemma 3.4, it follows that $I(\mathbf{M}) \subseteq L((0, V) \uparrow_f \sum_{j \in J} \mathcal{K}_j)$ for every countable (possibly finite) subset $\{\mathcal{K}_j \ ; \ j \in J\}$ of $\mathcal{C}$. Suppose that $(S, T)$ is $I(\mathbf{L})$-consistent. Then by Lemma 4.3, we have a $I(\mathbf{L})$-saturated extension $(S^*, T^*)$ of $(S, T)$. By Lemma 4.4, we have a Kripke-sheaf model with the intended property. This complete the proof of Theorem 4.1.

# 5 Concluding remarks

## 5.1 Remarks on the language

We can add countably many individual constants and function symbols to our basic language $\mathcal{L}$. We interpret individual constants and function symbols as 'global' constants and functions. That is, for every $n$-ary function symbol $f$, for every $a, b \in \mathbf{M}$ with $aRb$, and for every $\vec{u} \in D(a)^n$, it holds that $f^{I(a)}(\vec{u}) = f^{I(b)}(\vec{u})$. Here $f^{I(a)}$ and $f^{I(b)}$ are interpretations of $f$ at $a$ and $b$, respectively. Then, most results hold for this extended language, as well.

We can consider modal predicate logics with equality. Since one origin of Kripke sheaves is the Kripke frame with equality for intuitionistic predicate logic,[4] It is easy to modify the Kripke sheaf semantics suitable for modal predicate logics with equality. Namely, we have only to interpret the equality symbol $=$ as the identity relation $=$ in the domains. We shall however keep in mind that in Kripke sheaves with this interpretation of the equality, it holds that $\forall x \forall y (x = y \supset \Box(x = y))$.

## 5.2 Further research: What analogue is the best analogue?

Prof. Shimura gave me some comments on my talk presented in the meeting held at Research Institute of Mathematical Science, Kyoto University on August 2002. He suggested a possibility to retain the Kripke frame semantics with posing some condition on **L** in his Theorem (Fact 2.5). He stated one conjecture on the predicate extension of his Theorem.

**Conjecture (Shimura) Shimura's Analogue for the Predicate Logics** holds for **L** with the condition: $\mathbf{L} \subseteq \mathbf{Q\text{-}Triv} = \mathbf{S4}_* + p \supset \Box p$.

Note that the example $\mathbf{S4}^*$ does not deny his conjecture, since $\mathbf{S4}^* \not\subseteq \mathbf{Q\text{-}Triv}$. Here we have a counter example to this conjecture.

**Definition 5.1** Let $p$, $r_0$ and $r_1$ be propositional variables, and $q$ a unary predicate variable.

$Triv \ : \ p \supset \Box p,$

[4] See Dragalin [2] and Gabbay [3].

$$Z \;:\; \exists xq(x) \supset \forall xq(x),$$
$$H \;:\; r_0 \vee \Box(r_0 \wedge r_1 \supset \Box r_1).$$

Let $\omega$ be $\{0,1,\ldots\}$. Consider the Kripke frame $\langle\{0\},\omega\rangle$ whose Kripke base is the singleton $\{0\}$ and whose domain is $\omega$. Let $\mathcal{F}_2$ be the Kripke frame $\langle \mathbf{M}_2, U\rangle$ where $\mathbf{M}_2 = \{1,2\} \leq$ with $\leq$ being the natural order on it, and $U(1) = U(2) = \{0\}$. Define:

$$\mathbf{L_1} = L\langle\{0\},\omega\rangle,$$
$$\mathbf{L_2} = L(\mathcal{F}_2), \text{ and}$$
$$\mathbf{L} = \mathbf{L_1} \cap \mathbf{L_2}.$$

**Lemma 5.2** **L** *is Kripke-frame complete.*

*Proof.* By the above definition, $\{\langle\{0\},\omega\rangle, \mathcal{F}_2\}$ characterizes **L**. □

**Proposition 5.3** $I(\mathbf{L})$ *is Kripke-frame incomplete.*

**Lemma 5.4** $\exists xq(x) \vee \Box(\exists xq(x) \supset \forall xq(x) \vee Triv) \in I(\mathbf{L})$.

*Proof.* Since $\exists xq(x) \supset \forall xq(x) \vee Triv \in \mathbf{L}$, we have $\exists xq(x) \vee \Box(\exists xq(x) \supset (\exists xq(x) \supset \forall xq(x) \vee Triv)) \in I(\mathbf{L})$. □

**Lemma 5.5** *If* $\exists xq(x) \vee \Box(\exists xq(x) \supset \forall xq(x) \vee Triv)$ *is valid in a Kripke frame, then so is* $H \vee \Box(\exists xq(x) \supset \forall xq(x) \vee Triv)$.

*Proof.* We show the statement by proving the contraposition. Suppose $H \vee \Box(\exists xq(x) \supset \forall xq(x) \vee Triv)$ is not valid in a Kripke frame $\mathcal{F} = \langle \mathbf{M}, U\rangle$. Then there are a valuation $\models$ on $\mathcal{F}$ and $a \in \mathbf{M}$ such that
(1) $a \not\models H$, and
(2) $a \not\models \Box(\exists xq(x) \supset \forall xq(x) \vee Triv)$.
By (2), there exists an element $b \in \mathbf{M}$ such that $aRb$, $b \models \exists xq(x)$ and $b \not\models \forall xq(x) \vee Triv$. If $a \neq b$, then we change $\models$ at $a$ as $a \not\models q(u)$ for all $u \in U(a)$. Then $a \not\models \exists xq(x)$, and hence we have $a \not\models \exists xq(x) \vee \Box(\exists xq(x) \supset \forall xq(x) \vee Triv)$. Suppose that $a = b$. Then, from $b \models \exists xq(x)$ and $b \not\models \forall xq(x) \vee Triv$, it follows that there exist $\alpha, \beta \in U(b) = U(a)$ with $\alpha \neq \beta$. By (1), By (2), there exist elements $c_0, c_1 \in \mathbf{M}$ such that $aRc_0$, $c_0Rc_1$, and
(1-1) $a \not\models r_0$,
(1-2) $c_0 \models r_0 \wedge r_1$, and
(1-3) $c \not\models r_1$.
It is clear that $a \neq c_0$ and $c_0 \neq c_1$. Note that $\{\alpha,\beta\} \subseteq U(a) \subseteq U(c_0) \subseteq U(c_0)$. Define a valuation $\models'$ at $a$, $c_0$ and $c_1$ by
$a \not\models' q(u)$ for all $u \in U(a)$
$c_0 \models' q(u)$ if and only if $u = \alpha$

$c_0 \models' p$
$c_1 \not\models' p$
Then, we have $a \not\models' \exists x q(x)$, $c_0 \models' \exists x q(x)$, $c_0 \not\models' \forall x q(x)$, and $c_0 \not\models' Triv$. Therefore, $a \not\models \exists x q(x) \vee \Box(\exists x q(x) \supset \forall x q(x) \vee Triv)$. □

**Lemma 5.6** $H \vee \Box(\exists x q(x) \supset \forall x q(x) \vee Triv) \notin I(\mathbf{L})$.

*Proof.* There is a unique mapping $\pi : \omega \to \{0\}$. Since $\mathbf{L} \subset L(\mathcal{F}_2)$, we have $I(\mathbf{L}) \subset L((0,\omega) \uparrow_\pi \mathcal{F}_2)$ by Lemma 3.4. Let us define a valuation $\models$ on $(0,\omega) \uparrow_\pi \mathcal{F}_2$ by:

$a \models p$ if and only if $a = 0$, $\quad a \models q(u)$ if and only if $u = 0$,
$a \models r_i$ if and only if $a = 1$ $(i = 0, 1)$.

Then we have (1) $0 \not\models Triv$, (2) $0 \models \exists x q(x)$, and (3) $0 \not\models \forall x q(x)$. Therefore, we have (4) $0 \not\models \Box(\exists x r(x) \supset \forall x r(x) \vee Triv)$. Moreover, we have (5) $1 \models r_0 \wedge r_1$ and (6) $1 \not\models \Box r_1$. Therefore we have (7) $1 \not\models r_0 \wedge r_1 \supset \Box r_1$. Note that (8) $0 \not\models r_0$. Hence, by (4), (7) and (8), we have $0 \not\models H \vee \Box(\exists x q(x) \supset \forall x q(x) \vee Triv)$. □

As we have seen, **Shimura's Analogue for the Predicate Logics** does not work well. If we change our semantical setting into the Kripke sheaf semantics, and if we put strong condition on the completeness of a logic, then we can prove a not-simple but certain analogue (Theorem 4.1). Shall we be contented with Theorem 4.1 as a good analogue? Of course NO! Hence one agenda for the research comes as follows:

**What analogue is the best analogue of Shimura's Theorem?**

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