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The Prime Graph of a Sporadic Simple Group

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1. Introduction

Let G be a finite group and S a sporadic simple group. We denote by $\pi(G)$ the set of all primes dividing the order of G. The prime graph $\Gamma(G)$ of G is defined in the usual way connecting p and q in $\pi(G)$ when there is an element of order pq in G. The main purpose of this paper is to determine finite group G satisfying $\Gamma(G)=\Gamma(S)$. G. Chen characterized S by $\Gamma(S)$ and $|S|$. Let $\pi_e(G)$ be the set of orders of all elements in G. W. Shi proved that G satisfying $\pi_e(G)=\pi_e(S)$ is isomorphic to S, except for J_2, Co_1. Our main theorem generalizes their results. Moreover, we prove that a simple group G satisfying $|G|=|S|$ is isomorphic to S as a corollary of our main theorem.

2. Theorem

Let S be a sporadic simple group. Suppose that G is a finite group satisfying $\Gamma(G)=\Gamma(S)$.

(1) If S is one of J_1, M_{22}, M_{23}, M_{24}, Co_2, then G is isomorphic to S.

(2) If S is M_{11}, then $G\simeq M_{11}$ or $L_2(11)$.

(3) If S is one of J_3, J_4, Suz, $O'N$, Ly, Fi_{23}, Fi'_{24}, M, BM, Th, Ru, Co_1, then $G/O_\pi(G)\simeq S$ where π is a subset of the numbers in the 2nd column of the following.

S	π
J_3, Suz	$2,3,5$
J_4	$2,3,11$
Ly,Fi'_{24},Th,Co_1	$2,3$
M	3
$O'N$,Fi_{23},BM,Ru	2

(4) If S is HS, He, M^cL, Co_3, Fi_{22} or HN, then $G/O_\pi(G)$ is one of the groups in the 2nd column where π is a subset of the numbers in the 3rd column of the following.

S	$G/O_\pi(G)$	π
HS	HS, $U_6(2)$	$2,3,5$
He	$L_2(16)$, $L_2(16).2$, $L_2(16).4$, $O_8^-(2)$, $O_8^-(2).2$, $S_8(2)$, He or $He.2$	$2,3,5,7$
M^cL	M_{22}, $M_{22}.2$, HS, $HS.2$, $U_6(2)$, $U_6(2).2$, M^cL	2,3,5
Co_3	M_{24}, Co_3	2
Fi_{22}	Suz, $Suz.2$, Fi_{22}, $Fi_{22}.2$	2,3,5
HN	HN, $HN.2$	2,3,5,7

(5) If S is M_{12}, then one of the following holds : (a) $G \simeq 11^{2n} : SL_2(5)$ for any $n \in \mathbf{Z}$, $G \simeq 11^{2n} : SL_2(5).2$ for any $2 \leq n \in \mathbf{Z}$, (b) $G/O_\pi(G) \simeq L_2(11)$, $L_2(11).2$, M_{11}, M_{12} or $M_{12}.2$ where $\pi \subseteq \{2,5\}$.

(6) If S is J_2, then one of the following holds : (a) G is solvable and G is a Frobenius group or a 2-Frobenius group, (b) $G/O_\pi(G) \simeq J_2$, $L_3(4)$, $L_3(4).2_1$, $L_3(4).2_3$, $S_6(2)$, $O_8^+(2)$, $U_3(5)$, $U_3(5).2$, $U_4(3)$, $U_4(3).2_2$, $U_4(3).2_3$, A_7, $A_7.2$, A_8, $A_8.2$, A_9, $L_2(7)$, $L_2(7).2$, $L_2(8)$, $L_2(8).3$, $U_3(3)$ or $U_3(3).2$ where $\pi \subseteq \{2,3,5\}$.

参考文献

[1] M. Hagie, The prime graph of a sporadic simple group, Communications in algebra, accepted.