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Bounds for effective speeds of traveling fronts in spatially periodic media

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1. INTRODUCTION

We consider the following reaction-diffusion-advection equation on $\mathbb{R}$:

$$u_t = \varepsilon u_{xx} + \varepsilon b(x)u_x + \frac{1}{\varepsilon}f(u), \quad x \in \mathbb{R}, \ t > 0, \tag{1.1}$$

where ε is a small positive parameter, b is a smooth periodic function with zero mean, and $f(u) = -W'(u)$ is a smooth function derived from a double-well potential $W(u)$ having different depth at its two wells. In the case where $b(x) \equiv 0$, it is well-known that there exists a traveling front solution which travels at a constant speed preserving its shape ([3]). On the other hand, if $b(x) \not\equiv 0$, then traveling front solutions in the usual sense can no longer exist. However, under suitable conditions on b and f, there exists a kind of front solutions whose shape and propagation speed vary periodically in time ([8, 9]). Our aim in this article is to study the influence of spatial inhomogeneity on the speed of front propagation in periodic diffusive media.

Equation (1.1) is related to problems on front propagation in an infinite tubular domain with a smooth and oscillating boundary. Let

$$\Omega_\sigma = \{(x, \sigma y_1, \ldots, \sigma y_{n-1}) \in \mathbb{R} \times \mathbb{R}^{n-1} \mid y = (y_1, \ldots, y_{n-1}) \in \omega(x)\},$$

where $\sigma > 0$ is a parameter and the map $x \mapsto \omega(x)$ is periodic. Matano [6] has considered the Allen-Cahn equation

$$u_t = \varepsilon \Delta u + \frac{1}{\varepsilon}f(u), \quad x \in \Omega_\sigma, \ t > 0 \tag{1.2}$$

under the homogeneous Neumann boundary conditions on $\partial\Omega_\sigma$ and has obtained some conditions on f and $\omega(x)$ for the existence and non-existence of traveling fronts for (1.2). Some numerical experiments imply that the characteristics of front propagation such as front speeds and front profiles are strongly influenced by the shape of $\partial\Omega_\sigma$ (See Figure 1). It follows from an argument in [4] that equation (1.2) is formally reduced in the limit $\sigma \to 0$ to equation (1.1) with $b(x) = a'(x)/a(x)$, where $a(x) > 0$ is the $(n-1)$-dimensional volume of $\omega(x)$.

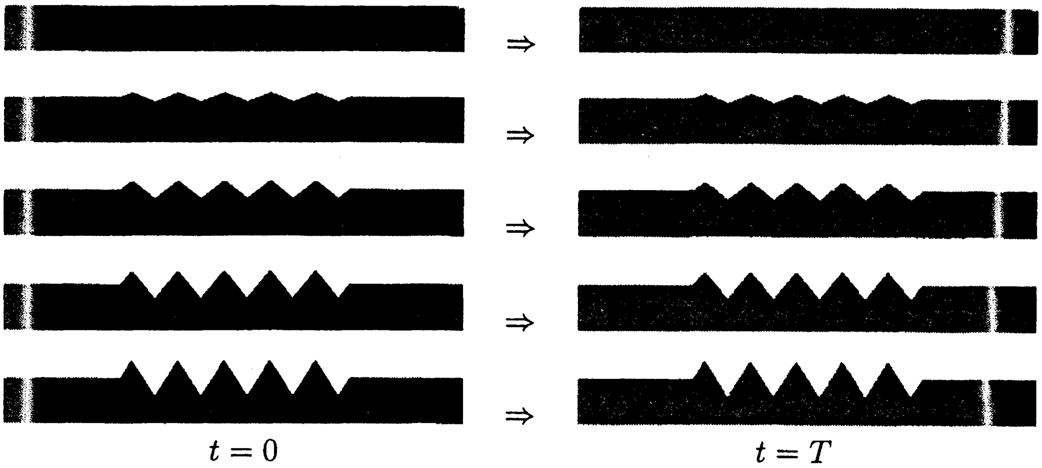

FIGURE 1. Numerical simulations of front propagation for (1.2) in five different domains when $\varepsilon = 0.050$, $f(u) = u(1-u)(u-1/4)$. The white region approximately represents the position of the front.

Since the effects of diffusion and advection are negligible for small ε, the solution of (1.1) is known to develop transition layers connecting two stable states within a short time. Let $P^\varepsilon(t)$ denote the position of one of the layers at time t. When the parameter ε is small, we will see that the speed of the layer at time t has the formal expansion

$$\frac{dP^\varepsilon}{dt} = c - \varepsilon b(P^\varepsilon) - \varepsilon^2 \gamma b'(P^\varepsilon) + \text{(higher order terms)}, \tag{1.3}$$

where $c > 0$ is the speed of the traveling front solution in the homogeneous case and γ is some constant. By constructing suitable supersolutions and subsolutions of (1.1), we justify the above formal expansion up to ε^2, provide sharp estimates for the propagation speed of the front solution and show that spatial inhomogeneity slows down the speed of front propagation for (1.1).

2. Main result

Our hypotheses are as follows:

(B1) $b(x)$ is a smooth periodic function with least period $L > 0$;

(B2) $\int_0^L b(x)dx = 0$;

(F1) f is smooth and has exactly three zeros $0, \alpha, 1$ with $0 < \alpha < 1$;
(F2) $f(u) < 0$ for $u \in (0, \alpha) \cup (1, +\infty)$, $f(u) > 0$ for $u \in (-\infty, 0) \cup (\alpha, 1)$;
(F3) $f'(0) < 0$, $f'(1) < 0$;
(F4) $\int_0^1 f(u)du > 0$.

A typical example of f is a cubic function $f(u) = u(1-u)(u-\alpha)$ where $0 < \alpha < 1/2$. In what follows we denote by $\langle g \rangle$ the mean of an L-periodic function g on $\mathbb{R}$, namely,

$$\langle g \rangle = \frac{1}{L} \int_0^L g(x)dx.$$

In the homogeneous case ($b(x) \equiv 0$), there exists a traveling front solution of (1.1) written in the form

$$u(x,t) = \phi\left(\frac{x-ct}{\varepsilon}\right)$$

where $(\phi(\xi), c)$ is a unique solution of

$$\left\{ \begin{array}{l} \phi_{\xi\xi} + c\phi_\xi + f(\phi) = 0, \quad \xi \in \mathbb{R} \\ \phi(-\infty) = 1, \ \ \phi(0) = \alpha, \ \ \phi(+\infty) = 0. \end{array} \right. \tag{2.1}$$

In addition, the speed c is positive by virtue of (F4). See [3] for details.

In the periodic case ($b(x) \not\equiv 0$), the notion of traveling fronts has to be replaced as follows:

Definition. A solution U^ε of (1.1) satisfying

$$\lim_{x \to -\infty} U^\varepsilon(x,t) = 1, \quad \lim_{x \to +\infty} U^\varepsilon(x,t) = 0 \quad \text{for all } t \in \mathbb{R} \tag{2.2}$$

is called a traveling front if there exists a $T_\varepsilon > 0$ such that

$$U^\varepsilon(x, t + T_\varepsilon) = U^\varepsilon(x - L, t) \quad \text{for all } x \in \mathbb{R}, \ \ t \in \mathbb{R}. \tag{2.3}$$

We define the effective speed (or the average speed) s_ε of U^ε by L/T_ε.

The main result of this article is:

Theorem 1. *Suppose that $b(x) \not\equiv 0$. Then there exist positive constants ε_0 and C such that for any $\varepsilon \in (0, \varepsilon_0)$, we have*

$$\left| s_\varepsilon - \left(c - \frac{\langle b^2 \rangle}{c} \varepsilon^2 \right) \right| \le C\varepsilon^3 |\log \varepsilon|^2. \tag{2.4}$$

Thus spatial inhomogeneity slows down the front propagation in this case.

Remark 2. Some existence results for traveling fronts of multidimensional reaction-diffusion-advection equations with bistable nonlinearities have been obtained by Xin [8, 9] on the supposition that diffusion and advection coefficients are nearly constant.

Applying his results to (1.1), we see that if ε is sufficiently small there exists a traveling front U^ε satisfying (2.2) and (2.3) with positive effective speed s_ε and that it is unique up to time shift.

Remark 3. In [5], Heinze, Papanicolaou and Stevens has given some variational formulas of the effective speeds of traveling fronts in the multidimensional case. However, it is rather difficult to obtain sharp estimates for s_ε like (2.4) by using the formulas.

3. Formal asymptotic expansions of the front speed

In this section we present a formal derivation of the propagation speed of the traveling front U_ε for (1.1).

Suppose that equation $U^\varepsilon(x,t) = \alpha$ has a unique solution $x = P^\varepsilon(t)$ for each t and that U^ε and P^ε have the expansions

$$\begin{aligned} U^\varepsilon(x,t) &= U_0(\xi,t) + \varepsilon U_1(\xi,t) + \varepsilon^2 U_2(\xi,t) + \cdots, \\ P^\varepsilon(t) &= P_0(t) + \varepsilon P_1(t) + \varepsilon^2 P_2(t) + \cdots, \end{aligned} \tag{3.1}$$

where $\xi = (x - P^\varepsilon(t))/\varepsilon$. The stretched space variable ξ gives the right spatial scaling to describe the sharp transition layer between the two stable states 0 and 1. Since $U^\varepsilon = \alpha$ at $P^\varepsilon(t)$, we normalize U_k in such a way that

$$U_0(0,t) = \alpha, \quad U_k(0,t) = 0 \ \ (k = 1,2,\dots) \tag{3.2}$$

for all t (normalization conditions). By (2.2), we also impose the following conditions as $\xi \to \pm\infty$:

$$U_0(-\infty,t) = 1, \quad U_0(+\infty,t) = 0, \quad U_k(\pm\infty,t) = 0 \ \ (k = 1,2,\dots) \tag{3.3}$$

for all t (limiting conditions).

Substituting the above expansions (3.1) into (1.1) and collecting the ε^0 terms, we obtain

$$U_{0\xi\xi} + P_{0t}U_{0\xi} + f(U_0) = 0.$$

From this together with (3.2) and (3.3), we find that

$$U_0(\xi,t) = \phi(\xi), \quad P_{0t} = c,$$

where (ϕ, c) is the unique solution of (2.1).

Collecting the ε^1 terms and recalling (3.2) and (3.3), we get

$$\begin{cases} U_{1\xi\xi} + cU_{1\xi} + f'(\phi(\xi))U_1 = -(P_{1t} + b(P_0))\phi'(\xi), \quad \xi \in \mathbb{R}, \\ U_1(-\infty,t) = 0, \ U_1(0,t) = 0, \ U_1(+\infty,t) = 0. \end{cases} \tag{3.4}$$

The following Fredholm type lemma gives us the solvability conditions for (3.4). For the case $c = 0$, similar statements have been appeared in [1].

Lemma 4. *Let $A(\xi)$ be given and assume that $A(\xi) = O(e^{-\mu|\xi|})$ as $|\xi| \to \infty$ for some $\mu > 0$. Then the following problem*

$$\begin{cases} \Psi_{\xi\xi} + c\Psi_\xi + f'(\phi(\xi))\Psi = A(\xi), & \xi \in \mathbb{R}, \\ \Psi(0) = 0, \end{cases}$$

has a bounded solution if and only if

$$\int_{\mathbb{R}} A(\xi)\phi'(\xi)e^{c\xi}d\xi = 0.$$

Moreover, the solution is written by

$$\Psi(\xi) = \phi'(\xi)\int_0^\xi \phi'(y)^{-2}e^{-cy}\left\{\int_{-\infty}^y A(z)\phi'(z)e^{cz}dz\right\}dy,$$

and satisfies $\Psi(\xi), \Psi'(\xi), \Psi''(\xi) = O(e^{-\mu|\xi|})$ as $|\xi| \to \infty$.

By this Lemma, the solvability condition for (3.4) yields $P_{1t} = -b(P_0)$ and thus $U_1(\xi, t) = 0$ for all (ξ, t).

In the same way as above, collecting the ε^2 terms, we get

$$\begin{cases} U_{2\xi\xi} + cU_{2\xi} + f'(\phi(\xi))U_2 = -(P_{2t} + b'(P_0)(P_1 + \xi))\phi'(\xi), & \xi \in \mathbb{R}, \\ U_2(-\infty, t) = 0, \ \ U_2(0, t) = 0, \ \ U_2(+\infty, t) = 0. \end{cases} \tag{3.5}$$

Again by Lemma 4, the solvability condition for (3.5) yields

$$P_{2t} = -b'(P_0)(P_1 + \gamma), \quad U_2(\xi, t) = b'(P_0(t))V(\xi),$$

where $\gamma \in \mathbb{R}$ is defined by

$$\gamma = \frac{\int_{\mathbb{R}} \phi'(\xi)^2 e^{c\xi}\xi d\xi}{\int_{\mathbb{R}} \phi'(\xi)^2 e^{c\xi}d\xi},$$

and

$$V(\xi) = \phi'(\xi)\int_0^\xi \phi'(y)^{-2}e^{-cy}\left\{\int_{-\infty}^y (\gamma - z)\phi'(z)^2 e^{cz}dz\right\}dy.$$

Consequently, we obtain the following formal asymptotic expansions

$$U^\varepsilon(x, t) = \phi\left(\frac{x - P^\varepsilon(t)}{\varepsilon}\right) + \varepsilon^2 b'(P^\varepsilon(t))V\left(\frac{x - P^\varepsilon(t)}{\varepsilon}\right) + \cdots, \tag{3.6}$$

and

$$\begin{aligned} \frac{dP^\varepsilon}{dt} &= c - \varepsilon b(P_0(t)) - \varepsilon^2 b'(P_0(t))(P_1(t) + \gamma) + \cdots, \\ &= c - \varepsilon b(P^\varepsilon(t)) - \varepsilon^2\gamma b'(P^\varepsilon(t)) + \cdots. \end{aligned} \tag{3.7}$$

4. Preliminaries

Let $\delta_0 > 0$ be such that for each $\delta \in [-\delta_0, \delta_0]$, $f_\delta(u) = f(u) + \delta$ satisfies the following conditions:

(a) f_δ has exactly three zeroes $\zeta_0(\delta), \zeta_\alpha(\delta), \zeta_1(\delta)$ with $\zeta_0(\delta) < \zeta_\alpha(\delta) < \zeta_1(\delta)$;
(b) $f'_\delta(\zeta_0(\delta)) < 0$, $f'_\delta(\zeta_\alpha(\delta)) > 0$ and $f'_\delta(\zeta_1(\delta)) < 0$;
(c) $\displaystyle\int_{\zeta_0(\delta)}^{\zeta_1(\delta)} f_\delta(u)du > 0$.

Then there exists a solution $(\psi, s) = (\psi(\xi;\delta), s(\delta))$ of

$$\begin{cases} \psi_{\xi\xi} + s\psi_\xi + f_\delta(\psi) = 0, & \xi \in \mathbb{R}, \\ \psi(-\infty) = \zeta_1(\delta), \;\; \psi(0) = \alpha, \;\; \psi(+\infty) = \zeta_0(\delta), \end{cases} \tag{4.1}$$

satisfying $\psi_\xi(\xi;\delta) < 0$ for all $\xi \in \mathbb{R}$ and $s(\delta) > 0$ ([3]). It is known that $\psi(\cdot;\delta) \to \phi$ uniformly on $\mathbb{R}$ and $s(\delta) = c + O(\delta)$ as $\delta \to 0$, where (ϕ, c) is the solution of (2.1).

For $\delta \in [-\delta_0, \delta_0]$, we define a function $V(\cdot;\delta)$ by

$$V(\xi;\delta) = \psi_\xi(\xi;\delta)\int_0^\xi \psi_\xi(\xi;\delta)^{-2}e^{-s(\delta)\eta}\left\{\int_{-\infty}^{\eta}(\gamma(\delta) - \zeta)\psi_\xi(\zeta;\delta)^2 e^{s(\delta)\zeta}d\zeta\right\}d\eta, \tag{4.2}$$

where

$$\gamma(\delta) = \frac{\displaystyle\int_{\mathbb{R}} \psi_\xi(\xi;\delta)^2 e^{s(\delta)\xi}\xi d\xi}{\displaystyle\int_{\mathbb{R}} \psi_\xi(\xi;\delta)^2 e^{s(\delta)\xi} d\xi}. \tag{4.3}$$

Then the function $V(\cdot;\delta)$ solves the problem

$$\begin{cases} V_{\xi\xi} + s(\delta)V_\xi + f'(\psi(\xi;\delta))V = (\gamma(\delta) - \xi)\psi_\xi(\xi;\delta), & \xi \in \mathbb{R}, \\ V(0;\delta) = 0. \end{cases} \tag{4.4}$$

Furthermore, $\psi(\xi;\delta)$ and $V(\xi;\delta)$ satisfy the following:

Lemma 5. *There exist positive constants M and Λ depending on δ_0 such that if $|\delta| \le \delta_0$, then*

$$\begin{aligned} \zeta_1(\delta) - Me^{\Lambda\xi} \le \psi(\xi;\delta) &< \zeta_1(\delta), && \textit{for } \xi \le 0, \\ \zeta_0(\delta) < \psi(\xi;\delta) &\le \zeta_0(\delta) + Me^{-\Lambda\xi}, && \textit{for } \xi \ge 0, \\ -Me^{-\Lambda|\xi|} \le \psi_\xi(\xi;\delta) &< 0, && \textit{for } \xi \in \mathbb{R}, \\ |\psi_{\xi\xi}(\xi;\delta)| &\le Me^{-\Lambda|\xi|}, && \textit{for } \xi \in \mathbb{R}, \\ |V(\xi;\delta)|,\; |V_\xi(\xi;\delta)|,\; |V_{\xi\xi}(\xi;\delta)| &\le Me^{-\Lambda|\xi|}, && \textit{for } \xi \in \mathbb{R}. \end{aligned} \tag{4.5}$$

5. Construction of supersolutions and subsolutions

This section is devoted to the proof of Theorem 1. In order to obtain sharp estimates for the effective speed s^ε, we construct suitable supersolutions and subsolutions of (1.1). The formal asymptotic expansions (3.6) and (3.7) provide us with useful information for the construction.

Let $\psi(\xi;\cdot)$, $V(\xi;\cdot)$, $\zeta_0(\cdot)$, $\zeta_1(\cdot)$, $\gamma(\cdot)$ and $s(\cdot)$ be as in the previous section and define $\phi_\varepsilon^\pm(\xi) = \psi(\xi;\pm h\varepsilon^3)$, $V_\varepsilon^\pm(\xi) = V(\xi;\pm h\varepsilon^3)$, $z_j^\pm(\varepsilon) = \zeta_j(\pm h\varepsilon^3)$ $(j = 0,1)$, $\gamma_\varepsilon^\pm = \gamma(\pm h\varepsilon^3)$ and $c_\varepsilon^\pm = s(\pm h\varepsilon^3)$, where h is a positive constant. In other words, $(\phi_\varepsilon^\pm, c_\varepsilon^\pm)$ and $V_\varepsilon^\pm(\xi)$ satisfy

$$\begin{cases} \dfrac{d^2\phi_\varepsilon^\pm}{d\xi^2} + c_\varepsilon^\pm \dfrac{d\phi_\varepsilon^\pm}{d\xi} + f(\phi_\varepsilon^\pm) \pm h\varepsilon^3 = 0, \quad \xi \in \mathbb{R}, \\ \phi_\varepsilon^\pm(-\infty) = z_1^\pm(\varepsilon), \ \ \phi_\varepsilon^\pm(0) = \alpha, \ \ \phi_\varepsilon^\pm(+\infty) = z_0^\pm(\varepsilon), \end{cases}$$

and

$$\begin{cases} \dfrac{d^2V_\varepsilon^\pm}{d\xi^2} + c_\varepsilon^\pm \dfrac{dV_\varepsilon^\pm}{d\xi} + f'(\phi_\varepsilon^\pm)V_\varepsilon^\pm = (\gamma_\varepsilon^\pm - \xi)\dfrac{d\phi_\varepsilon^\pm}{d\xi}, \quad \xi \in \mathbb{R}, \\ V_\varepsilon^\pm(\pm\infty) = V_\varepsilon^\pm(0) = 0, \end{cases}$$

respectively. In what follows we assume that $h\varepsilon^3 < \delta_0$, where δ_0 is the positive constant in the previous section.

Let K be a positive constant and define functions $W_\varepsilon^\pm$ by

$$W_\varepsilon^\pm(x,t) = \phi_\varepsilon^\pm\left(\frac{x - R_\varepsilon^\pm(t)}{\varepsilon}\right) + \varepsilon^2 b'(R_\varepsilon^\pm(t))V_\varepsilon^\pm\left(\frac{x - R_\varepsilon^\pm(t)}{\varepsilon}\right), \tag{5.1}$$

where $R_\varepsilon^\pm$ are solutions of

$$\frac{dR_\varepsilon^\pm}{dt} = c_\varepsilon^\pm - \varepsilon b(R_\varepsilon^\pm) - \varepsilon^2\gamma_\varepsilon^\pm b'(R_\varepsilon^\pm) \pm K\varepsilon^3|\log\varepsilon|^2. \tag{5.2}$$

Proposition 6. *There exist positive constants h, K and ε_0 depending on δ_0 such that if $\varepsilon \in (0,\varepsilon_0)$, then W_ε^+ is a supersolution of* (1.1) *and W_ε^- is a subsolution of* (1.1).

Outline of proof. We only show that W_ε^+ is a supersolution of (1.1) since the assertion for W_ε^- can be proved in the same manner. In the rest of the proof, we drop the subscript ε for simplicity. We define

$$I(x,t) = \varepsilon^2 W_{xx}^+ + \varepsilon^2 b(x) W_x^+ + f(W^+) - \varepsilon W_t^+ \tag{5.3}$$

and $\eta(x,t) = (x - R^+(t))/\varepsilon$. Substituting (5.1) into (5.3), we get

$$\begin{aligned} I(x,t) = &-h\varepsilon^3 + \{R_t^+ - c^+ + \varepsilon b(x)\}\phi_\xi^+(\eta) + f(\phi^+(\eta) + \varepsilon^2 b'(R^+)V^+(\eta)) - f(\phi^+(\eta)) \\ &+ \varepsilon^2 b'(R^+)V_{\xi\xi}^+(\eta) + \varepsilon^2 b'(R^+)\{R_t^+ + \varepsilon b(x)\}V_\xi^+(\eta) - \varepsilon^3 b''(R^+)R_t^+ V(\eta). \end{aligned}$$

By Lemma 5, there exists a positive constant μ depending on δ_0 satisfying

$$|\phi_\xi^+|,\ |V^+|,\ |V_\xi^+|,\ |V_{\xi\xi}^+| < \varepsilon^4 \quad \text{for } |\xi| > \mu|\log\varepsilon|.$$

Therefore, $I(x,t) = -h\varepsilon^3 + O(\varepsilon^4) < 0$ for $|x - R^+(t)| > \mu\varepsilon|\log\varepsilon|$ when ε is small.

In the case where $|x - R^+(t)| \le \mu\varepsilon|\log\varepsilon|$, we see that

$$I(x,t) \le \left\{K - \frac{\mu^2}{2}|b''(R^+)|\right\}\phi_\xi^+\varepsilon^3|\log\varepsilon|^2 - \{h - c|b''(R^+)V^+|\}\varepsilon^3 + O(\varepsilon^4|\log\varepsilon|^3).$$

Since $\phi_\xi^+ < 0$, if $h > \mu^2\|b''\|_\infty/2$ and $K > cM\|b''\|_\infty$, then $I(x,t) < 0$ for small ε. □

Corollary 7. *Fix $\varepsilon \in (0, \varepsilon_0)$. Then for any $t \in \mathbb{R}$, we have*

$$\inf_{x\in\mathbb{R}} W_\varepsilon^+(x,t) > 0, \quad \sup_{x\in\mathbb{R}} W_\varepsilon^-(x,t) < 1.$$

Since the right-hand sides of (5.2) are L-periodic and positive for small ε, there exist positive constants $\tau_\varepsilon^\pm$ satisfying

$$R_\varepsilon^\pm(t + \tau_\varepsilon^\pm) = R_\varepsilon^\pm(t) + L \tag{5.4}$$

for all $t \in \mathbb{R}$.

Let x_0 be such that $U^\varepsilon(x_0, 0) = \alpha$. By (2.2) and Corollary 7, we can choose $R_\varepsilon^-(0) < x_0 < R_\varepsilon^+(0)$ such that

$$W_\varepsilon^-(x,0) \le U^\varepsilon(x,0) \le W_\varepsilon^+(x,0).$$

Hence the comparison theorem yields that

$$W_\varepsilon^-(x,t) < U^\varepsilon(x,t) < W_\varepsilon^+(x,t)$$

for all $t > 0$. Thus we obtain:

Lemma 8. *Let $s_\varepsilon = L/T_\varepsilon$ be the effective speed of U^ε. Then $L/\tau_\varepsilon^- \le s_\varepsilon \le L/\tau_\varepsilon^+$.*

Proof of Theorem 1. Let $A_\varepsilon^\pm = c_\varepsilon^\pm \pm K\varepsilon^3|\log\varepsilon|^2$ and $B_\varepsilon^\pm(R) = \varepsilon b(R) + \varepsilon^2\gamma_\varepsilon^\pm b'(R)$. Then it follows from (5.2) and (5.4) that

$$\frac{L}{\tau_\varepsilon^\pm} = \frac{1}{\langle (A_\varepsilon^\pm - B_\varepsilon^\pm(\cdot))^{-1}\rangle}.$$

Since $A_\varepsilon^\pm = c + O(\varepsilon^3|\log\varepsilon|^2)$ and since $B_\varepsilon^\pm$ are L-periodic functions with $\langle B_\varepsilon^\pm\rangle = 0$,

$$\langle (A_\varepsilon^\pm - B_\varepsilon^\pm(\cdot))^{-1}\rangle = \frac{1}{c}\left(1 + \frac{\langle b^2\rangle}{c^2}\varepsilon^2 + O(\varepsilon^3|\log\varepsilon|^2)\right),$$

and hence

$$\frac{L}{\tau_\varepsilon^\pm} = c - \frac{\langle b^2\rangle}{c}\varepsilon^2 + O(\varepsilon^3|\log\varepsilon|^2). \tag{5.5}$$

The assertion of the theorem immediately follows from (5.5) and Lemma 8. □

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