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Coherence of the Double Negation in Linear Logic *

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Many formulations of proof nets and sequent calculi for *Classical Linear Logic (CLL)* [7, 8] take it for granted that a type A is *identical* to its double negation $A^{\perp\perp}$. On the other hand, since Seely [13], it has been assumed that $*$-*autonomous categories* [1, 2] are the appropriate semantic models of (the multiplicative fragment of) CLL. However, in general, in a $*$-autonomous category an object A is only *canonically isomorphic* to its double involution A^{**}. For instance, in the category of finite dimensional vector spaces and linear maps, a vector space V is only isomorphic to its double dual V^{**}. This raises the questions whether $*$-autonomous categories do not, after all, provide an accurate semantic model for these proof nets and whether there could be semantically non-identical proofs (or morphisms), which must be identified in any system which assumes a type is identical to its double negation. Whether this can happen is not completely obvious even when one examines purely syntactic descriptions of proofs with the isomorphism between A and $A^{\perp\perp}$ present such as [11, 9] or the alternative proof net systems of [4] which are faithful to the categorical semantics.

Fortunately, there is no such semantic gap: in this talk we provide a *coherence theorem* on the double involution on $*$-autonomous categories, which tells us that there is no difference between the up-to-identity approach and the up-to-isomorphism approach, as far as this double-negation problem is concerned.

> **Theorem.** *Any free $*$-autonomous category is strictly equivalent to a free $*$-autonomous category in which the double-involution $(-)^{**}$ is the identity functor and the canonical isomorphism $A \simeq A^{**}$ is an identity arrow for all A.*

This remains true under the presence of linear exponential comonads and finite products (the semantic counterpart of exponentials and additives respectively). Our proof is fairly short and simple, and we suspect that this

*Joint work with J.R.B. Cockett and R.A.G. Seely [5] is folklore among specialists (at least everyone would expect such a result), though we are not aware of an explicit treatment of this issue in the literature.

This result should be compared with the classical coherence theorem for monoidal categories, as found e.g. in [12, 10]. In fact, we follow the proof strategy by Joyal and Street in [10]. We first show a weaker form of coherence theorem which turns a $*$-autonomous category into an equivalent one with "strict involution" (where A^{**} is identical to A), for which we make use of (a simplified version of) a construction of Cockett and Seely [6]. We then strengthen it to a form of "all diagrams commute" result by some additional arguments on the structure-preserving functors. In this way, this work also demonstrates the applicability of the Joyal-Street argument (which actually can be seen an instance of a general flexibility result on free algebras of 2-monads developped by Blackwell, Kelly and Power [3]) to other sorts of coherence problems.

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