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Scattering Phenomena for Traveling Breathers

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Abstract

Scattering process between 1D traveling breathers (TBs) for the three-component reaction-diffusion system and complex Ginzburg-Landau equation (CGLE) with external forcing is studied. A hidden unstable solution called the scattor plays a crucial role to understand the scattering dynamics. The input-output relation depends in general on the phase of two TBs at collision point, which makes a contrast to the case for the steady traveling pulses. The phase-dependency of input-output relation comes from the fact that the profiles at collision point make a loop parametrized by the phase and it traverses the stable manifold of the scattor. A global bifurcation viewpoint is quite useful not only to understand how TBs emerge but also to detect scattors.

1 Introduction

Spatially localized moving objects such as pulses and spots form a representative class of dynamic patterns in dissipative systems. A qualitative change for the pattern may occur either by interaction with other patterns through collision or due to intrinsic instabilities such as splitting and destruction by itself. [7, 8] There is a variety of collision process for particle-like patterns in dissipative systems even restricted to head-on ones. [4, 12] A key issue is to classify the input-output relation before and after collision and clarify its underlying mechanism for the scattering process. One of the difficulties comes from the large deformation due to strong interaction. A new viewpoint was presented to clarify the process of head-on collisions among traveling pulses and spots, [9, 10] especially a notion of "scattor" was introduced to understand the input-output relation.

The scattor itself is just an unstable steady or time-periodic solution (i.e., saddle) and its center of mass does not move, however once there occurs collision, the solution deforms significantly and approaches a part of the unstable manifold of the scattor and is driven by it. The final output is therefore determined by the destination of the unstable manifold. Scattors are in general highly unstable and a variety of outputs originates from those of destinations of unstable manifolds. The issue is reduced, to some extent, to finding the scattors and their dynamic behaviors along unstable manifolds, however it is in general difficult to detect a scattor even by numerics, since it is not an attractor. To overcome this difficulty, the

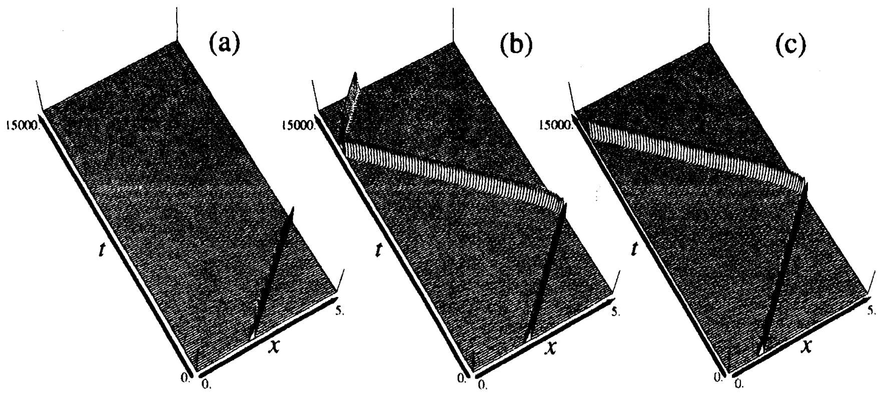

Figure 1: Symmetric head-on collisions for the three-component system (1)(i.e., hitting the boundary with the Neumann condition). (a) Annihilation $D_v = 2.5 \times 10^{-5}, r_c = 7.16 \times 10^{-4}$. (b) Preservation $D_v = 3.2 \times 10^{-5}, r_c = 9.72 \times 10^{-4}$. (c) The output changes depending on the phase at collision: the first one is of preservation and the second one is of annihilation. $D_v = 3.19 \times 10^{-5}, r_c = 9.60 \times 10^{-4}$ Only u component is displayed here.

following observation turns out to be quite useful[9, 10]: when parameters are close to a transition point where input-output relation changes qualitatively, the orbit becomes very close to the scattor by adjusting an appropriate number of parameters. Once a scattor is obtained at some particular point, then it can be continuated to other parameter regions by using, for instance, AUTO software.[2] The aim of this paper is to study the scattering process for the oscillatory-propagating pulses. Since the profile of the pulse is time-periodic, the input-output relation in general depends on the phase at collision, which makes a sharp contrast with the non-oscillatory case. Our goal is to clarify the phase-dependency of the input-output relation. In what follows we use the terminology "traveling breather"(or TB in short) for the oscillatory-propagating pulse. This work was carried out in collaboration with Takashi Teramoto and Kei-Ichi Ueda.

2 Phase-dependent outputs for scattering between traveling breathers

The following three-component system (1) has a TB for appropriate parameter values. In this paper we investigate the input-output relation of the symmetric head-on collisions for (1), which is equivalent to consider the collision process with the Neumann wall (see Fig.1).

$$\begin{cases} u_t &= D_u u_{xx} + \dfrac{su^2v}{(s_b + s_c w)(1 + s_a u^2)} - r_a u \\ v_t &= D_v v_{xx} + b_b - \dfrac{su^2v}{(s_b + s_c w)(1 + s_a u^2)} - r_b v \\ w_t &= r_c(u - w) \end{cases} \tag{1}$$

This model can be regarded as a 3-by-3 system by adding the effect of the inhibitor w to the 2-by-2 actibator-substrate system like the Gray-Scott model. For more details, see for

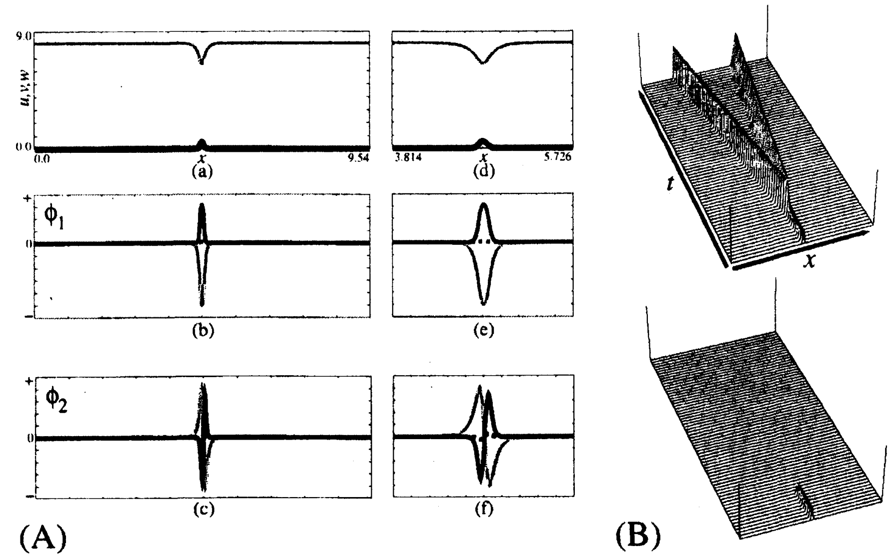

Figure 2: (A) (a) The scattor of static type obtained by the Newton method taking the profile at symmetric collision point as an initial seed. The scattor is of codim 2. We note that the profile of w is same as that of u. (b)(c) The unstable eigenfunctions ϕ_1 and ϕ_2 are depicted. The associated eigenvalues are 0.04636078 and 0.01404154. (d)-(f) The magnified pictures of central parts of (a)-(c) respectively. The solid, gray and dotted lines indicate u, v and w components respectively. (B) top (bottom): Outputs from the scattor. A small positive (negative) perturbation of ϕ_1 is added to the scattor.

instance Meinhardt [3]. We adopt two parameters D_v and r_c as bifurcation parameters. Note that steady states do not depend on r_c. Other parameters are fixed as $D_u = 1.0 \times 10^{-5}, r_a = 0.082, r_b = 0.0123, b_b = 0.1, s_a = 1.11, s_b = 1.55, s_c = 1.115, s = 0.08$. For numerical computation, we set to $\Delta x = 0.04$ and $\Delta t = 0.1$ unless otherwise said. We employ a well-settled TB as an initial data for numerics. Here the "well-settled TB" means that it is obtained after a long-run simulation on a large interval. This makes sence because the concerning TB is asymptotically stable.

A transition from preservation to annihilation can be understood in such a way [9, 10] that the orbit crosses the stable manifold of the scattor at the transition point and the orbits are sorted out according to which side of the stable manifold it belongs. A remarkable thing happens as shown in Fig.1(c), namely the TB is reflected at the first collision, while, the annihilation occurs at the second collision despite the parameters is fixed as $D_v = 3.19 \times 10^{-5}$ and $r_c = 9.60 \times 10^{-4}$. This not only makes a sharp contrast with the steady scattor case, but also implies that the bifurcation parameter is not sufficient to determine the input-output relation. To control the phase at collision, we change the distance between the initial pulse and boundary, since it is equivalent to shifting the phase of the initial oscillatory-pulse. To detect a scattor near the transition point, orbital behaviors are traced carefully by changing the collision-phase with keeping the parameters being fixed.

Take a closer look at the two collisions, we see that the orbits become very close to a quasi-steady state for certain time before preservation or annihilation as, in fact an unstable

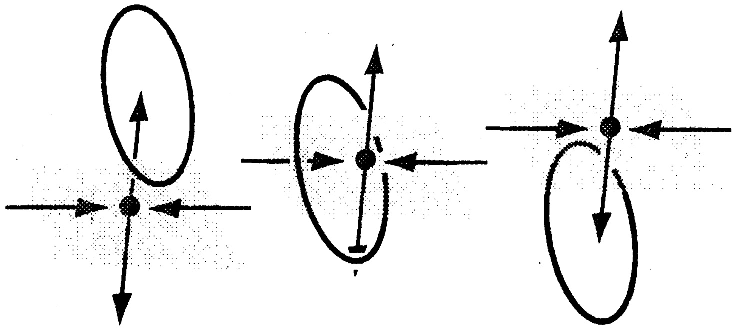

Figure 3: Schematic picture of the relation between the profiles of the orbit right after collision and the stable manifold of scattor. The rectangle shows a stable manifold of scattor. The solid circle indicates the plot of profiles of the orbit as the collision-phase varies from 0 to π. It crosses the stable manifold of scattor transversally, which explains the outputs of Fig. 1(c).

steady state can be found by the Newton method and it has two unstable eigenvalues as shown in Fig.2(A). This steady state is deserved to be called a scattor, because it emits the exactly the same outputs (see Fig.2) when it is perturbed along the first (symmetric) unstable direction (see Fig.2(B)). The local dynamics around the scattor and global behaviors of their unstable manifolds are the keys to link input to output during the scattering process. This steady state is deserved to be called a scattor, because it emits the exactly the same outputs as in Fig.2 when it is perturbed along the first (symmetric) unstable direction (see Fig.2(B)).

Now we are ready to explain why we have different outputs depending on the phase of collision as in Fig. 1(c). If we plot the profiles of the orbit right after collision in the appropriate phase near the scattor, then it becomes a closed curve as the phase at collision varies from 0 to 2π, moreover the closed curve generically crosses the stable manifold of scattor transversally. The output after collision is therefore determined by looking at on which side of the stable manifold the closed curve belongs. In view of Fig.3, it is clear that two different types of outputs come out depending on the phase. See the reference [11] for details.

3 Breathing Scattor in Complex Ginzburg-Landau Equation

In this section, we consider the following complex Ginzburg-Landau equation (CGLE) with a parametric forcing term.[1, 5, 12]

$$\begin{aligned} W_t = \; & (1+ic_0)W + (1+ic_1)W_{xx} \\ & -(1+ic_2)|W|^2W + c_3\overline{W}, \end{aligned} \tag{2}$$

where c_0, c_1, c_2, and c_3 are real parameters. The equation (2) becomes bistable in an appropriate parameter region where there exists a pair of stable homogeneous states W_0 and $-W_0$. When c_3 is large, the stationary front connecting W_0 to $-W_0$ is stable. Note that the

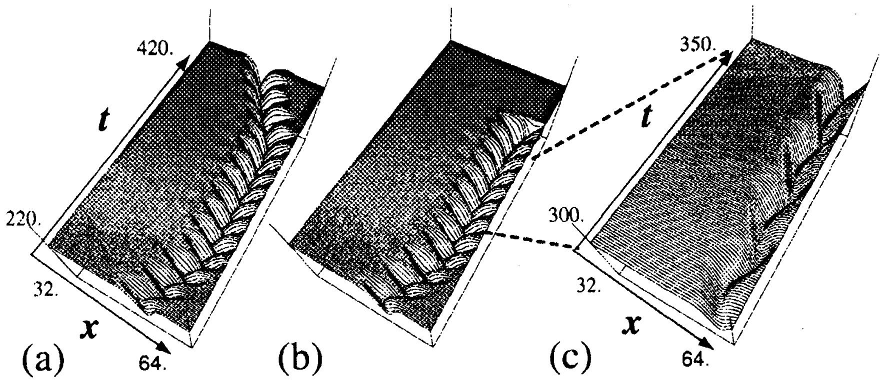

Figure 4: Magnified figures of transition from reflection (a) to annihilation (b) at $c_3 \approx 0.142429$, as c_3 is slightly decreased. (c) A quasi-time-periodic pulse observed at the transition point.

magnitude of $1-|W|^2$ (or the modulus $|W|$) is localized in space, so we call it a pulse rather than a front (or domain wall) in the sequel.

The scattor for the three component system (1) turns out to be a steady state as was discussed in previous sections, however this is not always the case. In fact we will see in the sequel that an unstable time-periodic solution plays a role of scattor for TBs. In order to have a TB, we employ here a particular set of parameters $c_0 = 1.0, c_1 = -0.5, c_2 = 1.1$ and take c_3 as a bifurcation parameter in the bistable regime. In this section we reveal the nature of quasi-time-periodic solution as depicted in Fig.4.

As c_3 is decreased further to 0.148, its center of mass starts to drift, which indicates the onset of TB. The drift velocity of TB is slow near the onset, since the bifurcation from the standing oscillating pulse to the TB is super-critical. The TB bounces off at the wall, therefore the input-output relation is preservation, namely an incoming TB emits an outgoing TB. When c_3 is decreased to 0.140, the velocity of TB becomes larger, and it annihilates at the collision to the Neumann wall. It is clear that transition of input-output relation must occur in between 0.140 and 0.148.

It turns out that the orbit stays very close to a quasi-time-periodic state for certain time when $c_3 \approx 0.142429$ as indicated in Fig.4. Such a phase-dependent output occurs over a range of c_3 including $0.141 \sim 0.143$ and the quasi-time-periodic objects like Fig.4 are observed near the transition point. It indicates the existence of a c_3-parameter family of unstable time-periodic solutions called breathing scattors (BSs) and those objects play a role of separatrix and should be responsible for the transition of input-output relations for the system.

Although BSs can be obtained approximately by tuning the parameter c_3, this approach has several drawbacks, for instance, it only works near the transition point of codim 1, and it does not give a precise profile to study the linearized spectrum around it. In what follows we present a more systematic and powerful method to detect BSs based on a global bifurcation viewpoint and clarify the origin of phase-dependent output.

Firstly we find stationary solutions for larger values of c_3. Once the stationary solution

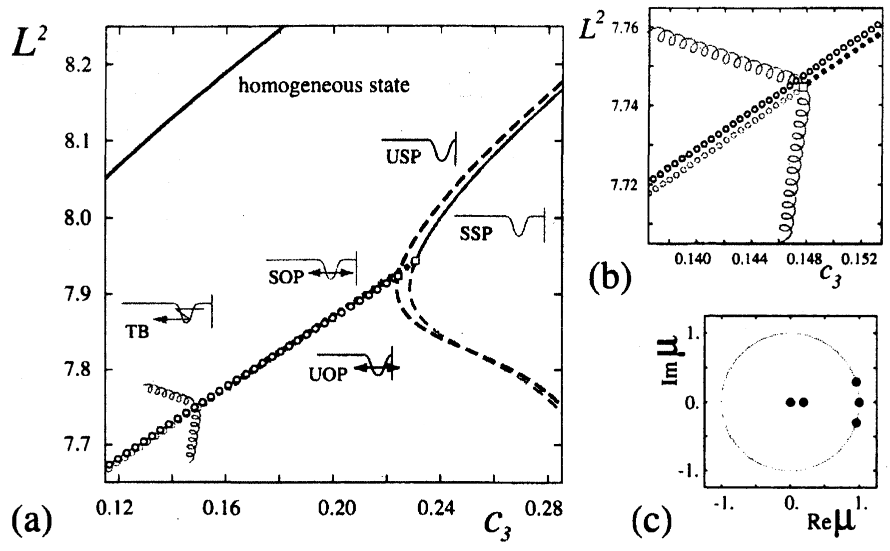

Figure 5: (a) Global bifurcation diagram for the breathing scattor. The bifurcation parameter is c_3. The black open (resp. gray filled) circles indicate the unstable oscillating pulse (UOP) (resp. stable (SOP)) connected to USP (resp. SSP). TB emerges below $c_3 \approx 0.148$. The solid line of the top of the figure indicates the stable uniform state. (b) A detail near the NS bifurcation point. (c) The distribution of multipliers μ of SOP at $c_3 \approx 0.148022$. The gray circle shows $|\mu| = 1$.

has been detected, we compute the branches globally by continuation by using AUTO. Following the branch of the stable steady pulses SSP (resp. unstable one denoted by USP), there occurs a saddle-node (SN) bifurcation at $c_3 \approx 0.22859$ (resp.0.22375) as in Fig.5(a). As c_3 is further decreased, a Hopf bifurcation occurs supercritically on SSP (resp. USP) near the SN-point at $c_3 = 0.23075$ (resp. 0.22431) shown as circles in Fig.5(a). This is the onset of the stable(resp.unstable) standing breather SOP (resp.UOP). The USP has only one real positive eigenvalue even on a whole interval and the associated one-dimensional unstable manifold is connected to the SOP and the homogeneous trivial state, respectively. The two Hopf branches SOP and UOP are extended to the range of c_3 in which numerical simulations of Fig.4 are carried out. The Neimark-Sacker (NS) bifurcation takes place on the stable Hopf branch SOP at $c_3 \approx 0.148022$, namely, a pair of multiplier $\mu_{1,2} = 0.955 \pm 0.297i$ crosses the unit circle as depicted in Fig.5(c). The Floquet multipliers μ can be used for the criterion of the stability of a periodic orbit. The SOP becomes unstable and the stable oscillatory-propagating pulse, i.e., TB takes over instead. The c_3 value of the NS point is in good agreement with that of the onset of TB. TBs originate from the NS-point of the SOP and we can observe a scattering among them for $c_3 < 0.148022$. On the other hand, the UOP is a hyperbolic saddle of codim 1, so that it has only one real unstable multiplier $\mu > 1$. It turns out that the quasi time-periodic behaviors like Fig. 4(c) are realized by the UOP. In other words the UOPs are the breathing scattors (BSs) and their unstable manifolds are connected to TBs and the homogeneous state as in Fig. 6. In views of Fig. 6, the destinations of the unstable manifold are homogeneous state (annihilation), if the

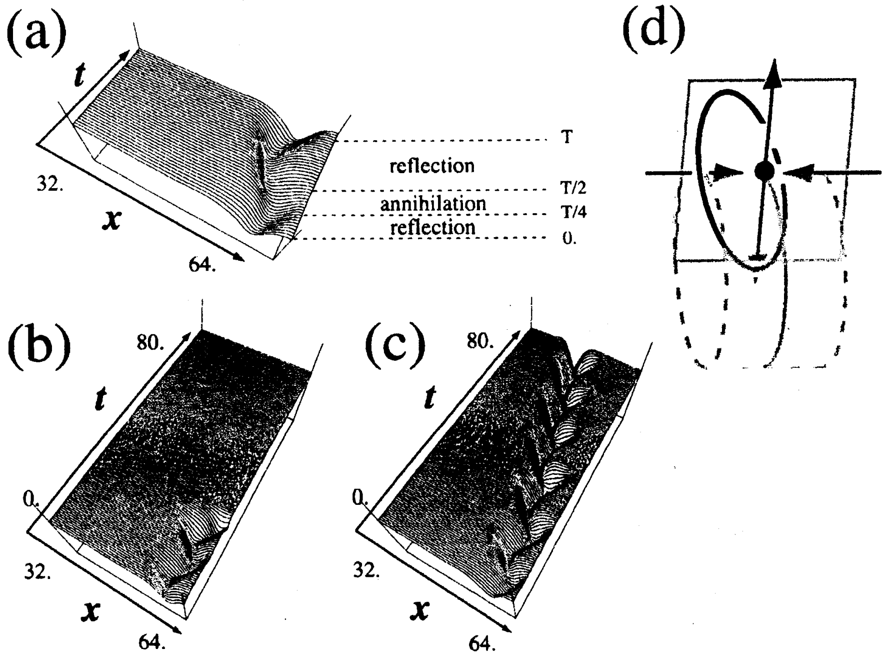

Figure 6: (a) Spatio-temporal pattern of the breathing scattor of $T \approx 12.8730$ when $c_3 \approx 0.142429$. (b) (resp. (c)) Response of the breathing scattor by adding a small constant-multiple perturbation of Φ to the snapshot of the unstable orbit at $t = 3T/8$ (resp. $t = 5T/8$). (d) Schematic picture of the relation for the loop right after collisions and the stable manifold of the breathing scattor. Generically there is non-empty interval of c_3 in which the loop belongs to both sides of the stable manifold.

perturbation is added to a quarter of a period of the breathing scattor between $T/4$ and $T/2$. Otherwise they are outgoing pulses (preservation). Accordingly, the coexistence of the annihilation and preservation for the fixed c_3 value is caused by the difference of phase at collision. The details will be discussed in our forthcoming articles.[11]

4 Conclusion

Scattering phenomena of oscillatory-propagating pulses (TBs) are studied for the three-component reaction diffusion system and the CGLE case. The transition of input-output relation like from annihilation to preservation can be explained from the scattor's viewpoint. The scattor for the three-component system (1) takes a form of unstable steady solusion, however this is not always the case for the CGLE case as discussed in the previous section.

The solution profile right after collision is a function of the collision-phase, and it makes a closed loop generically. This loop intersects transversaly with the stable manifold of scattor near the transition point of input-output relation, which causes the different outputs for the same parameter values. Such scattors can be found systemaically by adopting a global bifurcation viewpoint with the aid of the path-tracking software like AUTO. The origin of a diversity of input-output relations can be reduced to the local dynamics around scattors, in fact, when the orbit approaches a scattor right after collision, then it is sorted out along one of the unstable directions of it. Overall the response of scattors play a pivotal role to understand the transient aspect of scattering dynamics in dissipative systems.

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