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Targeting Debt: Some Implications for Growth and Inflation*

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1 Introduction

The fiscal authority determines the total quantity of the government's liabilities by issuing bonds, whereas the role of the monetary authority is to determine the composition of the liabilities via open market operations. In the literature, most research focuses on how monetary policy actions – changes in the composition of the government's liabilities – affect the real economy and the rate of inflation. This paper is intended to add some new insights into the interaction between debt, growth, and inflation. To do so, this paper takes a monetary endogenous growth model and considers a rather unusual policy tool: control of the public debt.

The conventional wisdom is that the fiscal authority affects the government' total liabilities and all the monetary variables such as the nominal interest rate and the inflation rate are under complete control of the monetary authority. Such a conventional view has been challenged by Sargent and Wallace's (1981) "some unpleasant monetarist arithmetic" and more recently by the school of the "fiscal theory of the price level." Their main message is that the monetary policy alone cannot control inflation because fiscal and monetary policies are connected by a single government's budget constraint so the fiscal authority's actions limit the monetary authority's degree of freedom. This paper takes this view further and asks: can the fiscal authority control inflation by targeting the quantity of bonds it issues? In a broad sense it studies implications of debt targeting for growth and inflation.

The analytical framework is an extension of Kudoh (2002), who presents a one-sector endogenous growth model with money and bonds. The model is in principle an endogenous growth version of Sargent and

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Wallace's (1981) overlapping generations economy. I adopt a quick way to endogenize the output growth rate. Namely, I follow Smith (1991) and more recently Espinosa and Yip (1999) to assume that the production technology exhibits some degree of externality so that the aggregate production function is of AK type.

I adopt Lucas and Stokey's (1983) cash-in-advance formulation to introduce money that is demanded even when it is dominated by other assets in rates of return. One of the reasons why I avoid the money-in-the-utility-function (MIUF) formulation, which is another famous (and tractable) short-cut method of modeling return-dominated money, is that model builders must take extra care of the issue of the timing of trades, as pointed out by Carlstrom and Fuerst (2001). In the standard MIUF model, the real money balance *after* all transactions took place enters the utility function. Carlstrom and Fuerst (2001) named this situation "cash-when-I'm-done timing." It will be shown that Lucas and Stokey's (1983) cash-in-advance formulation gives rise to a well-defined money demand function that is decreasing in the nominal interest rate.

Using the framework, I first study a model with debt targeting and interest rate pegging. Because of the presence of budget deficits, there are two distinct balanced growth equilibria. As pointed out by Evans *et al.* (1998), the standard stability analysis does not apply to such a model because the model's initial condition (stated as a level of capital stock) does not pin down a time path of capital stock. For this reason, I utilize an *adaptive learning scheme* as an equilibrium selection device. The basic idea of adaptive learning adopted in this paper is to describe behaviors of agents outside of equilibria under a particular adaptive learning mechanism, and find a mapping that maps from the PLM (perceived law of motion) to the ALM (actual law of motion). A perfect-foresight equilibrium is said to be E-stable (expectationally stable) if such a learning scheme converges to that equilibrium.

A primary finding regarding stability is that it is the *low-growth equilibrium* that is E-stable. The high-growth equilibrium is either E-stable or E-unstable, depending on the level of primary deficit and the targeted debt-GDP ratio. If the level of primary deficit is low or the targeted debt-GDP ratio is high, then both equilibria become E-stable, causing *expectational indeterminacy*, the possibility raised in Kudoh (2002).

It is well-known in the literature that *sunspot equilibria* of various forms exist in a model with multiplicity or indeterminacy. Recently Evan and Honkapohja (1994, 2001b) addressed the issue of *stability*, rather than existence, of sunspot equilibria using the technique of adaptive learning. In the spirit of Evan and Honkapohja (1994, 2001b), this paper takes up the issue of stability of possible sunspot equilibria of the model. In the model with nominal interest rate pegging, sunspots around the two distinct balanced growth equilibria exist, but the sunspots are not stable under learning.

I extend the basic model to consider a somewhat extreme issue: can the fiscal authority control inflation without an active central bank? The answer seems affirmative in the sense that there is a unique balanced growth equilibrium in a model in which the central bank lets all the monetary variables – the nominal interest rate and the inflation rate – be determined by the market. It will be shown that there is a unique balanced growth equilibrium, and that it is stable under learning. This suggests that it is possible that the fiscal authority's actions alone determine the long-run rates of growth and inflation. This would add another support for the view that the fiscal authority's coordinated actions are required for price (and output) stability.

Recently Evan and Honkapohja (1994, 2001b) consider existence and stability of sunspot equilibria near *a single indeterminate steady state*. Their results imply that although uniqueness obtains, the economy without an active monetary authority can be subject to *adaptively stable sunspot fluctuations*.

The rest of the paper is organized as follows. Section 2 describes the model economy. Section 3 presents equilibria under debt targeting and interest rate pegging. Section 4 asks whether the fiscal authority can control inflation by targeting debt. Section 5 concludes.

2 The Model

2.1 Environment

Consider a growing economy consisting of an infinite sequence of two-period lived overlapping generations, an initial old generation, and an infinitely-lived government. Let $t = 1, 2, ...$ index time. At each date t, a new generation comprised of N_t identical members appears where I normalize $N_t = 1$ for all t. Each agent is endowed with one unit of labor when young and is retired when old. In addition, the initial old agents are endowed with $M_0 > 0$ units of fiat currency and $K_1 > 0$ units of capital.

There is a single final good produced using the production function $Y_t = A\overline{K}_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha}$, where $A > 1$ is a constant, $\alpha \in (0, 1)$ is the capital's share, $\overline{K}_t$ is the aggregate capital stock, K_t denotes the capital input, and L_t denotes the labor input at t. The aggregate capital stock enters the production function because of externality; the labor productivity rises as the society accumulates capital stock. Note that $\overline{K}_t = K_t$ holds in equilibrium. In addition, capital is assumed to depreciate 100% between periods.

2.2 Factor Markets

Factor markets are perfectly competitive. Thus, factors of production receive their marginal product. Young agents supply their labor endowment inelastically in the labor market. Thus, $L_t = 1$ in equilibrium. When make decisions, firms take the stock of aggregate capital, $\overline{K}_t$, as given. Then the gross return on capital, r_{t+1}, and the real wage rate, w_t, are given by

$$r_t = \alpha A \overline{K}_t^{1-\alpha} K_t^{\alpha-1} L_t^{1-\alpha} = \alpha A, \tag{1}$$

$$w_t = (1-\alpha) A \overline{K}_t^{1-\alpha} K_t^{\alpha} L_t^{-\alpha} = (1-\alpha) A K_t = (1-\alpha) Y_t. \tag{2}$$

2.3 Consumers

Let c_{1t} (c_{2t}) denote the consumption of the final good by a young (old) agent born at date t. In order to simplify the analysis as much as possible, I assume that agents care consumption only when old. This immediately follows that $c_{1t} = 0$ for all t so all income will be saved.

Following Lucas and Stokey (1983) and more recently Woodford (1994), I assume that consumption goods are divided into two types: "cash goods" and "credit goods." Cash goods must be purchased by cash, so agents wishing to consume cash goods need cash in advance. On the other hand, agents do not need cash to purchase credit goods. I follow Lucas and Stokey's (1983) interpretation that at some stores an agent is known to the producer so credit is available, while at other stores the agent is unknown to the seller so cash must be used to make a transaction. Let c_{mt} (c_{nt}) denote the amount of cash (credit) goods consumed when old. Then, $c_{2t} = c_{mt} + c_{nt}$ must hold. The cash-in-advance constraint is therefore

$$p_{t+1} c_{mt} \le M_t, \tag{3}$$

where p_t denotes the time t price level and M_t denotes the nominal money balance. According to (3), a young agent must set aside cash in advance in order to purchase cash goods when old.

It is assumed that agents may hold money and non-monetary assets. The non-monetary assets, denoted by Z_t, are assumed to yield the gross nominal return of $I_{t+1} \ge 1$ in the next period. I assume that agents do not have access to any other storage technology. The budget constraint for a young agent born at date t is therefore

$$M_t + Z_t \le p_t w_t - T_t, \tag{4}$$

where T_t is the amount of tax paid. (4) states that a young agent of generation t receives nominal wage income and allocates all disposable income to monetary and non-monetary assets (because no one consumes when young). Throughout, I consider only symmetric equilibria in which all agents of the same generation have the same amount of assets. Since the nominal interest rate on money is zero, the budget constraint when old is

$$p_{t+1}c_{2t} \leq M_t + I_{t+1}Z_t. \tag{5}$$

The cash-in-advance constraint binds if and only if money is dominated by non-monetary assets in rates of return. In other words, the cash-in-advance constraint binds as long as the net nominal interest rate is non-negative, or equivalently, $I_{t+1} \geq 1$. Under the biding cash-in-advance constraint, $c_{mt} = M_t/p_{t+1}$ and $c_{nt} = I_{t+1}Z_t/p_{t+1}$.

Following Chari *et al.* (1991), I specify the utility function as[1]

$$U(c_{mt}, c_{nt}) = \ln \left\{ \left[(1-\sigma) c_{mt}^{1-\rho} + \sigma c_{nt}^{1-\rho} \right]^{\frac{1}{1-\rho}} \right\}, \tag{6}$$

where $0 < \sigma < 1$ and $0 < \rho < 1$. Each young agent chooses c_{mt} and c_{nt} to maximize (6) subject to $c_{mt} = M_t/p_{t+1}$, $c_{nt} = I_{t+1}Z_t/p_{t+1}$, and $M_t + Z_t = p_t w_t - T_t$. The first order necessary condition for the maximization problem gives the real money demand function,

$$\frac{M_t}{p_t} = \gamma(I_{t+1}) \left(w_t - \frac{T_t}{p_t} \right), \tag{7}$$

where

$$\gamma(I) \equiv \left[1 + \left(\frac{\sigma}{1-\sigma} \right)^{\frac{1}{\rho}} I^{\frac{1-\rho}{\rho}} \right]^{-1}. \tag{8}$$

It is important to check the properties of the money demand function just derived.

Lemma 1 *$\gamma(I)$ satisfies (a) $\gamma'(I) < 0$ for $0 < \rho < 1$, (b) $\lim_{I\to\infty} \gamma(I) = 0$ for $0 < \rho < 1$, (c) $0 < \gamma(I) < 1$, and (d) $I\gamma'(I)/\gamma(I) = -[1-\gamma(I)](1-\rho)/\rho$.*

Proof. (a) From (8),

$$\gamma'(I) = -\left[1 + \left(\frac{\sigma}{1-\sigma} \right)^{\frac{1}{\rho}} I^{\frac{1-\rho}{\rho}} \right]^{-2} \left(\frac{\sigma}{1-\sigma} \right)^{\frac{1}{\rho}} \frac{1-\rho}{\rho} I^{\frac{1-\rho}{\rho}-1}. \tag{9}$$

It is then easy to check that $\gamma'(I) < 0$ for $0 < \rho < 1$. (b) Immediate from (8). (c) Obvious from (b). (d) Straightforward from (9). ∎

Lemma 1 (a) states the condition under which the real money demand is decreasing in the nominal interest rate. As the nominal interest rate increases, the household substitutes away from money, which

[1] According to Chari *et al.* (1991), $\sigma = 0.57, \rho = 0.17$ for the U.S. economy. Note, however, that the parameter values are for their model economy in which there is an infinitely lived agent, rather than a series of overlapping generations.

reduces money demand. An increase in the nominal rates, at the same time, raises earning from bond holding, which raises money demand through income effect. The former dominates the latter if $0 < \rho < 1$, which I assume to hold throughout. In addition, I assume that $(1-\rho) I < 1$ holds, which is plausible and easily satisfied.

2.4 The Government

The government's flow budget constraint is

$$G_t = T_t + B_t - I_t B_{t-1} + M_t - M_{t-1} \tag{10}$$

for $t \geq 2$ and $G_1 + M_0 = T_1 + M_1 + B_1$ for $t = 1$, where the initial stock of bonds is assumed to be zero. I assume that the government simply consumes G_t and that it does not affect utility of any generation or the production process at any date. In order to simplify the analysis, divide (10) by $p_t Y_t$ to obtain

$$g_t = \tau_t + b_t - \frac{R_t}{\theta_t} b_{t-1} + m_t - \frac{p_{t-1}}{p_t} \frac{1}{\theta_t} m_{t-1}, \tag{11}$$

where $\theta_t \equiv Y_t / Y_{t-1}$, $g_t \equiv G_t/(p_t Y_t)$, $\tau_t \equiv T_t/(p_t Y_t)$, $b_t \equiv B_t/(p_t Y_t)$, $m_t \equiv M_t/(p_t Y_t)$. Throughout, I assume that the government spending per GDP is constant over time, or, $g_t = g \in [0, 1)$ for all t.

3 Equilibria with Debt Targeting and Interest Rate Pegging

3.1 Characterization

This section considers a scenario in which the fiscal authority targets the debt-GDP ratio and the central bank pegs the nominal interest rate. Let $\bar{b}$ denote the targeted debt-GDP ratio where $0 \leq \bar{b} < \infty$. It follows therefore that $b_t = \bar{b}$ and $I_t = I$ for all t. Thus, the tax rate, τ_t, is endogenous. Before proceeding, note that $\tau_t < 1 - \alpha$ is imposed to ensure $T_t < w_t = (1-\alpha) Y_t$, otherwise the household will be bankrupted by taxes.

A monetary equilibrium is defined as a set of sequences for allocations $\{m_t\}$, $\{z_t\}$, $\{k_t\}$, $\{b_t\}$, prices $\{r_t\}$, $\{w_t\}$, $\{p_t\}$, and the initial conditions $M_0 > 0$, $K_1 > 0$, $B_0 = 0$ such that (a) the factor markets clear, *i.e.*, (1) and (2) hold, (b) the asset market clears: $K_{t+1} + B_t/p_t = Z_t/p_t$, (c) the allocations solve agents' utility maximization problem, (d) the cash-in-advance constraint (3) binds, or equivalently, $I_t > 1$ holds, (e) the government's budget constraints, $p_1 g + M_0 = M_1 + B_1$ for $t = 1$ and (10) for $t > 1$, hold, (f) $I_t = I$ and $g_t = g$ for all t, and (g) $b_t = \bar{b}$ for all t.

The money market equilibrium requires that

$$\frac{M_t}{p_t} = \gamma(I)\left(w_t - \frac{T_t}{p_t}\right) = (1-\alpha-\tau_t)\,\gamma(I)\,Y_t. \tag{12}$$

Divide (12) by Y_t to obtain

$$m_t = (1-\alpha-\tau_t)\,\gamma(I) \tag{13}$$

Since all income is saved, the asset market equilibrium requires that

$$K_{t+1} + \frac{B_t}{p_t} = w_t - \frac{T_t}{p_t} - \frac{M_t}{p_t} = (1-\alpha-\tau_t)\,[1-\gamma(I)]\,Y_t. \tag{14}$$

Divide (14) by Y_t and substitute $b_t = \bar{b}$ to obtain

$$\theta_{t+1} = A\,(1-\alpha-\tau_t)\,[1-\gamma(I)] - A\bar{b}. \tag{15}$$

(15) immediately implies that at any equilibrium θ_{t+1} and τ_t are negatively related. In addition, the condition $\tau_t < 1-\alpha$, combined with (15), requires that $\theta_{t+1} + A\bar{b} > 0$. Substitute $R_t = \alpha A$, $b_t = \bar{b}$, (13), and the Fisher equation into (11) to obtain

$$g = \tau_t + \bar{b} - \frac{\alpha A}{\theta_t}\bar{b} + (1-\alpha-\tau_t)\,\gamma(I) - \frac{\alpha A}{I}\frac{1}{\theta_t}\,(1-\alpha-\tau_{t-1})\,\gamma(I)\,. \tag{16}$$

Substitute (15) into (16) and solve it for θ_{t+1} as a function of θ_t alone as

$$\theta_{t+1} = [1-g-\alpha H(I)]\,A - \frac{\alpha A^2\bar{b}H(I)}{\theta_t} \equiv \Omega(\theta_t)\,, \tag{17}$$

where

$$H(I) \equiv \frac{1-\gamma(I)+\gamma(I)/I}{1-\gamma(I)} = 1 + \frac{\gamma(I)/I}{1-\gamma(I)} > 1. \tag{18}$$

Equation (17) describes the equilibrium law of motion of the output growth rate. It will be helpful to study some properties of the function H.

Lemma 2 *The function H satisfies a) $H'(I) < 0$, and b) $H(1) = 1 + \left(\frac{1-\sigma}{\sigma}\right)^{\frac{1}{\rho}}$.*

3.2 Balanced Growth Equilibria

At any balanced growth equilibrium, $\theta_t = \theta$ and $\tau_t = \tau < 1-\alpha$ for all t. Thus, a balanced growth equilibrium solves

$$\theta = [1-g-\alpha H(I)]\,A - \frac{\alpha A^2\bar{b}H(I)}{\theta} \equiv \Omega(\theta)\,. \tag{19}$$

Lemma 3 *The function Ω satisfies (a) $\Omega'(\theta) > 0$, (b) $\lim_{\theta\to 0}\Omega'(\theta) = \infty$, (c) $\lim_{\theta\to\infty}\Omega'(\theta) = 0$, and (d) $\lim_{\theta\to\infty}\Omega(\theta) = [1-g-\alpha H(I)]\,A$.*

Proof. From (19), it is easy to compute

$$\Omega'(\theta) = \frac{\alpha A^2 \bar{b} H(I)}{\theta^2} > 0, \text{ and } \Omega''(\theta) = -\frac{2\alpha A^2 \bar{b} H(I)}{\theta^3} < 0.$$

The rest of the proof is immediate. ■

Proposition 4 *(a) There exist two distinct balanced growth equilibria if* $[1-g-\alpha H(I)]^2 / [4\alpha H(I)] \geq \bar{b}$, *(b) there exists a unique balanced growth equilibrium if* $[1-g-\alpha H(I)]^2 / [4\alpha H(I)] = \bar{b}$, *and (c) there exists no balanced growth equilibrium if* $[1-g-\alpha H(I)]^2 / [4\alpha H(I)] \leq \bar{b}$.

Proof. Rewrite (19) as the following quadratic form,

$$\theta^2 - [1-g-\alpha H(I)] A\theta + \alpha A^2 \bar{b} H(I) = 0,$$

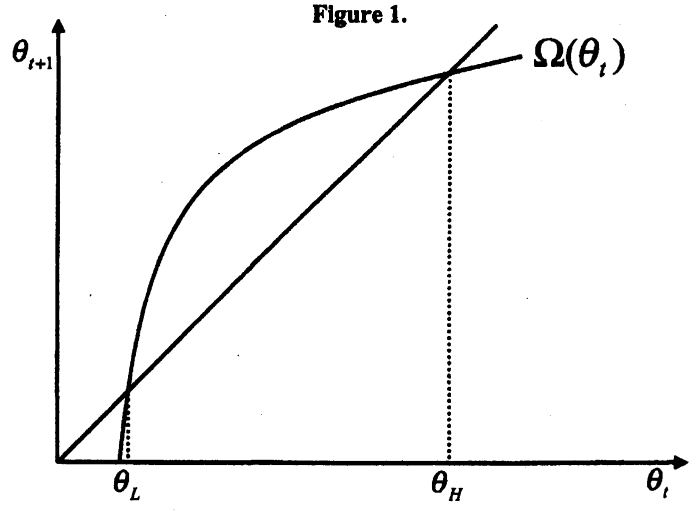
which has the roots

$$\theta = \frac{[1-g-\alpha H(I)] A \pm A\sqrt{[1-g-\alpha H(I)]^2 - 4\alpha\bar{b}H(I)}}{2}. \tag{20}$$

Thus, the roots are real if and only if $[1-g-\alpha H(I)]^2 \geq 4\alpha\bar{b}H(I)$. ■

Figure 1 shows equilibria of the model. As is shown in the figure, there are normally two balanced growth equilibria, the high-growth equilibrium, denoted by θ_H, and the low-growth equilibrium, θ_L. Obviously, $\Omega'(\theta_H) > 1$ and $\Omega'(\theta_L) < 1$. It is important to check whether the condition $\tau < 1 - \alpha$ is ever violated. Notice that, in this economy, the targeted public debt must be large enough for a given level of the fiscal deficits.

Figure 1.



Example 5 *Let* $A = 3$, $\alpha = 0.33$, $\rho = 0.2$, $\sigma = 0.6$, $g = 0.02$, $I = 1.03$, *and* $\bar{b} = 0.25$. *The corresponding inflation rate is* $\Pi = 1.04$. *Under this specification, there are two balanced growth equilibria with* $\theta = 0.79$ *and* $\theta = 1.05$.

Proposition 6 *(a) An increase in the nominal interest rate reduces (raises) the output growth rate at the low-growth (high-growth) equilibrium. (b) An increase in the targeted debt-GDP ratio raises (reduces) the output growth rate at the low-growth (high-growth) equilibrium. (c) An increase in the fiscal spending per GDP raises (reduces) the output growth rate at the low-growth (high-growth) equilibrium.*

Proof. From (19) it is easy to compute

$$\frac{d\theta}{dI} = \frac{-1 - A\bar{b}/\theta}{1 - \Omega'(\theta)} \alpha A H'(I),$$

where $\Omega'(\theta_H) > 1$ and $\Omega'(\theta_L) < 1$. Since $H'(I) < 0$, the Ω locus shifts up. The rest is immediate. (b) From (19) it is easy to compute

$$\frac{d\theta}{d\bar{b}} = -\frac{\alpha A^2 H(I)/\theta}{1 - \Omega'(\theta)}.$$

The rest is immediate. (c) Omitted. ■

3.3 Stability under Learning

This subsection studies dynamic properties of equilibria. As is pointed out by Evans *et al.* (1998), one cannot apply the standard stability analysis to the model. The reason is because the model's initial condition, K_1, does not pin down the next-period's capital stock in this model. Thus, I follow Evans *et al.* (1998) to use an *adaptive learning* scheme as an equilibrium selection device.[2] The basic idea is to consider behavior of the economy *outside* of the perfect-foresight equilibria and to ask to which equilibrium agents' expectations converge.

Consider the following adaptive learning scheme,

$$\theta^e_{t+1} = \theta^e_t + \delta_{t+1}(\theta_{t-1} - \theta^e_t), \tag{21}$$

where θ^e_{t+1} is a point expectation of the output growth rate and $\delta_t = \delta/t$ is called the *gain sequence*. It is said in the learning literature that information is *lagged* if the adaptive learning scheme is described by (21). Alternatively, one could replace the learning rule with $\theta^e_{t+1} = \theta^e_t + \delta_{t+1}(\theta_t - \theta^e_t)$, which corresponds to the

[2] Good discussions on learning as an equilibrium selection device can be found in Evans and Honkapohja (2001) and Lettau and Van Zandt (forthcoming). See also Kudoh (2002).

case in which information is *current.* Lettau and Van Zandt (forthcoming) point out that the use of current information in the learning process could drastically change stability. I adopt here the standard assumption that information is lagged.

Solve (15) for the tax-GDP ratio to obtain

$$\tau_t = 1 - \alpha - \frac{\theta^e_{t+1} + A\bar{b}}{A\left[1 - \gamma(I)\right]}, \tag{22}$$

where θ^e_{t+1} is a point expectation of the output growth rate and $h(I) \equiv 1 - \gamma(I) + \gamma(I)/I$. Equation (22) states that the real tax revenue is determined once a point expectation on the future output growth rate is formed. Substitute (22) into (16) and solve it for the actual output growth rate to obtain

$$\theta_t = \frac{\alpha A}{1 - \gamma(I)} \frac{h(I)\, A\bar{b} + \theta^e_t \gamma(I)/I}{(1 - \alpha - g)\, A - \theta^e_{t+1}} \equiv \Phi\left(\theta^e_{t+1}, \theta^e_t\right), \tag{23}$$

which defines an important function that maps from point expectations on the output growth rates, θ^e_{t+1} and θ^e_t, to the actual output growth. Agents revise expectations using (21). Under perfect foresight, (23) implies (19).

Substitute (23) into (21) to obtain $\theta^e_{t+1} = \theta^e_t + \delta_{t+1}\left(\Phi\left(\theta^e_t, \theta^e_{t-1}\right) - \theta^e_t\right)$, which is a second-order system in the expectations. Rewrite it as $(\theta^e_{t+\Delta} - \theta^e_t)/\Delta = \Phi\left(\theta^e_t, \theta^e_{t-\Delta}\right) - \theta^e_t$, where $\Delta = \delta_{t+1}$. Notice that $\lim_{t\to\infty} \Delta = 0$. It follows therefore the differential equation $d\theta^e/ds = \Gamma(\theta^e) - \theta^e$, where

$$\Gamma(\theta^e) \equiv \lim_{t\to\infty} \Phi\left(\theta^e_t, \theta^e_{t-1}\right) = \frac{\alpha A}{1 - \gamma(I)} \frac{A\bar{b}h(I) + \theta^e \gamma(I)/I}{(1 - \alpha - g)\, A - \theta^e}. \tag{24}$$

The mapping from the PLM (perceived law of motion) to the ALM (actual law of motion) is therefore given by $\Gamma(\theta^e)$.

Lemma 7 *Let* $\mathbf{b} \equiv (1 - \alpha - g)\, A$. *Then, the mapping* Γ *satisfies (a)* $\lim_{\theta\to\mathbf{b}} \Gamma'(\theta) = \infty$, *(b)* $\lim_{\theta\to\infty} \Gamma'(\theta) = 0$, *and (c)* $\Gamma'(\theta) > 0$ *for all* θ *if and only if* $\bar{b}h(I)\, I/\gamma(I) + 1 - \alpha > g$.

Proof. From (24), it is easy to show that

$$\Gamma'(\theta) = \frac{\alpha A^2}{1 - \gamma(I)} \frac{\bar{b}h(I) + (1 - \alpha - g)\, \gamma(I)/I}{\left[(1 - \alpha - g)\, A - \theta\right]^2}.$$

The rest is immediate. ■

The condition for E-stability is $\Gamma'(\theta^e) < 1$. Figures 2a and b depict the map Γ. As shown in these figures, there are two distinct fixed points and the curve cuts the 45 degree line from above (below) at the low-growth (high-growth) equilibrium if g is small (large) relative to $\bar{b}$. This establishes that $\Gamma'(\theta^e) < 1$ (> 1) at the low-growth (high-growth) equilibrium. Formally,

Figure 2a.

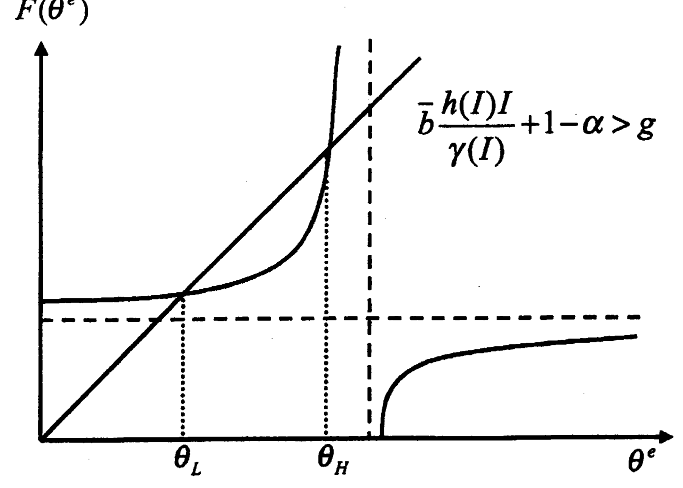
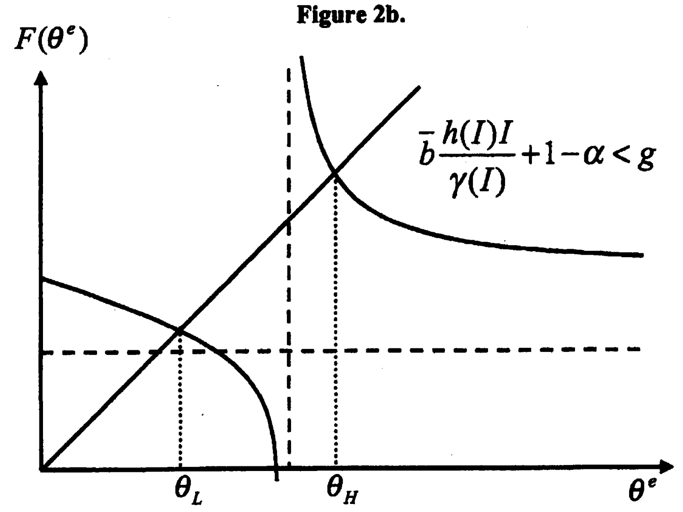


Figure 2b.



Proposition 8 *(a) The low-growth equilibrium is E-stable, and (b) the high-growth equilibrium is E-stable if and only if* $\bar{b}h(I)\,I/\gamma(I)+1-\alpha<g$.

According to proposition 8, the low-growth equilibrium is selected as adaptively stable one. Further, the high-growth equilibrium becomes stable if the deficit is small or the targeted debt-GDP ratio is high. An important policy implication is that if the monetary authority pegs the nominal interest rate, then there arises the possibility of *expectational indeterminacy* in the sense of Evans *et al.* (1998). This proposition also implies that such indeterminacy can be overcome by appropriately chosen fiscal parameters, $\bar{b}$, and g.

3.4 Stability of Sunspot Equilibrium

This subsection considers existence and stability of possible sunspot equilibria (SSEs) of the economy. In order to make the model comparable to Evans and Honkapohja's (1994, 2001b) results, rewrite (17) as

$$\theta_t = \Omega^{-1}(\theta_{t+1}) = \frac{\alpha A^2 \bar{b} H(I)}{[1-g-\alpha H(I)]A - \theta_{t+1}} \equiv F(\theta_t), \tag{25}$$

which defines the temporary equilibrium map from expectations about the next period output growth rate to the current one. Consider a sunspot variable s_t which follows a Markov chain with transition probabilities π_{ij}, which is the probability that the current state is i and the next state is j. For the case of a 2-state Markov chain, $\pi_{12} = 1-\pi_{11}$ and $\pi_{21} = 1-\pi_{22}$. The following definitions are due to Evans and Honkapohja (1994).

Definition 9 *(θ_1, θ_2) is a 2-state Markov sunspot equilibrium with transition probabilities $0 < \pi_{ij} < 1$ if $\theta_1 = \pi_{11}F(\theta_1) + (1-\pi_{11})F(\theta_2)$ and $\theta_2 = (1-\pi_{22})F(\theta_1) + \pi_{22}F(\theta_2)$.*

Definition 10 *Local animal spirits sunspots are stationary sunspot equilibria (SSEs) for which the two states are near distinct rest points θ_L and θ_H, such that $\theta_L = F(\theta_L)$ and $\theta_H = F(\theta_H)$.*

According to Evans and Honkapohja (1994), the conditions for the existence of local animal spirits sunspots are $F'(\theta_L) \neq 1$ and $F'(\theta_H) \neq 1$. Since $\Omega'(\theta_L) > 1 > \Omega'(\theta_H)$ and $F'(\theta) = 1/\Omega'(\theta)$ hold in this economy, it is easy to check that $F'(\theta_L) < 1 < F'(\theta_H)$, which ensures $F'(\theta_L) \neq 1$ and $F'(\theta_H) \neq 1$. Thus, local animal spirits sunspots exist near the balanced growth equilibria θ_L and θ_H.

Proposition 11 (Evans and Honkapohja (1994)) *Local animal spirits sunspots are (a) weakly E-stable if and only if $F'(\theta_L) < 1$ and $F'(\theta_H) < 1$, and (b) strongly E-stable if and only if $|F'(\theta_L)| < 1$ and $|F'(\theta_H)| < 1$.*

Proof. See Evans and Honkapohja (1994). ■

The terminology "weak E-stability" is used here to denote the standard E-stability notion in order to distinguish this from a stronger notion of E-stability, "strong E-stability." The notion of strong E-stability, which is concerned with stability of an overparameterized system, is suggested and discussed at length in Evans and Honkapohja (1994, 2001). In what follows, I will use proposition 11 as the stability conditions for the model. The distinction between weak and strong E-stability, however, will not be emphasized.

Proposition 12 *Local animal spirits sunspots near θ_L and θ_H are not E-stable.*

Proof. The result is easily checked because $F'(\theta_L) < 1 < F'(\theta_H)$ holds. ∎

4 Targeting Debt to Control Inflation

4.1 Characterization

This section considers an alternative scenario in which the fiscal authority is active in the sense that it targets the tax rate as well as the debt-GDP ratio. Thus, the central bank has to adjust the nominal interest rate so as to be consistent with all other equilibrium conditions. Accordingly, the gross nominal interest rate is endogenous.

The money market equilibrium requires $m_t = (1-\alpha-\tau)\gamma(I_{t+1})$ and the capital market equilibrium implies

$$\theta_{t+1} = A(1-\alpha-\tau)[1-\gamma(I_{t+1})] - A\bar{b} \equiv \Theta(I_{t+1}). \tag{26}$$

It is easy to check that $\Theta'(I) = -A(1-\alpha-\tau)\gamma'(I) > 0$. In words, the nominal interest rate and the output growth rate are positively related. The government's budget constraint can be rewritten as

$$g = \tau + \bar{b} - \frac{\alpha A}{\theta_t}\bar{b} + (1-\alpha-\tau)\gamma(I_{t+1}) - \frac{\alpha A}{I_t}\frac{1}{\theta_t}(1-\alpha-\tau)\gamma(I_t), \tag{27}$$

The evolution of the economy is described by (26) and (27).

4.2 Balanced Growth Equilibrium

At any balanced growth equilibrium, $\theta_t = \theta$ and $I_t = I$ for all t. Thus, a balanced growth equilibrium solves

$$I = \frac{\alpha A(1-\alpha-\tau)\gamma(I)}{\left[\tau - g + \bar{b} + (1-\alpha-\tau)\gamma(I)\right]\Theta(I) - \alpha A\bar{b}} \equiv J(I). \tag{28}$$

It will be helpful to study some properties of the function J.

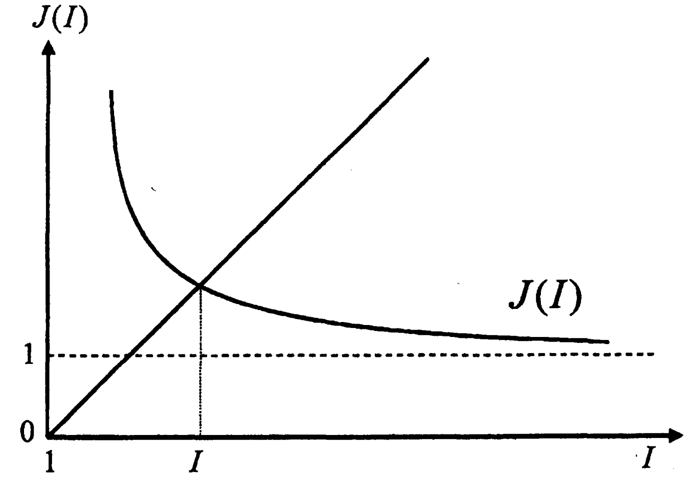
Lemma 13 $J'(I) < 0$ *holds.*

Proof. From (28), it is straightforward to show that

$$J'(I) = \frac{[A(1-\alpha-\tau)]^2\alpha\gamma'(I)\left\{(1-\alpha-\tau)[\gamma'(I)]^2 + \left(\tau - g + \bar{b}\right)\right\}}{\left\{\left[\tau - g + \bar{b} + (1-\alpha-\tau)\gamma(I)\right]\Theta(I) - \alpha A\bar{b}\right\}^2}$$

Since $\gamma'(I) < 0$, $J'(I) < 0$ holds. ∎

Figure 3.



From lemma 13 it is easy to show that the mapping J has a unique fixed point. The typical configuration of the function J is depicted in figure 3. The question here is whether the fixed point constitutes an equilibrium. Namely, one needs to check if the fixed point satisfies $I \geq 1$, which is the natural lower bound for the gross nominal interest rate or equivalently the nominal interest factor.

Example 14 *Let* $A = 3$, $\alpha = 0.33$, $\rho = 0.2$, $\sigma = 0.6$, $g = 0.02$, $\tau = 0$, *and* $\bar{b} = 0.25$. *Then, there is a unique balanced growth equilibrium with* $I = 1.03$ *and the corresponding output growth rate and inflation rate are* $\theta = 1.05$ *and* $\Pi = 1.04$.

Proposition 15 *(a) An increase in the government spending raises the nominal interest rate, the inflation rate, and the output growth rate. (b) An increase in the target debt-GDP ratio reduces the nominal interest rate, the inflation rate, and the output growth rate if and only if* $\theta > A(1-g)/2$.

Proof. (a) Omitted. (b) From (28),

$$\frac{dI}{d\bar{b}} = \frac{\partial J/\partial \bar{b}}{1 - J'(I)},$$

where $J'(I) < 0$ and

$$\frac{\partial J}{\partial \bar{b}} = \frac{-\alpha A (1-\alpha-\tau)\gamma(I)\left[A(1-\alpha-\tau)[1-2\gamma(I)] - 2A\bar{b} - (\alpha+\tau-g)A\right]}{\left[\tau - g + \bar{b} + (1-\alpha-\tau)\gamma(I)\right]\left[\Theta(I) - \alpha A\bar{b}\right]^2}.$$

So $\partial J/\partial \bar{b} < 0$ if and only if

$$(1-\alpha-\tau)[1-\gamma(I)] + \frac{g-1}{2} > \bar{b},$$

which, using (26), can be rewritten as $\theta > A(1-g)/2$. ■

4.3 Dynamics: Local Indeterminacy

I now turn to describing dynamic properties of the economy. Substitute (26) into (27) to obtain

$$\gamma\left(I_{t+1}\right)=\frac{\alpha A\bar{b}+\alpha A\left(1-\alpha-\tau\right)\gamma\left(I_t\right)/I_t}{\left(1-\alpha-\tau\right)\left[A\left(1-\alpha-\tau\right)\left[1-\gamma\left(I_t\right)\right]-A\bar{b}\right]}-\frac{\tau-g+\bar{b}}{1-\alpha-\tau}\equiv\Phi\left(I_t\right), \tag{29}$$

from which one can define the perfect-foresight dynamics, $I_{t+1}=\gamma^{-1}\left(\Phi\left(I_t\right)\right)$. Linearize (29) around the steady state to obtain $dI_{t+1}=DdI_t$, where

$$D\equiv\frac{\left(1-\alpha-\tau\right)\left[\gamma'\left(I\right)-\gamma\left(I\right)\left[1-\gamma\left(I\right)\right]\right]+\bar{b}\left[\gamma\left(I\right)-\left(1-I\right)\gamma'\left(I\right)I\right]}{\left[A\left(1-\alpha-\tau\right)\left[1-\gamma\left(I\right)\right]-A\bar{b}\right]^2 I^2\gamma'\left(I\right)}\alpha A^2.$$

Then, the unique balanced growth equilibrium is locally determinate if and only if $|D|>1$. This implies that sunspots near the single balanced growth equilibrium exist if and only if $|D|<1$.

4.4 Stability under Learning

This section studies stability of the balanced growth equilibrium. Consider the following adaptive learning scheme,

$$I^e_{t+1}=I^e_t+\delta_{t+1}\left(I_{t-1}-I^e_t\right). \tag{30}$$

From (13),

$$m_t=\left(1-\alpha-\tau\right)\gamma\left(I^e_{t+1}\right), \tag{31}$$

where I^e_{t+1} is a point expectation of the gross nominal interest rate. (13), (15), and (11) combined with $b_t=\bar{b}$ yield

$$\theta_{t+1}=A\left(1-\alpha-\tau\right)\left[1-\gamma\left(I^e_{t+1}\right)\right]-A\bar{b}\equiv\Theta\left(I^e_{t+1}\right). \tag{32}$$

Substitute (31), (32), and the Fisher equation into the government's budget constraint (11) to obtain the actual gross nominal interest rate

$$I_t=\frac{\alpha A\left(1-\alpha-\tau\right)\gamma\left(I^e_t\right)}{\left[\left(1-\alpha-\tau\right)\gamma\left(I^e_{t+1}\right)+\bar{b}+\tau-g\right]\Theta\left(I^e_t\right)-\alpha A\bar{b}}\equiv J\left(I^e_{t+1},I^e_t\right). \tag{33}$$

It follows therefore the differential equation $dI^e/ds=\Gamma\left(I^e\right)-I^e$, where

$$\Gamma\left(I^e\right)=\lim_{t\to\infty}J\left(I^e_{t+1},I^e_t\right)=\frac{\alpha A\left(1-\alpha-\tau\right)\gamma\left(I^e\right)}{\left[\left(1-\alpha-\tau\right)\gamma\left(I^e\right)+\bar{b}-g\right]\Theta\left(I^e\right)-\alpha A\bar{b}}=J\left(I^e\right).$$

The condition for E-stability is $\Gamma'\left(I^e\right)<1$. Thus, it is now obvious that the following result is true.

Proposition 16 *The unique balanced growth equilibrium is E-stable.*

4.5 Stability of Sunspot Equilibrium

I have pointed out that the model generates sunspot fluctuations around the unique balanced growth path if it is indeterminate. This subsection is concerned with stability of such sunspot equilibria. Rewrite (29) as $I_t = \Phi^{-1}(\gamma(I_{t+1})) \equiv F(I_{t+1})$, where I use the same notation F in order to make it easier to apply Evans and Honkapohja's (1994, 2001b) results to this model. It is well-known that sunspot equilibrium exists if $|F'(I)| > 1$. The question here is whether such SSEs are E-stable. Let (I_1, I_2) be a 2-state Markov sunspot equilibrium with transition probabilities $0 < \pi_{ij} < 1$ if $I_1 = \pi_{11}F(I_1) + (1-\pi_{11})F(I_2)$ and $I_2 = (1-\pi_{22})F(I_1) + \pi_{22}F(I_2)$. Further, Let Þ solves Þ $= F($Þ$)$.

Proposition 17 (Evans and Honkapohja (2001b)) *(a) If $F'($Þ$) < 1$, then every SSE sufficiently near the steady state is E-unstable. (b) If $F'($Þ$) < -1$, then there exists an E-stable SSE (I_1, I_2) near the steady state.*

Proof. See Evans and Honkapohja (2001b). ■

5 Conclusion

This paper studied a monetary-fiscal policy regime where the fiscal authority targets the debt-GDP ratio. If the monetary authority is active and targets the nominal interest rate, then two balanced growth equilibria are shown to exist. The low-growth equilibrium is stable under learning. If there is no active monetary authority, then a unique balanced growth equilibrium obtains. The unique balanced growth path is stable under learning. This suggests that it is at least theoretically possible that the fiscal authority's actions alone determine growth and inflation in the long run. This would add another support for the view that the fiscal authority's coordinated actions are required for price (and output) stability. Although uniqueness obtains, the economy without an active monetary authority can be subject to adaptively stable sunspot fluctuations.

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